بسم الله الرحمن الرحیم

Markov phenomena.

$\boxed{\text{Monte Carlo methods}}$

① Decaying Simulation — واپاشی ذرات

② Information Inference

- Normalization
- Marginalization — بُعدسازی

$$p(x) = \int p(x,y)\,dy$$

- Partition function — تابع پارش

Z

- Expectation Value — $\langle f \rangle = ?$

ρ — Z

- $\boxed{\text{PDF generator}}$ — Box-Muller

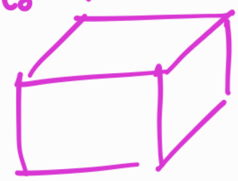
Von-Neumann Method

③ Optimization — Ground state.

- Variational Monte Carlo
- Markov-Chain Monte-Carlo MCMC
- Hamiltonian Monte-Carlo HMC
- Bootstrap algorithm
- Demon algorithm

Ex: Decaying model.

$t = t_0$ N_0



Half-Time

Half

$t=0 \quad N_0 \quad , \quad t_M$ نیمه عمر $\longrightarrow \boxed{N(t=t_M)=\frac{N_0}{2}}$

$N(t)$ = # of Particles at t

$\Delta N(t)$ = # of Decaying Particles during t and $t+\Delta t$

$$\boxed{\Delta N(t) = P(t) \times N(t)} \rightarrow \boxed{p(t) = \frac{\Delta N(t)}{N(t)}}$$

احتمال واپاشی کردن در بازه $t, t+\Delta t$

تعداد ذرات در زمان t

Probability of Decaying Particle in t and $t+\Delta t$

A: $\lambda \Delta t$ — p دلخواه

B: $\frac{\lambda}{t}\Delta t$ — دلخواه

A: $\Delta N(t) = \lambda \, \Delta t \, N(t)$

B: $\Delta N(t) = \frac{\lambda}{t} \Delta t \, N(t)$

(A) $p(t) = cte = \lambda \Delta t$

(B) $p(t) = \lambda \Delta t \, t^{-1}$

$$\lim_{\Delta t \to 0} \frac{N(t+\Delta t) - N(t)}{\Delta t} = -\frac{\Delta N}{\Delta t} = -\frac{p(t) \times N(t)}{\Delta t}$$

$$A: \frac{dN}{dt} = -\frac{\lambda \Delta t \, N(t)}{\Delta t} = -\lambda N(t)$$

$$\boxed{\frac{dN}{dt} = -\lambda N(t)} \rightarrow \boxed{N(t) = N_0 e^{-\lambda t}}$$

Theory (A)

$$B: \frac{dN}{dt} = -\frac{\lambda}{t}\frac{\Delta t}{\Delta t} N(t) = -\frac{\lambda N(t)}{t}$$

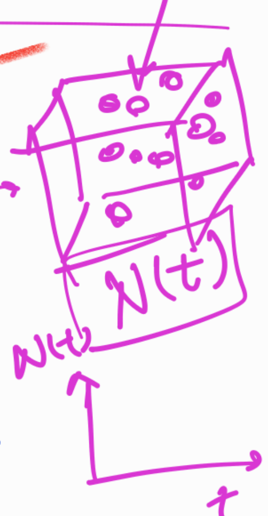
$$\boxed{N(t) = N_0 \, t^{-\lambda}}$$

Theory (B)

What about Simulation ?

Ⓐ : $\boxed{N(t) = N_0 e^{-\lambda t}}$, $t_s = s\,\Delta t$

شمونه‌های دیگر



$N(t) \longrightarrow N(s) = N_0 e^{-\lambda s \Delta t} = N_0 e^{-\lambda \Delta t\, s}$

p

$\boxed{N(s) = N_0 e^{-ps}}$

$\log N(t) = -ps + \log N_0$

t

$\boxed{0 < P < 1}$ احتمال واپاشی

بنابر تعریف

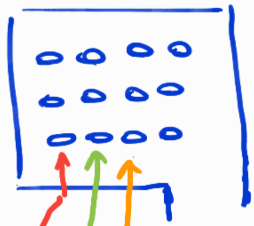
A: Numerical Algorithm

احتمال در طول زمان ثابت است

$P = cts$

```
p = ✓ ,  N_old = ✓ ,  S = 0 ,  Δt = ✓
N = N_old
Do while ( N > 0 )
   ΔN = 0
   loop i = 1 , N
      R = Call Random          (0,1)
      if ( R < p ) Then        fixed  0.8
         ΔN = ΔN + 1
      End if
   End loop
   S = S + 1
   N = N - ΔN
   Write  S×Δt , N   ——>  t , N
```

کل ذرات بمانده برای شوت بعدی مرحله زمانی جلو می‌رود



80%

20%

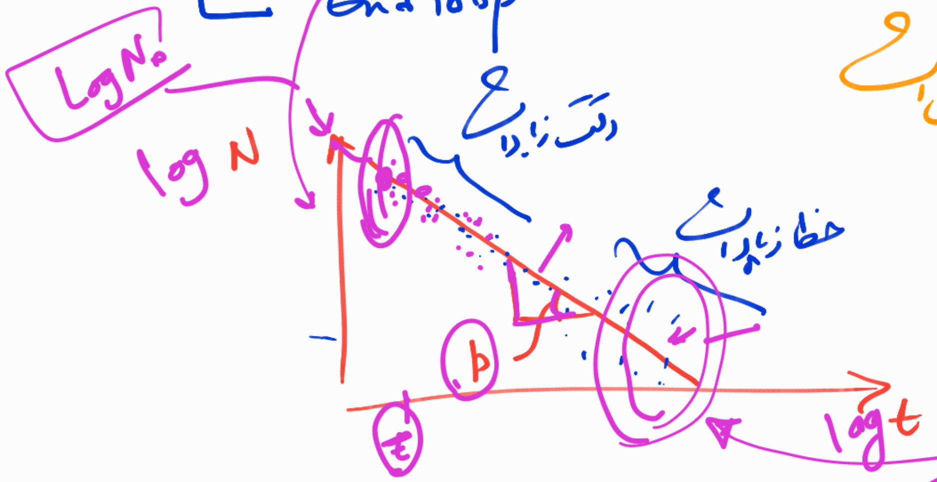
Ⓑ $p = \dfrac{\lambda}{s}$

از زمان

$N(t)$

t

End loop



N>>1 جواب دقیق و

N ↘ t ↗ $\boxed{O\left(\frac{1}{\sqrt{N}}\right)}$

$N(t)=N_0 e^{-ps}$ ← $p=0$ $p=1$ برای حالت

$p=1$: همان اول همگی یکی واپاشی کنند

$N(t)=N_0 \underbrace{e^{-\infty t}}_{0} \rightarrow 0$

$p=0$: هیچ کدام واپاشی نکنند

$N(t)=cts$

$$p=\lambda\Delta t$$

$$\lim_{\Delta t\to 0}\frac{\Delta N}{\Delta t}=\lim_{\Delta t\to 0}-\frac{p(t)\,N(t)}{\Delta t}$$

$\rightarrow$ for $p(t)=1$ $\lim_{\Delta t\to 0}\frac{\Delta N}{\Delta t}=-\lim_{\Delta t\to 0}\frac{N(t)}{\Delta t}=-\infty$

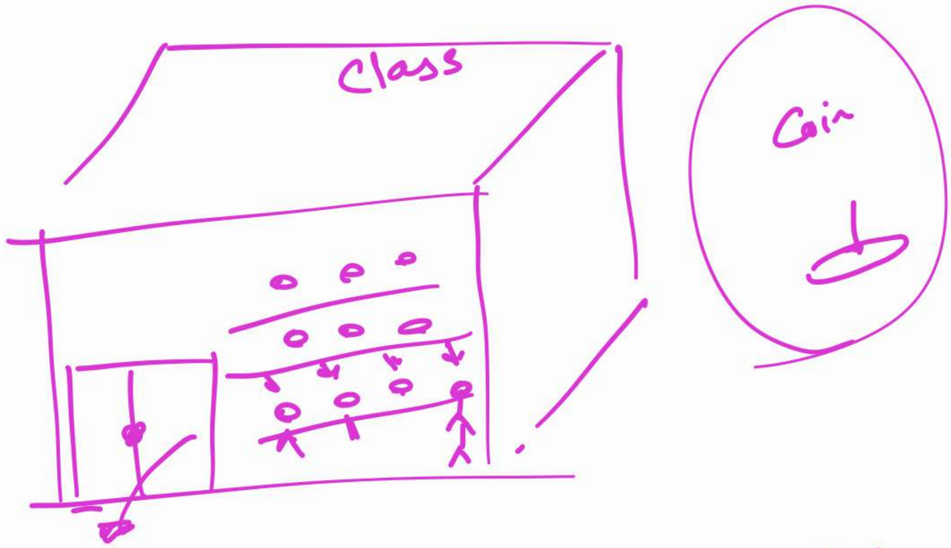
یعنی همان لحظه اول واپاشی کنند $N(t)=N_0 e^{-\infty}$

for $p(t)=0$ $\frac{dN}{dt}=0 \longrightarrow N=cts$

هیچ تغییری نمی کند

$0 < P < 1$ ... در الگوریتم $\boxed{P = \lambda \Delta t}$ ...

$N(t) = N_0 e^{-\lambda t}$ for $\begin{cases} P \neq 0 \\ P \neq 1 \end{cases}$



$\boxed{P(t) = cts.}$ → Probability of Decaying

$P = 0.8$

$q = 0.2$

$\boxed{P + q = 1}$

$H = H_0 + V \rightarrow P(t)$

$H = H_0 + \cancel{V}$

... $= cts$

$\varphi \rightarrow \boxed{p(t) =}$