بسم الله الرحمن الرحیم

۱۴۰۳/۳/۱۷
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Part A: Different Classifications of Data modeling.

- Theory-Based
- Data-Based

In the second part: We are not able to construct a well-defined Relation between desired Parameter (Model's free Parameters) and those Quantities measured (observed) in observation (Exp.)

Ex 3: (Data-Based approach): Imprint of Massive Neutrino On the Large-Scale Structure.

Imprint of massive neutrinos on Persistent Homology of large-scale structure

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The Story : Massive Neutrino Can contribute in the LSS in some sophisticated approach : for $L > \lambda_{\text{free-streaming}}$

characteristic length scale.

Massive Neutrino Can be similar to Dark matter

"Power spectrum" → Weighted TPCF → Can mainly explain the Properties of LSS + Massive Neutrino

Small scale $(K \gg 1)$

For $L < \lambda_{\text{free-streaming}}$ → Hot Dark Matter

So, we expect that LSS in the presence of Massive Neutrino to be suppresed

Small, scale Power-spectrum has many limitation

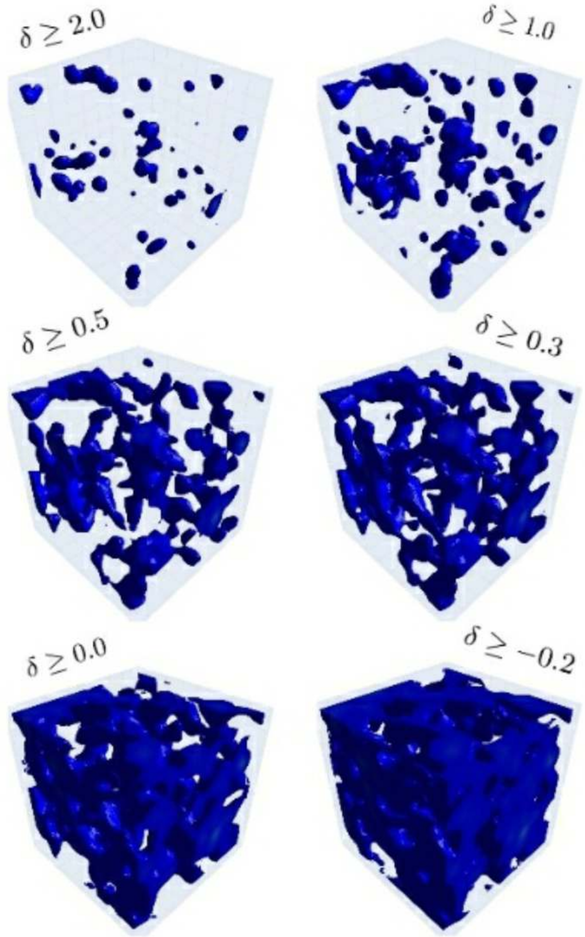


Figure 2. Excursion sets were made with super-level filtration on the density field for a fiducial realization from Quijote simulations. We cropped box of 156 Mpc h^{-1} size from the original volume. The density field is constructed based on the particle position at $z = 0$ using the cloud-in-cell scheme performed by `Pylians`. Here we consider six threshold levels, $\vartheta = \{2.0, 1.0, 0.5, 0.3, 0.0, -0.2\}$ in such that $\delta(\boldsymbol{r}, z = 0) \geq \vartheta$. To smooth the constructed density field, we use the Gaussian window function with smoothing scale $R = 5$ Mpc h^{-1}.

Number of Dark matter Particle

$$\delta \equiv \frac{n(r) - \langle n \rangle}{\langle n \rangle}$$

number Density Contrast

and we can make a smoothed field by Convolution

$$\delta_R(r) = \int dr' \, \underbrace{W_R(r-r')}_{\text{Window function}} \, \delta(r')$$

according to Topological invariants

$\beta_0 \rightarrow$ Clusters $\rightarrow$ living in high threshold

$\beta_1 \rightarrow$ filemants $\rightarrow$ living in intermediate threshold

$\beta_2 \rightarrow$ Voids $\rightarrow$ living in negative threshold

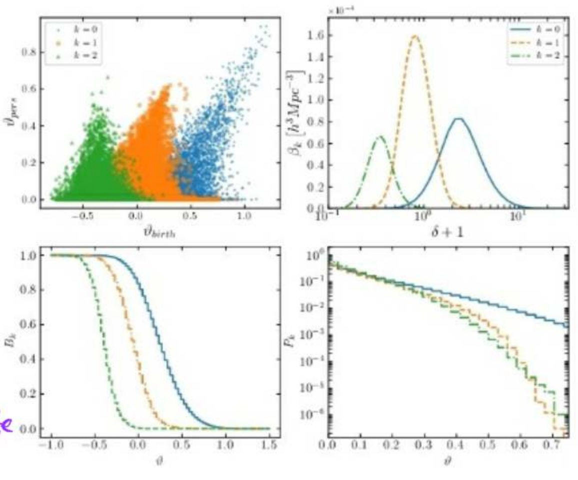


Figure 3. The extracted persistence diagram of a fiducial realization from Quijote simulations. The upper left panel reveals the persistence diagram in the scatter plot for 0-, 1- and 2-holes. The β_k as a function of density threshold $(1 + \delta)$ is represented in the upper right panel. The complementary representations of the persistence diagram, namely B_k and P_k are indicated in the lower left and lower right panels, respectively. It is worth mentioning that the ϑ for the B_k and P_k quantifies respectively the birth and persistency thresholds.

For this approach we used following Data Vector:

$$(\beta_k, B_k, P_k)$$

Betti Numbers

$$\beta_k(\vartheta) = \sum_{i=1}^{n_k} \Theta\left(\vartheta^{(k)}_{(i),birth} - \vartheta\right) \Theta\left(\vartheta - \vartheta^{(k)}_{(i),death}\right)$$

$$B_k(\vartheta) \equiv \sum_{i=1}^{n_k} \Theta\left(\vartheta^{(k)}_{(i),birth} - \vartheta\right)$$

Corresponds to those Components whose birth threshold is higher than given threshold

$$P_k(\vartheta) \equiv \sum_{i=1}^{n_k} \Theta\left(\vartheta^{(k)}_{(i),pers} - \vartheta\right)$$

Corresponds to those Component whose persistence threshold is higher than a given threshold

- Persistent Homology → { Topological features of LSS }

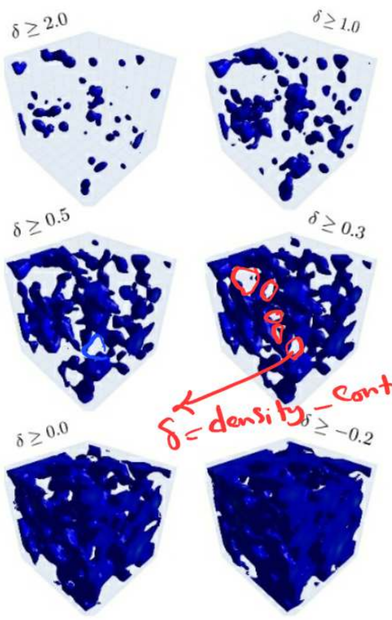


Figure 2. Excursion sets were made with super-level filtration on the density field for a fiducial realization from Quijote simulations. We cropped box of 156 Mpc h^{-1} size from the original volume. The density field is constructed based on the particle position at $z = 0$ using the cloud-in-cell scheme performed by Pylians. Here we consider six threshold levels, $\vartheta = \{2.0, 1.0, 0.5, 0.3, 0.0, -0.2\}$ in such that $\delta(\mathbf{r}, z = 0) \geq \vartheta$. To smooth the constructed density field, we use the Gaussian window function with smoothing scale $R = 5$ Mpc h^{-1}.

Super-level filtratn.

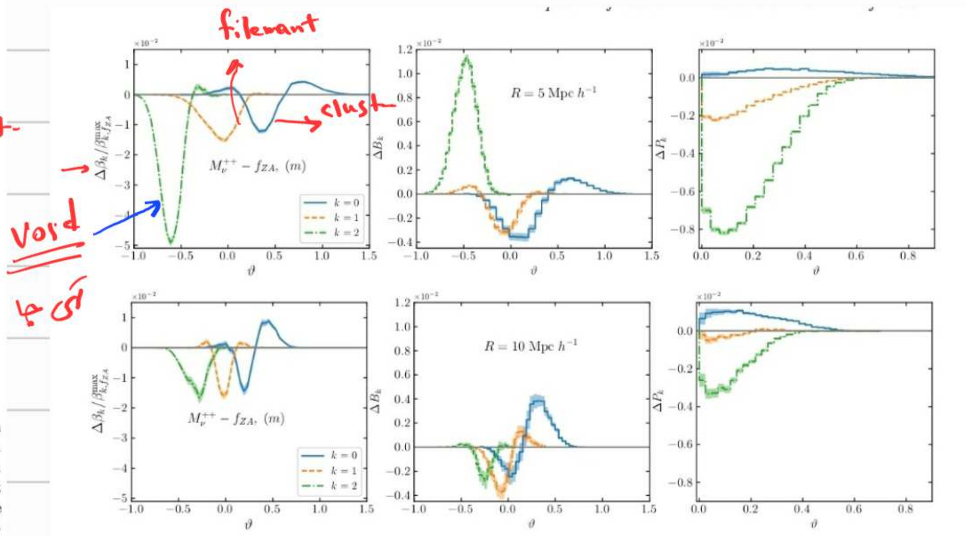


Figure 4. The PH vectorization for the m field when the massive neutrinos particles with the total mass $M_\nu^{++} = 0.2$ eV are added compared to the fiducial cosmology. The left panels represents the Betti curves divided by the corresponding maximum value in the fiducial case. The middle and right panels are devoted to the differences in the (B_k, P_k) with respect to fiducial cosmology, respectively. The blue solid, orange dashed and green dashed-dotted lines represent the 0-hole, 1-hole, and 2-hole homology groups, respectively. The shaded areas are associated with the 2σ confidence interval errors. The upper and lower rows are devoted to smoothing scales $R = 5$ Mpc h^{-1} and $R = 10$ Mpc h^{-1}, respectively.

β_K : Topological Invariant of Underlying field (Matter_field)

Density Contrast → $\delta = \dfrac{\rho - \langle \rho \rangle}{\langle \rho \rangle}$

$$\begin{cases} \delta < 0 \rightarrow \text{Void} \\ \delta > 0 \rightarrow \text{Cluster.} \end{cases}$$

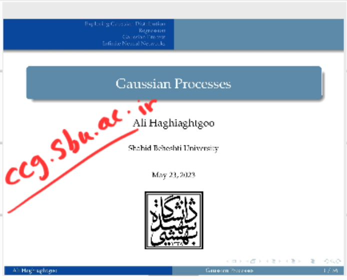
$$P_K(\Omega_m, \Omega_R, H_0, n_s, \sigma_8, M_\nu \ldots)$$

$\{\theta\}$: Model's free Parameter

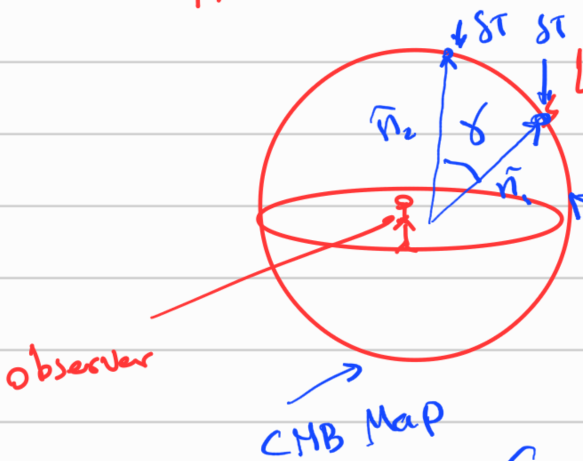
$$\boxed{\chi^2(\{\theta\}) = P_{obs} - P_{Theory}(\{\theta\})}$$

We don't know the well Define form of this Quantity

Simulation-Based Inference (SBI)



Suppose that → Cosmic Microwave Background (CMB)



Last scattering surface

$T(\theta, \varphi) \longrightarrow$ (1+2)-Dimensional field

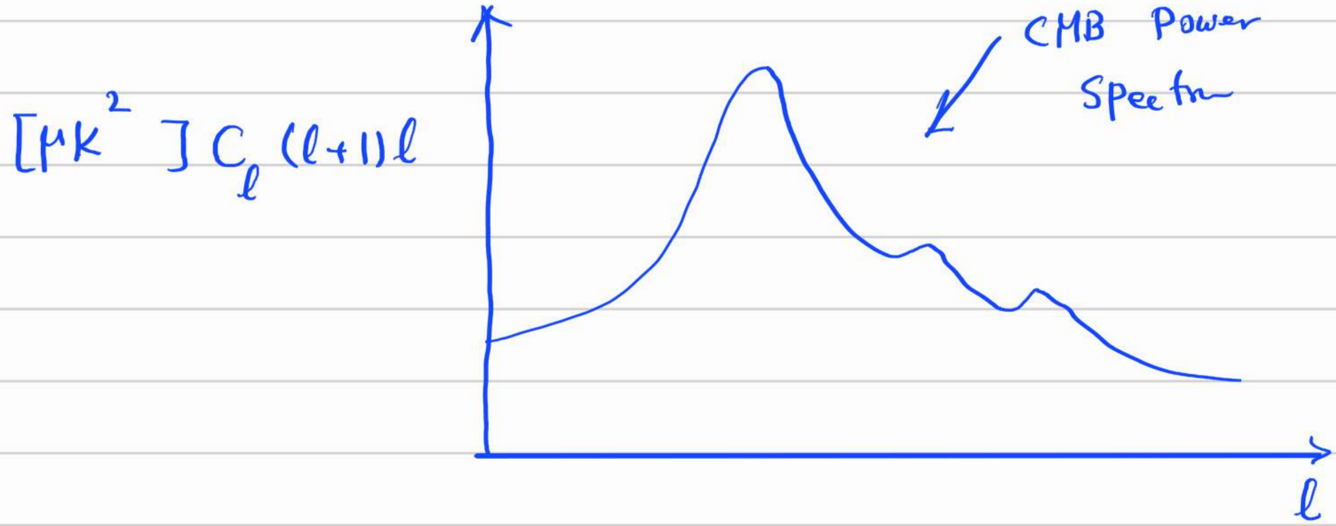
Isotropic field

Correlation function (Weighted TPCF)

$$\gamma = \cos^{-1}|\hat{n}_1 \cdot \hat{n}_2|$$

$$C_{\delta T \delta T}(\gamma) = \underbrace{\langle \delta T(\hat{n}_1)\, \delta T(\hat{n}_2) \rangle}_{\text{ensemble}}$$

$$= \sum \frac{(2\ell+1)}{4\pi} C_\ell P_\ell(\cos\gamma)$$

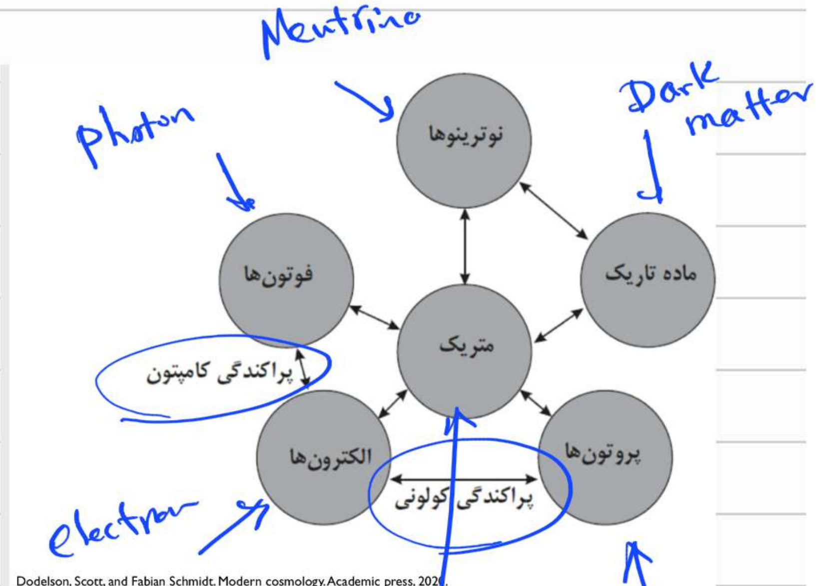


$$[\ell] = r^{-1}$$

$$\{\theta\} = \{\Omega_m, \Omega_b, \Omega_R, H_0, \omega, \Omega_\lambda, n_s, A, \dots\}$$

$$C_\ell(\{\theta\}) =$$

CMB map



Dodelson, Scott, and Fabian Schmidt. Modern cosmology. Academic press, 2020.

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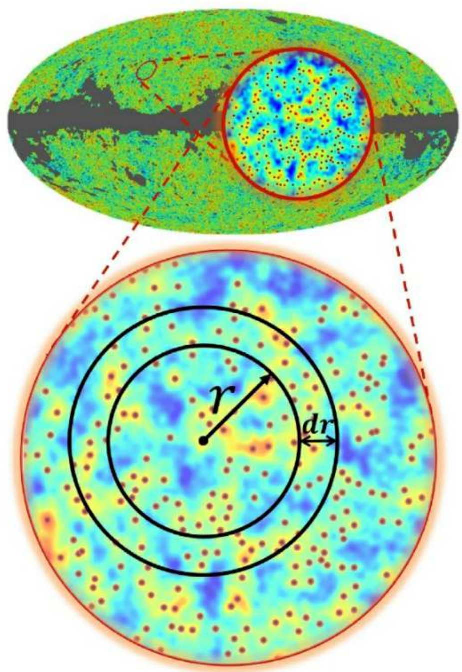

Figure 1. Peak distribution on the NILC map for $N_{\rm side} = 512$ at threshold $\vartheta = 0.5$. In the enlarged plot, we indicate a sketch to illustrate clustering of local peaks separated by r.

Boltzman Eq. $$\frac{df}{dt} = C[f]$$

CMB: Einstein-Boltzmann hierarchy equations Perturbations

$$\frac{df}{dt} = C[f] \rightarrow$$

Boltzmann equation

1) $\dot{\Theta} + ik\mu\Theta = -\dot{\Phi} - ik\mu\Psi - \dot{\tau}[\Theta_0 - \Theta + \mu v_b - \frac{1}{2}\mathcal{P}_2(\mu)\Pi]$

2) $\dot{\Theta}_P + ik\mu\Theta_P = -\dot{\tau}[-\Theta_P + \frac{1}{2}(1-\mathcal{P}_2(\mu))\Pi]$

$\Pi = \Theta_2 + \Theta_{P2} + \Theta_{P0}$

3) $\dot{\delta} + ikv = -3\dot{\Phi}$

4) $\dot{v} + \frac{\dot{a}}{a}v = -ik\Psi$

$$g_{\mu\nu} = \begin{pmatrix} -(1+2\Psi) & 0 \\ 0 & a(t)^2\delta_{ij}(1+2\Phi) \end{pmatrix}$$

5) $\dot{\delta}_b + ikv_b = -3\dot{\Phi}$

6) $\dot{v}_b + \frac{\dot{a}}{a}v_b = -ik\Psi + \frac{\dot{\tau}}{R}[v_b + 3i\Theta_1]$

$\frac{1}{R} \equiv \frac{4\rho_\gamma^{(0)}}{3\rho_b^{(0)}}$

7) $\dot{\mathcal{N}} + ik\mu\mathcal{N} = -\dot{\Phi} - ik\mu\Psi$

8) $k^2\Phi + 3\frac{\dot{a}}{a}(\dot{\Phi} - \Psi\frac{\dot{a}}{a}) = 4\pi Ga^2[\rho_{CDM}\delta + \rho_b\delta_b + 4(\rho_\gamma\Theta_0 + \rho_\nu\mathcal{N}_0)]$

9) $k^2(\Phi + \Psi) = -32\pi Ga^2(\rho_\gamma\Theta_2 + \rho_\nu\mathcal{N}_2)$

(1,2) Photon equations

(3,4) Dark matter

(5,6) Baryon

(7) Neutrino

(8,9) Einstein perturbed equations

9_Coupled Differential Equation

$\boxed{\frac{\delta T}{T}(\hat{n})}$ from theory

Dodelson, Scott, and Fabian Schmidt. Modern cosmology. Academic press, 2020.

We have Well_Defined Relation between model's Parameters and observation

$\frac{\delta T}{T}(n)\Big|_{\text{observation}}$

$\boxed{\text{CAMB\_Software}}$

Topological Invariant

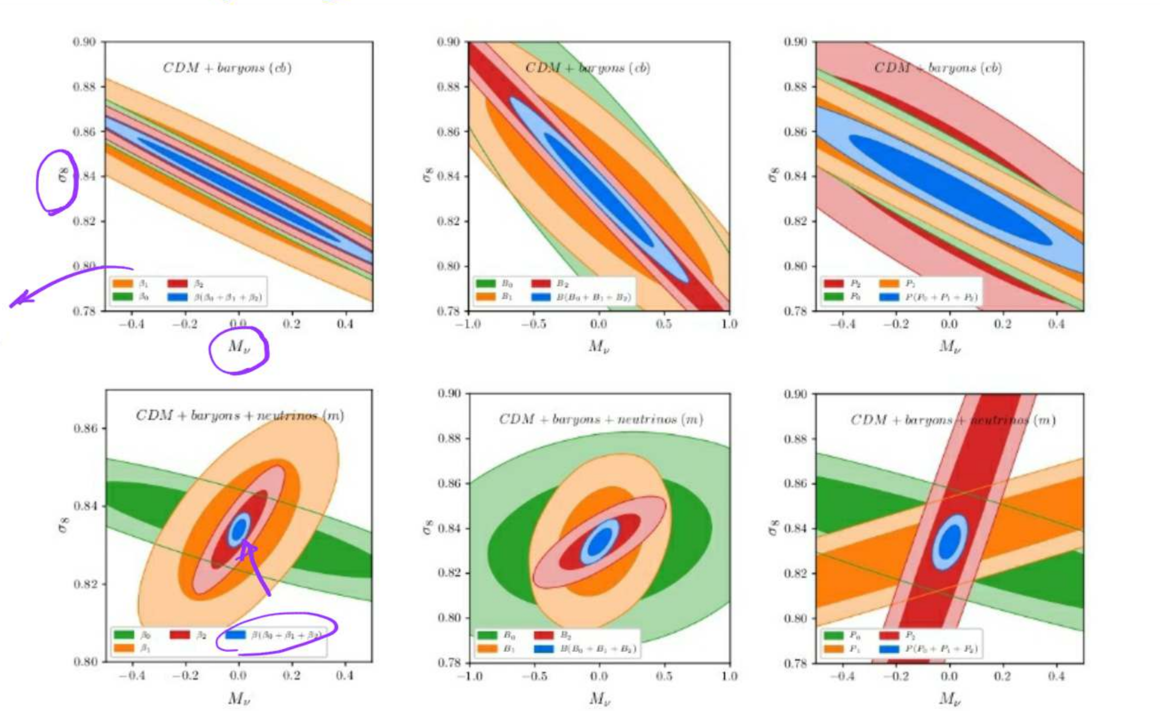

Figure 11. The 68% and 95% confidence contours for (M_ν, σ_8) obtained from various measures under the PH vectorization for both cb and m fields at redshift $z = 0$ by the Fisher information analysis. The upper row depict constraints for cb field, while the bottom row are devoted to m field constraints. Here we adopted $N_{bins} = 15$ and $R = 5$ Mpc h^{-1}.

Part B: ③ Different types of Data Modeling

④ Various Parts of a typical Pipeline of Data modeling.

Main streamline of "DM"
- → Frequentist's approach
- → Bayesian approach

بسم الله الرحمن الرحیم

Review on Data Analysis Methods & Cosmological Simulations

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Workshop on Computational Cosmology:
From Theory to Observation
1-2 August 2023

Computational Cosmology Group Of Shahid Beheshti University

COMPLEXLAB SHAHID BEHESHTI UNIVERSITY

تعیین مقدار برای کمیتهای آزاد

رهیافت Bayesian

۱) در این روش داده ها و نتایج بخشی از یک آنسامبل هستند

Unknown

۲) کمیت های آزاد مدل مجهول هستند که ما تنها به صورت احتمالی می توانیم مقدار آنها را تعیین کنیم

Prior information

۳) هر گونه اطلاعات اولیه در این رهیافت قابل استفاده هستند

۴) سازوکاری برای رهایی از کمیتهای اضافی (Nuisance) دارد

رهیافت Frequentist

۱) در این روش داده ها و نتایج ریزحالت یک پیکربندی به حساب می آیند و داده ها تکرار پذیرند

fixed

۲) کمیت های مدل مجهول ولی ثابت هستند

۳) هیچ گونه اطلاعی از مدل مورد استفاده نیست

۴) سازوکاری برای رهایی از کمیتهای اضافی (Nuisance) ندارد

In the Bayesian approach: $\{\theta\}$'s are Unknown Parameters and We Can determine the Best fit Values in the Context of Probability

But in the frequentist approach $\{\theta\}$: are Unknown but they are $\boxed{\text{fixed}}$

منطق استقرایی
Inductive logic: Bayesian

مشاهده رفتار تک تک اجزا منجر به قانون کلی می شود. تمام انسانهای مشاهده شده فانی بودند در نتیجه انسان فانی است

The evidence of Individual behavior leads to a law

منطق قیاسی
Deductive logic: Frequentist

مشاهده مبتنی بر واقعیت های شناخته شده که در کنار هم قرار می گیرند و نهایتا به نتیجه می رسند. انسانها فانی هستند سقراط انسان است لذا سقراط فانی است

Human are mortal

The evidence based on the Completely Known Knowledge makes the consequence

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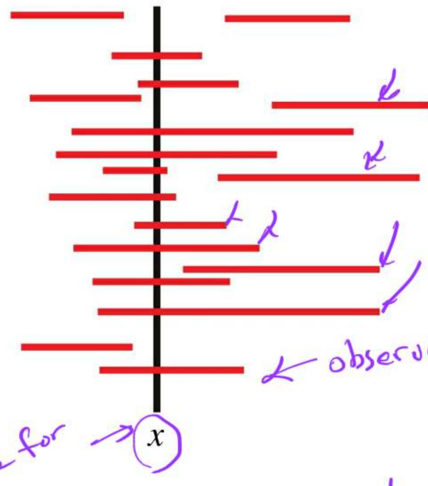
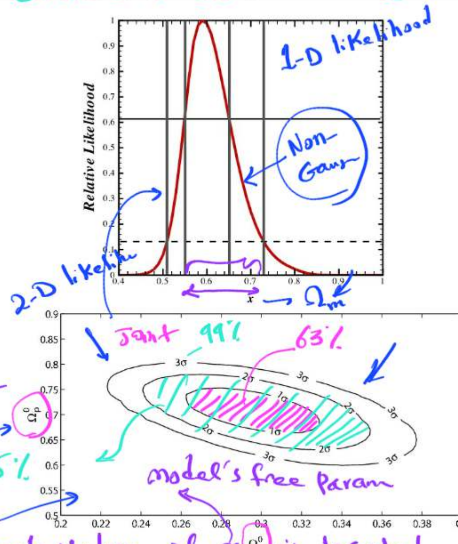
The 90% of observations is Consistent with a fixed value of $\underline{x}$



بر اساس نگرش frequentist برای مثال ۹۰ درصد مشاهدات به مقدار ثابت x منجر شده است



The Best Value of x is located in a given area

بر اساس نگرش Bayesian با احتمال ۹۰ درصد مقدار دقیق کمیت x دراین بازه قرار می گیرد

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$P(\Omega_m, \Omega_p) \rightarrow \underbrace{P(\Omega_m) = \int d\Omega_p \, P(\Omega_m, \Omega_p)}_{\text{Margnalizat (Integrating out)}}$

An other word: With a Probability of 90%. The best value of x is found in a given area. (region)

A history about Bayesian strategy

- By Thomas Bayes (1702-1761)
- Pierre Simon Laplace (1812)
- Fisher, Neyman, Wald,...
- Gelfand and Smith (1990)
- Now

Bayesian approach

Data: {observations & experiments}

$N \sim 50\,000000$ Pixels $T(\theta, \varphi)$ Planck

$D: \{x_i, y_i\} \quad i = 1, \ldots, N$ مشاهدات: داده ها

Model's free Parameters

$\Theta: \{\theta_j\} \quad j = 1, \ldots, M \quad \rightarrow \quad Y(x, \Theta) = \sum_{j=1}^{M} \theta_j f_j(x)$ برای مثال یک مدل

$y = ax + b$

Posterior Probability: $p(\Theta | D)$ احتمال یافتن مدل به شرطی که داده ها به دست آمده باشند. با چه احتمالی مشاهدات به مدل منجر میشود Posterior

Probability of having {Θ} given {D}

Likelihood Probability: $p(D | \Theta)$ احتمال به دست آمدن مشاهدات به شرطی که مدل مشخص باشد. مدل مشخص با چه احتمالی مشاهدات را به دست می دهد. Likelihood

$\mathcal{L}(D|\Theta)$: Probability of having {D} given {θ}

$p(\Theta)$ هر گونه اطلاعات اولیه در مورد مقادیر کمیتهای مدل در این تابع توزیع وجود دارد. Prior

Prior Information of Parameters

Bayes theorem:

$$P(x|y) = \frac{P(y|x)P(x)}{P(y)}$$

$$\boxed{P(x|y)P(y) = P(y|x)P(x)}$$

$$\underbrace{P(\{\theta\}|D)}_{\text{posterior}} = \frac{\mathcal{L}(D|\{\theta\})\, P(\{\theta\})}{\underbrace{\int d\{\theta\}\, \mathcal{L}(D|\{\theta\})\, P(\{\theta\})}_{\mathcal{L}(D)}}$$

Likelihood

Prior informat

evidence

ویژگی های رهیافت بیزی (II)

Bayesian approach

بهبود توزیع احتمال پارامترهای مدل

$D_1 : \{x_i, y_i\}$ ← observation {Data}

$\Theta : \{\theta_j\}$ ← Model's free parameter

$$p(\Theta | D_1) = \frac{\mathcal{L}(D_1 | \Theta) p(\Theta)}{\int d\Theta\ p(D_1 | \Theta) p(\Theta)}$$

Prior informat

$p(\Theta | D_1)$ به عنوان تابع اطلاعات اولیه در تحلیل جدید به حساب می آید

$$p(\Theta D_1 | D_2) = \frac{\mathcal{L}(D_2 | \Theta D_1) p(\Theta | D_1)}{p(D_2 | D_1)}$$

$$\mathcal{L}(D_2 | \Theta D_1) = \frac{p(D_2, D_1 | \Theta)}{p(D_1 | \Theta)}$$

$$p(\Theta D_1 | D_2) = \frac{p(D_2, D_1 | \Theta)}{p(D_1 | \Theta)} \times \frac{p(D_1 | \Theta) p(\Theta)}{p(D_1)} \times \frac{1}{p(D_2 | D_1)}$$

$$= \frac{p(D_2, D_1 | \Theta) p(\Theta)}{p(D_1, D_2)}$$

یعنی مشاهدات جدید را برای نتایج به دست آمده با مشاهدات قدیمی به کار برده تا نهایتاً مقادیر کمیت ها به دست آیند

$$\underbrace{P(D_2|D_1)P(D_1)}_{P(D_2, D_1)}$$

$$\underbrace{\{D\} = \{D_1\} \oplus \{D_2\}}$$

$$P(\{\theta\}|D) = \frac{\mathcal{L}(D|\{\theta\})\, P(\{\theta\})}{\mathcal{L}(D)}$$

$$= \frac{\mathcal{L}(D_1, D_2 | \theta)\, P(\{\theta\})}{\mathcal{L}(D_1, D_2)}$$

$P(D_1, D_2)$

$$p(\Theta | D) = \frac{p(\Theta, D)}{p(D)} = \frac{p(D, \Theta)}{p(D)} = \frac{p(D | \Theta) p(\Theta)}{p(D)}$$

$$p(D) = \int d\Theta\ p(D, \Theta) = \int d\Theta\ p(D | \Theta) p(\Theta)$$

$$p(\Theta | D) = \frac{\mathcal{L}(D | \Theta) p(\Theta)}{\int d\Theta\ p(D | \Theta) p(\Theta)}$$

در حالتی که اطلاعات اولیه ای برای کمیت های مدل نداشته باشیم

Posterior=Likelihood

$P(\{\theta\}) = cts.$

$$\boxed{P(\theta|D) \sim \mathcal{L}(D|\theta)}$$

ویژگی های رهیافت بیزی (I)

مثال: فرض کنید که برای یک دسته مشاهده داده شده، دو مدل داریم . کدامیک بامثال احتمال بیشتری انتخاب می شوند؟

Bayesian Model averaging (BMA)

$$D : \{x_i, y_i\}$$

$$\Theta_1 \quad \Theta_2$$

Model 1 $\rightarrow$ $$p(\Theta_1 | D) = \frac{p(D|\Theta_1)p(\Theta_1)}{\int d\Theta\, p(D|\Theta_1)p(\Theta_1)}$$

Model 2 $\rightarrow$ $$p(\Theta_2 | D) = \frac{p(D|\Theta_2)p(\Theta_2)}{\int d\Theta\, p(D|\Theta_2)p(\Theta_2)}$$

$$\frac{p(\Theta_1 | D)}{p(\Theta_2 | D)} = \frac{\dfrac{p(D|\Theta_1)p(\Theta_1)}{\int d\Theta\, p(D|\Theta_1)p(\Theta_1)}}{\dfrac{p(D|\Theta_2)p(\Theta_2)}{\int d\Theta\, p(D|\Theta_2)p(\Theta_2)}} = \frac{p(D|\Theta_1)p(\Theta_1)}{p(D|\Theta_2)p(\Theta_2)}$$

$$if \quad p(\Theta_1) = p(\Theta_2)$$

مدلی که درست نمایی بزرگتری داشته باشد با احتمال بیشتری توسط داده ها انتخاب می شود

مراحل مدل کردن داده ها

۱) اندازه گیری (مستقیم یا شبیه سازی) (Measurment)

۲) برآورد خطا و ارزیابی انتشار خطا بر روی کمیت های ثانویه (Error estimation and error propagation)

۳) تدوین مدل با توجه به تابع مناسب (model selection) (Merit function)

۴) انتخاب بینش تعیین مقادیر آزاد مدل (Bayesian or Frequentist)

۵) استفاده از روشهای تعیین مقادیر کمیت های آزاد مدل و البته حوزه اعتبار آنها (Parameter estimation and confidence interval)

۶) تعیین خوبی مدل Goodness of fit