In the name of God

Department of Physics
Shahid Beheshti University

ADVANCED COURSE ON COMPUTATIONAL PHYSICS AND OPTIMIZATION

Exercise Set 3

(Due Date: 1403/01/20)

1. Joint PDF:
A : For the input data set, compute $\Delta(\tau) \equiv \int dxdy|p(x,t;y,t+\tau) - p(x,t)p(y,t+\tau)|$ as a function of τ. Explain your results.
B : For the input data set, compute $\Delta(\tau) \equiv \int dx_1 dx_3 |p(x_3, t+2\tau; x_1, t) - \int dx_2 p(x_3, t+2\tau|x_2, t+\tau)p(x_2, t+\tau|x_1, t)p(x_1, t)|$ as a function of τ. Explain your results.

2. According to Box-Muller algorithm, generate Gaussian random field with $\sigma_0^2 = 2$ and $\langle x \rangle = 3$. Check your results by fitting a Gaussian function on the computed PDF of your generated data.

3. According to Von-Neumann method, generate a set of random data set in the range $x \in [1-5]$ with PDF as: $p(x) = \sin(x^2/100) + \frac{1}{\cos(x^3/100)} + x^{-3}$.

4. Suppose that x has the Pareto distribution, $p(x) = \frac{a}{x^{a+1}}$ for $1 \leq x < \infty$. Find the probability density function of each of the following random variables:

A : $y = x^2$.
B : $z = \frac{1}{x}$.
C : $T = \ln(x)$.

5. According to Pearson correlation coefficient, compute the degree of correlation between 0.2.txt and 0.5.txt as well as with themselves.

6. Compute $C(\tau) = \langle x(t+\tau)x(t) \rangle$ for 0.2.txt and 0.5.txt and 0.8.txt data sets. Interpret your results.

7. Non-linear correlation. There are many methods to compute non-linear correlation coefficient. According to Wang, Qiang, Yi Shen, and Jian Qiu Zhang. "A nonlinear correlation measure for multivariable data set." Physica D: Nonlinear Phenomena 200.3-4 (2005): 287-295, and use the Eqs. (1), (2) and (3) of mentioned paper, compute the mutual information between all pairs of 0.2.txt, 0.5.txt and 0.8.txt.

8. Linear and non-linear correlation coefficients. Pearson's coefficient is a familiar measure to quantify the linear-correlation, while for assessing non-linear relation and even to determine the degree of correlation in the presence of outliers the Spearman's correlation coefficient is used. For all available pairs of 0.2.txt, 0.5.txt and 0.8.txt data sets, compute Spearman's and Pearson's correlation coefficient compare your results. Where:

$$\rho_p \equiv \frac{\langle [x - \langle x \rangle][y - \langle y \rangle] \rangle}{\sigma_x \sigma_y}$$

$$\rho_s \equiv 1 - 6\frac{\sum_i d_i^2}{N(N^2-1)}$$

CONTINUED

and $d_i \equiv [Rank(x_i) - Rank(y_i)]$ and $Rank$ means the order of value of variable in a set. Suppose that for $\{x\} : \{20, 100, 30, 50, 160, 10\}$. Then the $Rank(x) : \{5, 2, 4, 3, 1, 6\}$.

9. Un-weighted TPCF:
A : Compute the un-weighted TPCF of peaks for (1+1)-Dimension data set (1d_data.txt) using Natural estimator.
B : Compute the un-weighted TPCF of peaks for (1+2)-Dimension data set (2d_data.txt) and (2d_datab.txt)using Natural estimator.

Good luck, Movahed