In the name of God

Department of Physics
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SELECTED TOPICS COURSE

Exercise Set 4

1. Bias definition: The relation between Unweighted TPCF and weighted TPCF can be considered as:

$$\Psi_{fg}(R) = \mathcal{B}_{fg}C(R)$$

(a) For Up-crossing feature and according to the theoretical form of un-weighted TPCF, determine $\mathcal{B}$ for 1+1-D stochastic field.

(b) For Up-crossing feature and according to the theoretical form of un-weighted TPCF, determine $\mathcal{B}$ for 1+2-D stochastic field.

(c) For *cmd* feature (introduced by Mohammad Jalali, (Kanafi, MH Jalali, and S. M. S. Movahed. "Probing the Anisotropy and Non-Gaussianity in the Redshift Space through the Conditional Moments of the First Derivative. The Astrophysical Journal 963.1 (2024): 31.)) and according to the theoretical form of un-weighted TPCF, determine $\mathcal{B}$ for 1+2-D stochastic field.

(d) For a sharp clipping feature, we have $f \equiv A\Theta(\alpha - \vartheta)$. Here A is a normalization coefficient. The average value of sharp clipping in a Gaussian random field can be written by:

$$\langle f(\alpha)\rangle = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} A\Theta(\alpha-\vartheta)e^{-\alpha^2/2}d\alpha$$

Now, we expand $\Theta(\alpha - \vartheta)$ in terms of Hermite polynomials defined by $H_n = e^{x^2/2}(d/dx)^n e^{-x^2/2}$ with $\langle H_n(x)H_m(x)\rangle\delta_{nm}m!$. Therefore we have

$$A\Theta(\alpha) = \sum_{k=0}^{\infty}\frac{J_k}{k!}H_k(\alpha)$$

Show that

$$J_k = \frac{\vartheta H_{k-1}(\vartheta)}{\vartheta\sqrt{\pi/2}\exp(\vartheta^2/2)\mathrm{erfc}(\vartheta/\sqrt{2})}$$

(e) Show $\lim_{\vartheta\to\infty} J_k = \vartheta^k$

(f) Show $\langle\Theta(\alpha(r))\Theta(\alpha(r'))\rangle = 1 + \Psi(R) = \sum_{k=0}^{\infty}\frac{J_k^2}{k!}C_{\alpha\alpha}^k(R)$, where $R = |r - r'|$.

(g) For $R \to \infty$ show $\langle\Theta(\alpha(r))\Theta(\alpha(r'))\rangle = 1 + \Psi(R) \sim J_1^2 C_{\alpha\alpha}(R)$. Here $J_1 = \vartheta^2$.

2. According to the expansion of tracer as a function of field, namely $n_{cmd}(\vartheta_{eff}) = n_{cmd}(\vartheta - \frac{\delta_b}{\sigma_0})$ and its Tylor expansion, compute the bias factor for *cmd* measure introduced by Mohammad Jalali, ("The Astrophysical Journal 963.1 (2024): 31.). Here we can consider the δ_b plays the role of underlying field.

3. In the Redshift space distortion, the β is very important quantity, according to the $\mathcal{B}_{cmd}$, find the β-dependancy of bias factor and according to the Eq. (46) of Mohammad Jalali, (The Astrophysical Journal 963.1 (2024): 31.), compute the relative uncertainty of β for the given error on the bias factor ($\sigma_{\mathcal{B}_{cmd}}$).

Good luck, Movahed