بسم الله الرحمن الرحیم

Subject: Transformation (3)

① Summary on Transformation (projection, ...) So far

☆ Continuous Fourier Transformation (F.T.) (1+1)-Dimension

Time Series

سری زمانی

Independent Parameter

$$x(t) = \int d\omega \, e^{+i\omega t} X(\omega) \qquad (1+1) \qquad \omega = \frac{2\pi}{t}$$

$$X(\omega) = \int dt \, e^{-i\omega t} x(t)$$

فرکانس

Dependent Par

* (R+D)-dimension $\quad \left\{ x^{(j)}(t^{(k)}) \right\} \quad j=1,\dots R$

$k=1,\dots D$

Ex 1: (2+3)-Dimension $\quad (r_x, r_y, r_z)$ — Independent Parameters

Temperature

Pressure

$$A^{(j)}(r^{(k)}) \qquad j=1,2 \quad (T, P)$$

$$K=1,2,3 \quad (x,y,z)$$

$$A^{(1)}(x,y,z) \xrightarrow{\text{Discrete Form}} A^{(1)}(x_i, y_i, z_i)$$

$$i=1,\dots N$$

$$A^{(2)}(x,y,z) \longrightarrow A^{(2)}(x_i, y_i, z_i)$$

$$i=1,\dots,N$$

x	y	z	$T(x,y,z)$	$P(x,y,z)$
x_1	y_1	z_1	T_1	P_1
x_2	y_2	z_2	T_2	P_2
$\vdots$	$\vdots$	$\vdots$		

N

$$A^{(R)}(t^{(D)}) \longleftarrow A(\vec{r})$$

$$\phi(\vec{r}) \longrightarrow (1+3)$$

$$\phi(\vec{r}) = \int d^3K \, e^{i\vec{K}\cdot\vec{r}} \, \tilde{\phi}(\vec{K})$$

$$|K| = \frac{2\pi}{\lambda} = \text{Wave Number}$$ عدد موج

$$\downarrow$$

$$\vec{E}(\vec{r}) \longrightarrow (3+3)\text{-Dimens.}$$

$$E^{(l)}(\vec{r}) = \int d^3K \, e^{i\vec{K}\cdot\vec{r}} \, \tilde{E}_l(\vec{K})$$

$$\begin{cases} l=x & E_x(\vec{r}) = \int d^3K \, e^{i\vec{K}\cdot\vec{r}} \, \tilde{E}_x(\vec{K}) \\ l=y & E_y(\vec{r}) = \int d^3K \, e^{i\vec{K}\cdot\vec{r}} \, \tilde{E}_y(\vec{K}) \\ l=z & E_z(\vec{r}) = \int d^3K \, e^{i\vec{K}\cdot\vec{r}} \, \tilde{E}_z(\vec{K}) \end{cases}$$

$$\tilde{E}_x(\vec{k}) = \int d^3r\, e^{-i\underbrace{\vec{k}}_{\frac{1}{m}}\cdot\vec{r}(m)}\, \underbrace{E_x(\vec{r})}_{}$$ Coefficient

$$\tilde{E}_y(\vec{k}) = \int d^3k\, e^{-i\vec{k}\cdot\vec{r}}\, E_y(\vec{r})$$

$$\tilde{E}_z(\vec{k}) = \int d^3k\, e^{i\vec{k}\cdot\vec{r}}\, E_z(\vec{r})$$

$$e^{i\vec{k}\cdot r} \quad — \quad e^{ik\cdot x}$$

$$\vec{E}(\vec{r}) = \begin{pmatrix} \cancel{E_x(x)} \\ \cancel{E_y(y)} \\ \cancel{E_z(z)} \end{pmatrix} \qquad \begin{matrix} E_x(x,y,z) \\ E_y(x,y,z) \\ E_z(x,y,z) \end{matrix}$$

$$e^{ik\cdot r} = \sum J_l \ldots$$

Ex 2: $T(r_x, r_y, r_z)$ $\underbrace{\text{میدان اسکالر است در ۳ بعد}}_{(1+3)\text{-Dimens.}}$ ✓

Ex 3: $i\hbar \frac{\partial}{\partial t}\psi = \mathcal{H}\psi$ 3-D spatial coordinate

1-D Temporal.

$$\psi(\underbrace{x, y, z; t}_{(1+4)\text{-Dimens}})$$

(1+3)

$$\psi(\vec{r},t)=\int d^3K\int d\omega\; e^{i\bar{K}\cdot r}\; e^{i\omega t}\;\tilde{\psi}(\vec{K},\omega)$$

$$\tilde{\psi}(\bar{K},\omega) \longleftarrow P=\chi_E E \longrightarrow \frac{\partial P}{\partial E(t,r)}$$

$$M=\chi_M B$$

$$\cancel{\chi}(\bar{r},t) \longrightarrow \tilde{\chi}(K,\omega)$$

Static Response function $\longleftarrow \tilde{\chi}(0,0)$

$\omega=0 \quad T\to\infty$

$K=0\to\lambda\to\infty$

☆ Discrete F.T. (Regular Data (Equidistant Sampling))

$\Delta t = cts$

$$x(t)\longrightarrow \{x_j\},\ j=1,\dots,N \quad \text{or} \quad \{x(t_1),x(t_2)\,\dots\,x(t_N)\}$$

$$t_{j+1}-t_j=\Delta t \quad \forall\, j\in[1,N]$$

$$\int d\omega \longrightarrow \sum \frac{\Delta\omega}{\left(\frac{2\pi}{T}\right)}$$

$$\rightarrow t_{min} = \Delta t$$

$$t_j = j\,\Delta t \qquad j = 1, \dots N \quad (= 0, \dots N-1)$$

اندیس

$$t_{Max} = N\,\Delta t$$

$$\omega_{max} = \frac{2\pi}{t_{min}} = \frac{2\pi}{\Delta t}$$

$$\omega_K = K\frac{2\pi}{N\Delta t} = K\omega_{min} = K\Delta\omega \qquad K = 1, \dots N\ (0, \dots N-1)$$

اندیس

$$\Delta\omega \equiv \omega_{min} = \frac{2\pi}{t_{max}} = \frac{2\pi}{N\Delta t}$$

$$f_N \geqslant 2 f_s$$

حداقل فرکانس داده برداری برای یافتن f_s

$$f_s \leqslant f_{N/2}$$

حداکثر فرکانس قابل دیدن

☆ Recall that

$$\langle x \rangle = 0$$

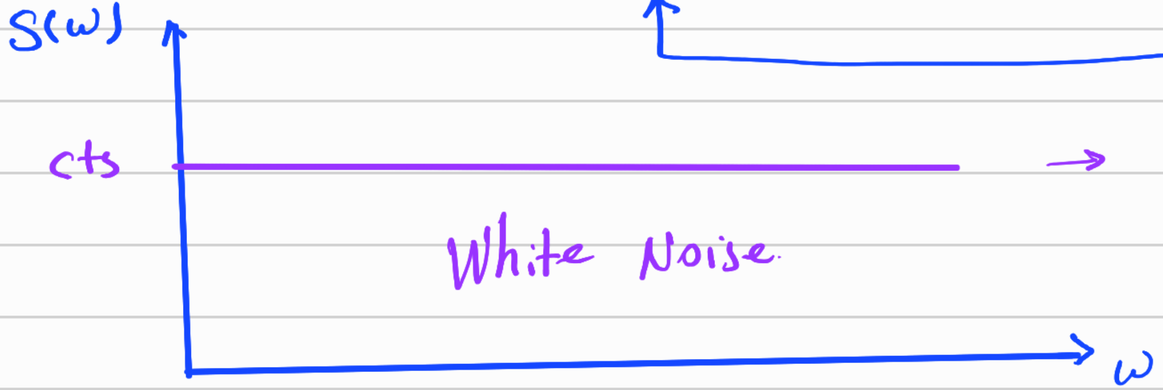
$$C_x(\tau) = \langle x(t)\, x(t+\tau)\rangle = \int d\omega\, e^{i\omega\tau}\, S_x(\omega)$$

Power Spectrum

(A) Random Number $\quad C_x(\tau) = \langle x(t)\, x(t+\tau)\rangle = \delta_D(\tau)$

$$\delta_D(\tau) = \int d\omega\, e^{i\omega\tau}\, S(\omega)$$

$$S(\omega) = \int d\tau\, e^{-i\omega\tau}\, \delta_D(\tau) = cts$$



→ Frequency-Independent

(B) $C_x(\tau) \neq \delta_D(\tau)$

Power-law $\tau^{-\alpha}$

$e^{-\tau}$

Colored Noise

نوفه رنگی

{ Blue Tilte / Red Tilte }

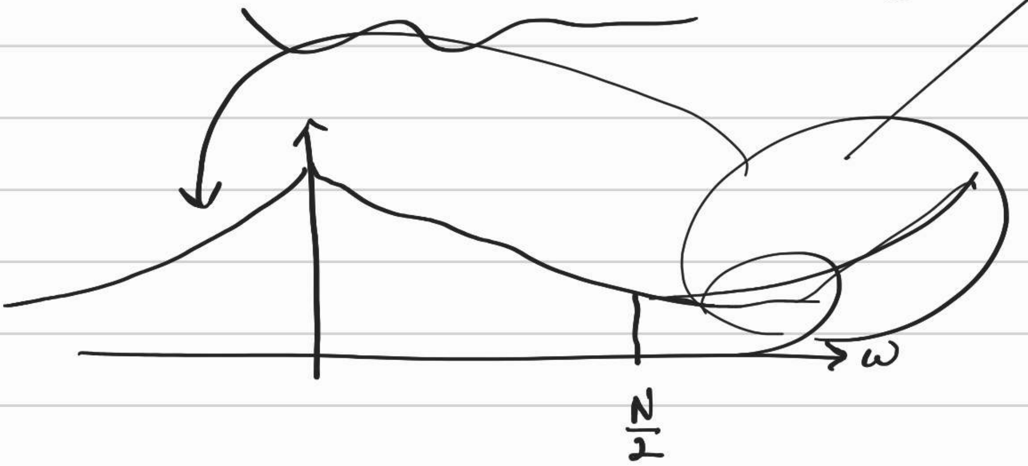
*

$$e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$$

$$C(\tau) = \int_{-\infty}^{+\infty} d\omega \left[\cos(\omega t) + i\sin(\omega t) \right] S_x(\omega)$$

$$= \int d\omega \cos(\omega t)\, S(\omega) + i\int d\omega\, \sin(\omega t)\, S(\omega)$$

even



$$S_x(\omega_k) = S_x\left(K \frac{2\pi}{N\Delta t}\right) \qquad K = 0, \; N-1$$

$$1, \; N$$

$$\boxed{S_x(K) = S_x\left(-\left(K - \underline{\underline{\tfrac{N}{2}}}\right)\right) \qquad K \geq N/2}$$

Nyquist Phenomenon . . . فرانی

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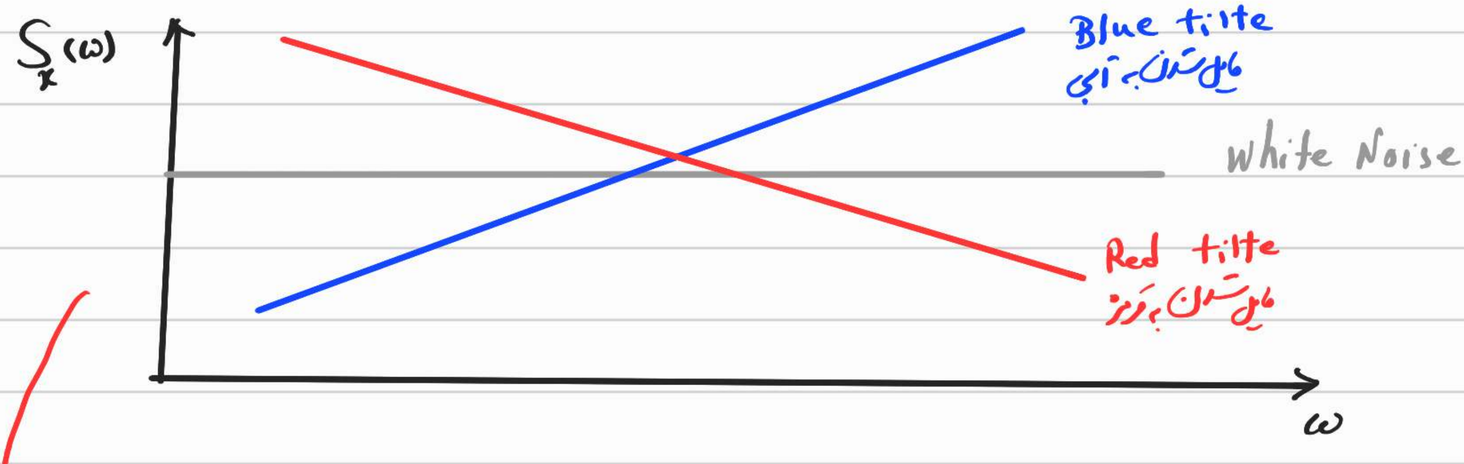


$$\mathcal{H} = \int_{a}^{\overset{L}{+\infty}} d^d x \left[t\, m^2 + K(\nabla m)^2 + L(\nabla^2 m)^2 \right]$$

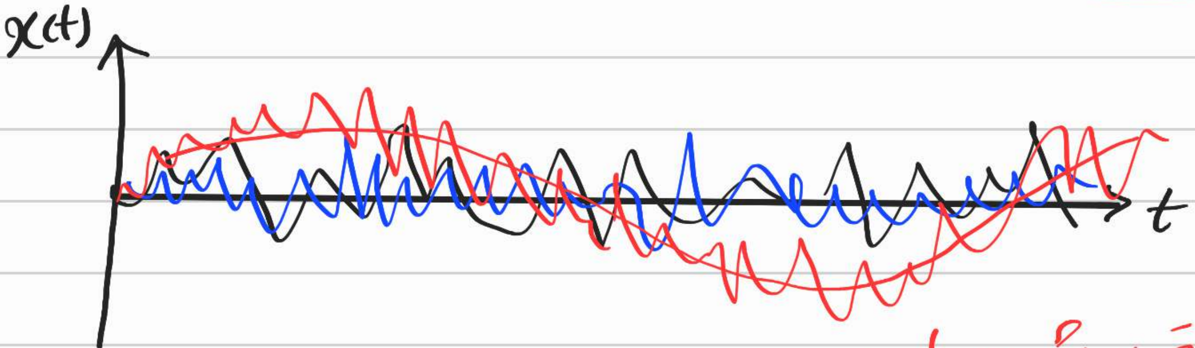
Field

$$\tilde{\mathcal{H}} = \int_{0\ \frac{2\pi}{L}}^{\frac{2\pi}{a}} \frac{d^d q}{(2\pi)^d} \left[t|\tilde{m}(q)|^2 + Kq^2|\tilde{m}(q)|^2 + Lq^4|\tilde{m}(q)|^2 \right]$$

$a \leq L < \infty \longrightarrow 0 < q \leq \frac{2\pi}{a} \longrightarrow$ UV-Cutoff

$q = \frac{2\pi}{\lambda}$ $\qquad \frac{2\pi}{L} \leq q < \frac{2\pi}{a} \longrightarrow$ UV-cutoff

IR-cut off



تصویرِ شهودی از داده

② Surrogate Data — گوسی شده $\rightarrow P(x) = \mathcal{N}(\bar{x}, \sigma_0)$

Shuffled Data — بُرخورده $\rightarrow C_x(\tau) = \delta_0(\tau)$

$S_x(\omega) = cts$

Import Data set $\{x(t)\}\ \{x_j\},\ j = 1, \underline{\underline{N}}$

loop MC

ℓ = Call Random Number(1, N)

m = Call Random Number (1, N)

$\Delta = x(\ell)$

$x(\ell) = x(m)$

$x(m) = \Delta$

End loop

loop τ

$C_{sh} = 0$

loop $j = 1, N-\tau$

$C_{sh} = C_{sh} + x(j)\,x(j+\tau)$

End loop

$C_{sh} = \dfrac{C_{sh}}{N-\tau}$

Write τ, C_{sh}

End loop

To make a Surrogate series, we use

Phase-randomized algorithm

$$\{x(t)\} \xrightarrow{F.T.} \{X(\omega)\}$$

Random phase

$$\rightarrow \quad X(\omega) \longrightarrow X'(\omega) = X(\omega)\, e^{i\underbrace{\phi(\omega)}}$$

$$X'(\omega) \xrightarrow{I.F.T} \{x'(t)\}$$

$$\boxed{\begin{array}{c} P(x') = \mathcal{N} \\ C_{x'}(\tau) = C_{x}(\tau) \end{array}}$$

$$X(\omega) = \frac{1}{N}\sum_{j=1}^{N} e^{i\omega t_j}\, x(t_j)$$

$$\downarrow$$

$$X'(\omega) = X(\omega)\, e^{i\phi(\omega)}$$

Random Number

$$\downarrow$$

$$(R \times 2\pi)$$

$$\downarrow$$

$$\phi(\omega_k) \in [0, 2\pi]$$

ساخته شده

از داده‌های اصلی

مخدوش

$$x'(t_j) = \frac{1}{N}\sum_{\omega_k} e^{-i\omega_k t_j}\, \underbrace{X(\omega_k)}\, \underbrace{e^{i\phi(\omega_k)}}$$

$$\underbrace{x'(t_j)}$$

تابع توزیع گوسی دارد.

$$S_x(\omega) = |X(\omega)|^2$$

$$\underline{S_{x'}(\omega)} = |X'(\omega)|^2 = \left(X(\omega)\, e^{i\phi}\right)\left(X^*(\omega)\, e^{-i\phi}\right)$$

$$= |X(\omega)|^2 = \underline{S_x(\omega)}$$

$$\Rightarrow \begin{cases} C_x(\tau) = \int d\omega\, e^{i\omega\tau}\, S_x(\omega) \\ C_{x'}(\tau) = \int d\omega\, e^{i\omega\tau}\, S_{x'}(\omega) \end{cases}$$