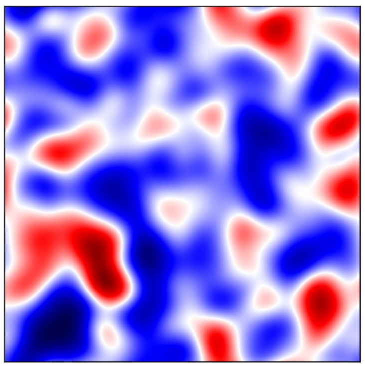
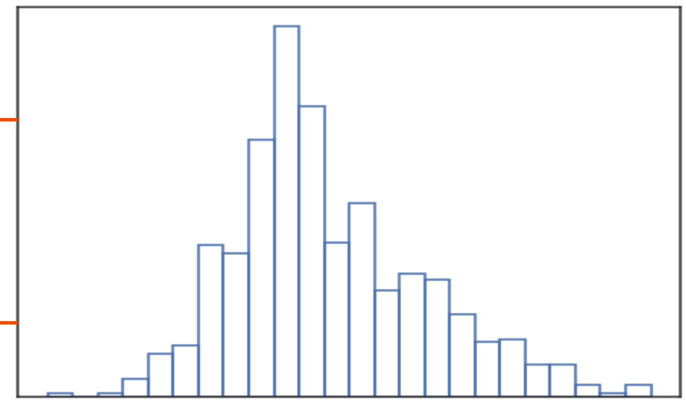
Parameter Estimation

an observation from
a part of Universe



Extracting Data Vector

$$\boldsymbol{\mathcal{D}}_{\diamond} = \{d_i\}_{i=1}^{N_{bin}}$$



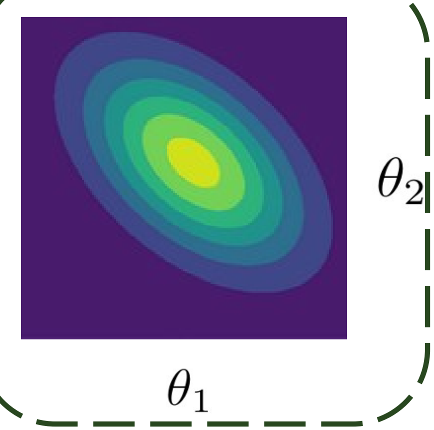
$\mathcal{M}$

represents the assumed model to describe this physical system with corresponding parameter set:

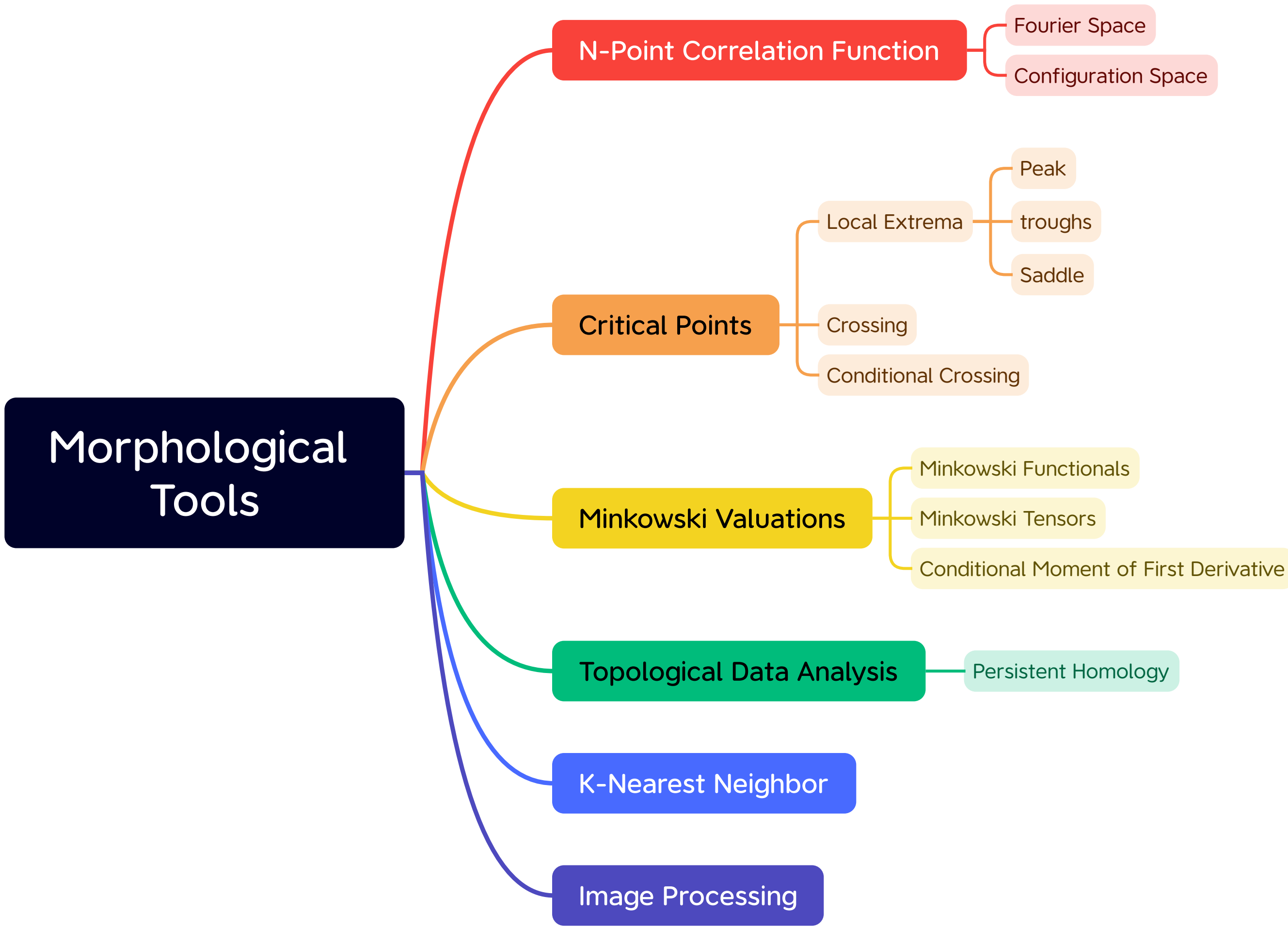
$$\boldsymbol{\Theta} = \{\theta_1, \theta_2, \cdots, \theta_n\}$$

$$\mathcal{P}(\boldsymbol{\Theta}|\boldsymbol{\mathcal{D}}_{\diamond}, \mathcal{M})$$

Finding Posterior



Parameter Estimation



Bayesian inference

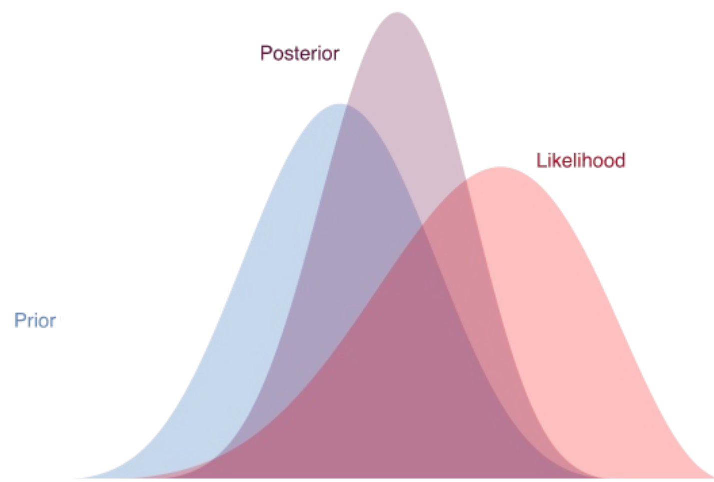
Bayesian theorem

$$\mathcal{P}(\boldsymbol{\Theta}|\boldsymbol{\mathcal{D}}_\diamond,\mathcal{M}) = \frac{\mathcal{P}(\boldsymbol{\mathcal{D}}_\diamond|\Theta,\mathcal{M})\,\mathcal{P}(\boldsymbol{\Theta}|\mathcal{M})}{\mathcal{P}(\boldsymbol{\mathcal{D}}_\diamond|\mathcal{M})}$$

$$\ln\mathcal{L}(\boldsymbol{\mathcal{D}}_\diamond|\boldsymbol{\Theta}) = \frac{1}{2}\left[\left(\boldsymbol{\mathcal{D}}_\diamond - \boldsymbol{\mathcal{D}}_\diamond^{(th)}(\boldsymbol{\Theta})\right)^T \cdot \boldsymbol{C}^{-1} \cdot \left(\boldsymbol{\mathcal{D}}_\diamond - \boldsymbol{\mathcal{D}}_\diamond^{(th)}(\boldsymbol{\Theta})\right)\right]$$

Fisher4cast | Monte Carlo | . . .

Posterior sampling (parameter estimation)



Fisher Forecast

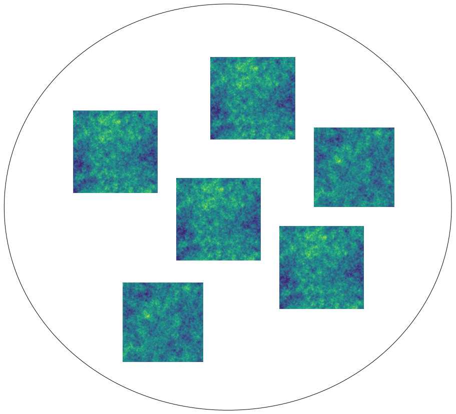
$$F_{mn} = -\left\langle \frac{\partial^2 \ln \mathcal{L}}{\partial \theta_m \, \partial \theta_n} \right\rangle$$

$$F_{mn} = \left[\frac{\partial \boldsymbol{\mathcal{D}}_{\diamond}^{(th)}(\boldsymbol{\Theta})}{\partial \theta_m} \right]^T \cdot \boldsymbol{C}^{-1} \cdot \frac{\partial \boldsymbol{\mathcal{D}}_{\diamond}^{(th)}(\boldsymbol{\Theta})}{\partial \theta_n}$$

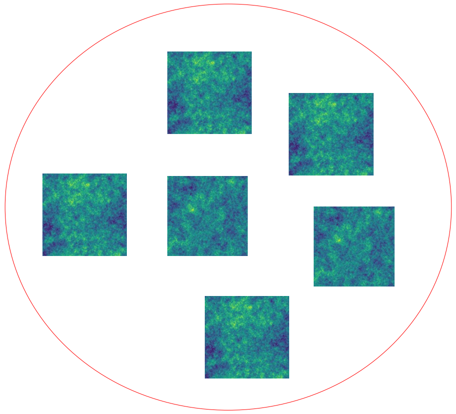
$$\boldsymbol{C}^{-1} = \frac{N_{sim} - N_{bin} - 2}{N_{sim}} \, \hat{\boldsymbol{C}}^{-1}$$

$$\hat{\boldsymbol{C}}_{ij} = \left\langle \left(d_i - \bar{d}_i\right) \left(d_j - \bar{d}_j\right) \right\rangle, \qquad \bar{d}_i = \langle d_i \rangle$$

Fiducial Simulation



Variable parameter simulation



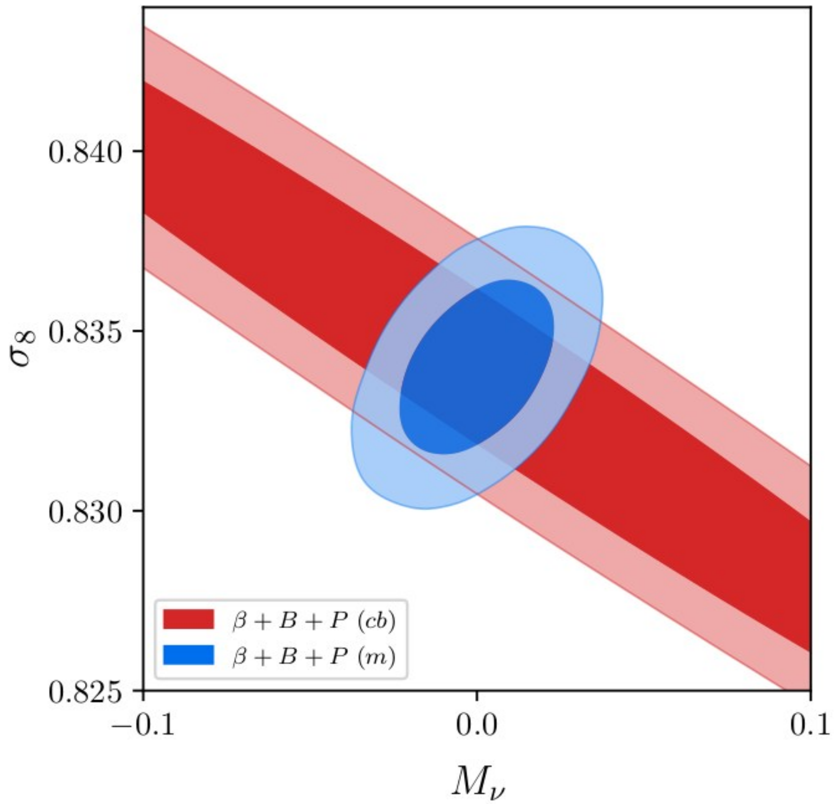
Fisher Forecast

$$F_{mn} = -\left\langle \frac{\partial^2 \ln \mathcal{L}}{\partial\theta_m\, \partial\theta \ldots} \right.$$

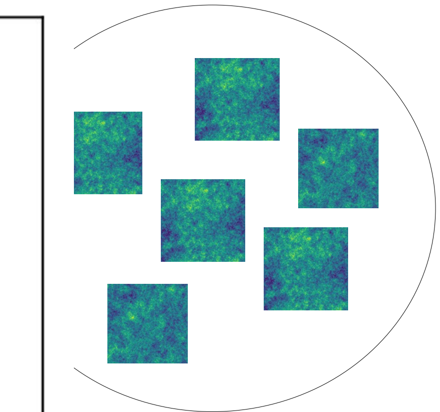
$$F_{mn} = \left[\frac{\partial \boldsymbol{\mathcal{D}}^{(th)}_{\diamond}(\boldsymbol{\Theta})}{\partial\theta_m} \right]^{T} \ldots$$

$$\boldsymbol{C}^{-1} = \frac{N_{sim} - N_{bin} \ldots}{N_{sim}}$$

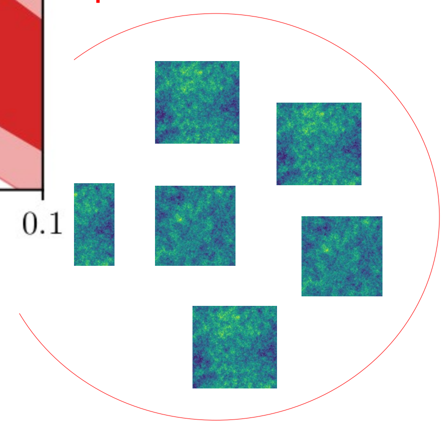
$$\hat{\boldsymbol{C}}_{ij} = \left\langle \left(d_i - \bar{d}_i\right) \left(d_j \ldots\right.\right.$$



Fiducial Simulation



parameter simulation

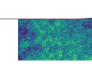


Fisher Forecast

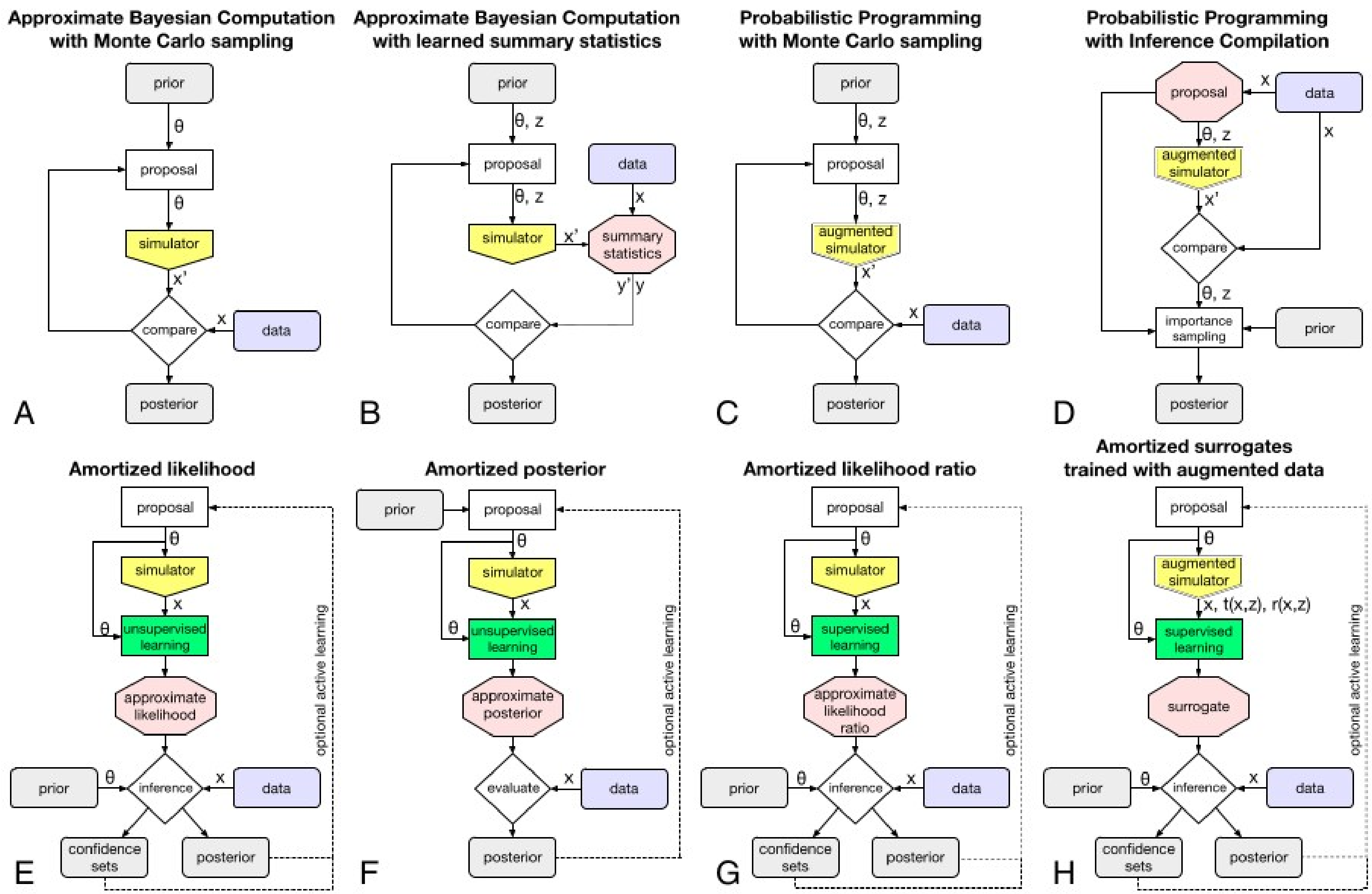
Disadvantages of traditional Bayesian inference:

- Assuming a model to describe the likelihood function
- Inefficient for high dimensional data vector
- Inefficient for extracting constraint from nonlinear (non-Gaussian) scales
- In some cases, it depends on the analytical prediction of the data vector

These are motivations for applying simulation-based or free-likelihood inference methods

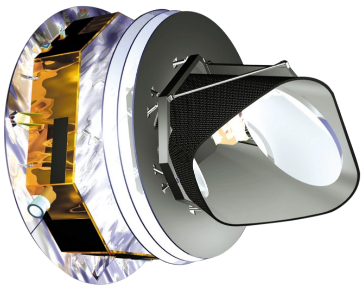


Overview of different approaches to simulation-based inference



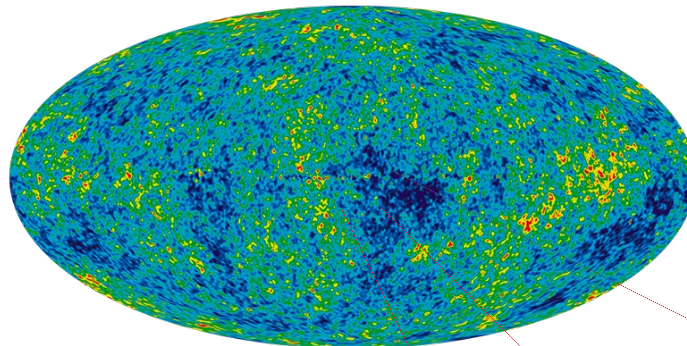
Cranmer, Kyle, Johann Brehmer, and Gilles Louppe. "The frontier of simulation-based inference." Proceedings of the National Academy of Sciences

Starting point to SBI

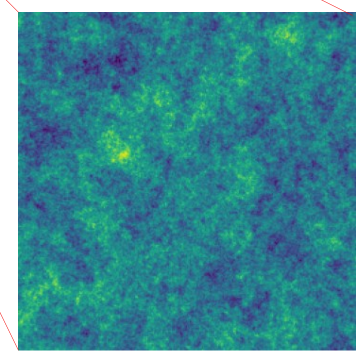


Observing the early universe

Cosmic Microwave Background



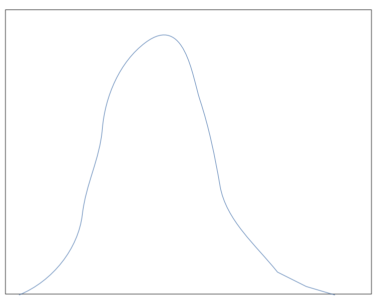
Planck Cosmic Microwave Background European Space Agency Herschel Space Observatory Space Telescope



Observation

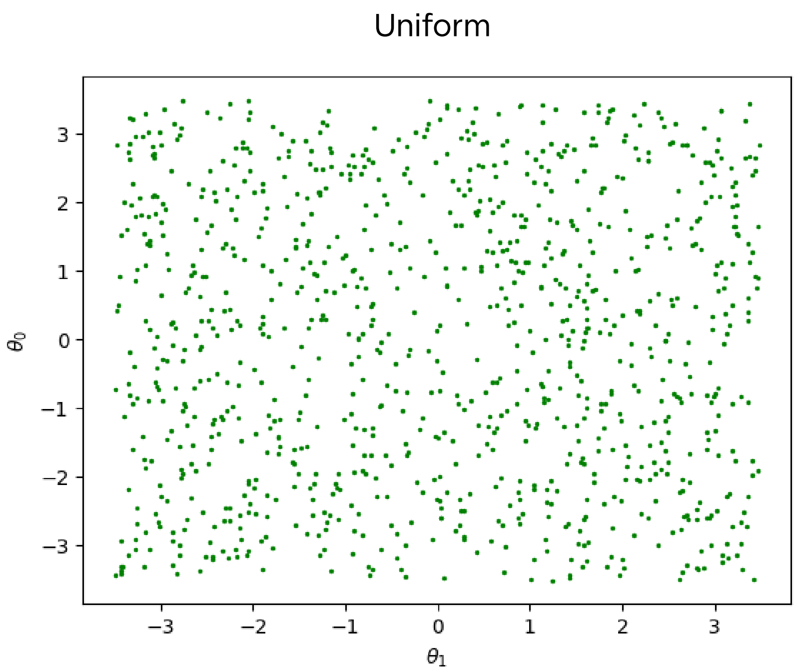
Compute summary statistic from observation

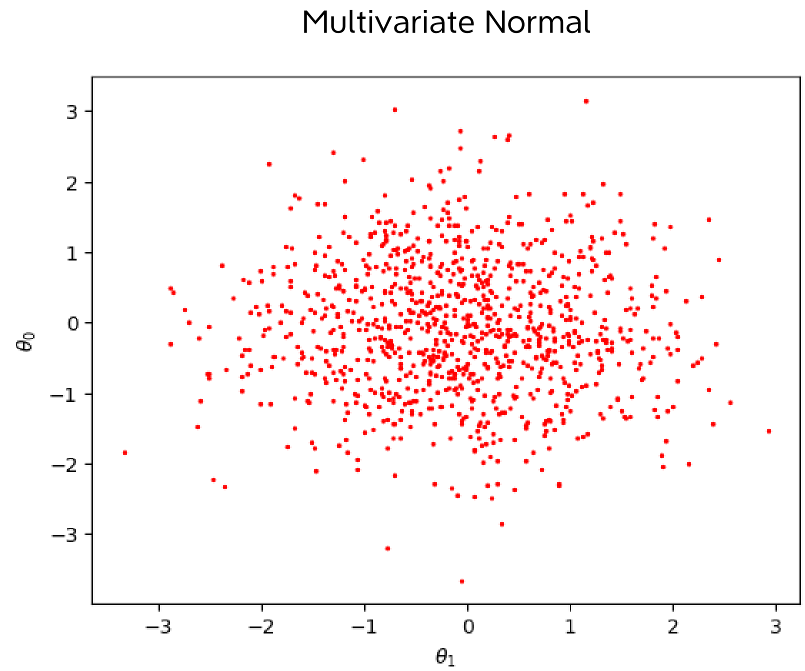
$\mathcal{D}_{obs}$



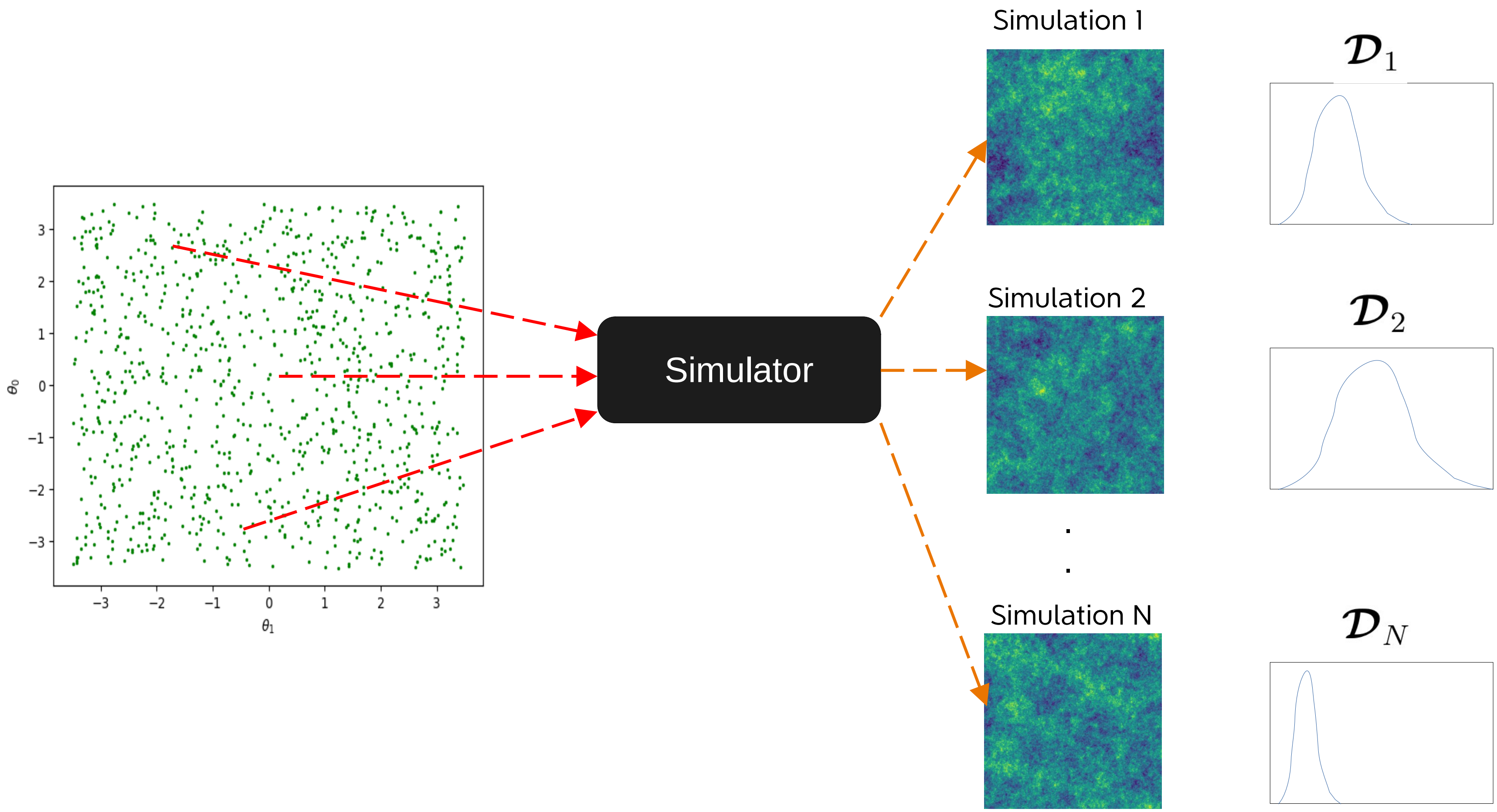
Starting point to SBI

The prior represents the state of belief on the parameters that will be inferred, prior to any influence by the new observed data.

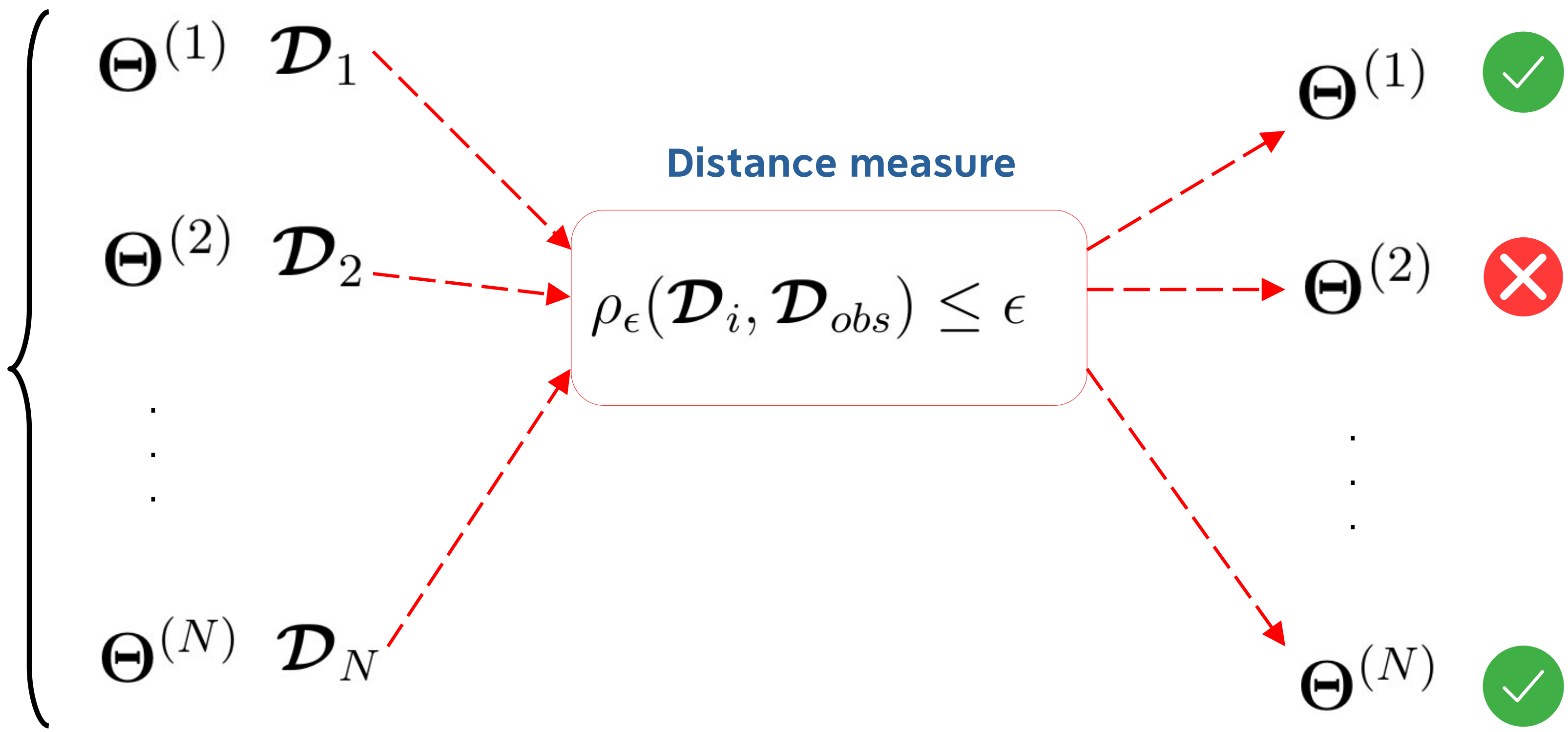




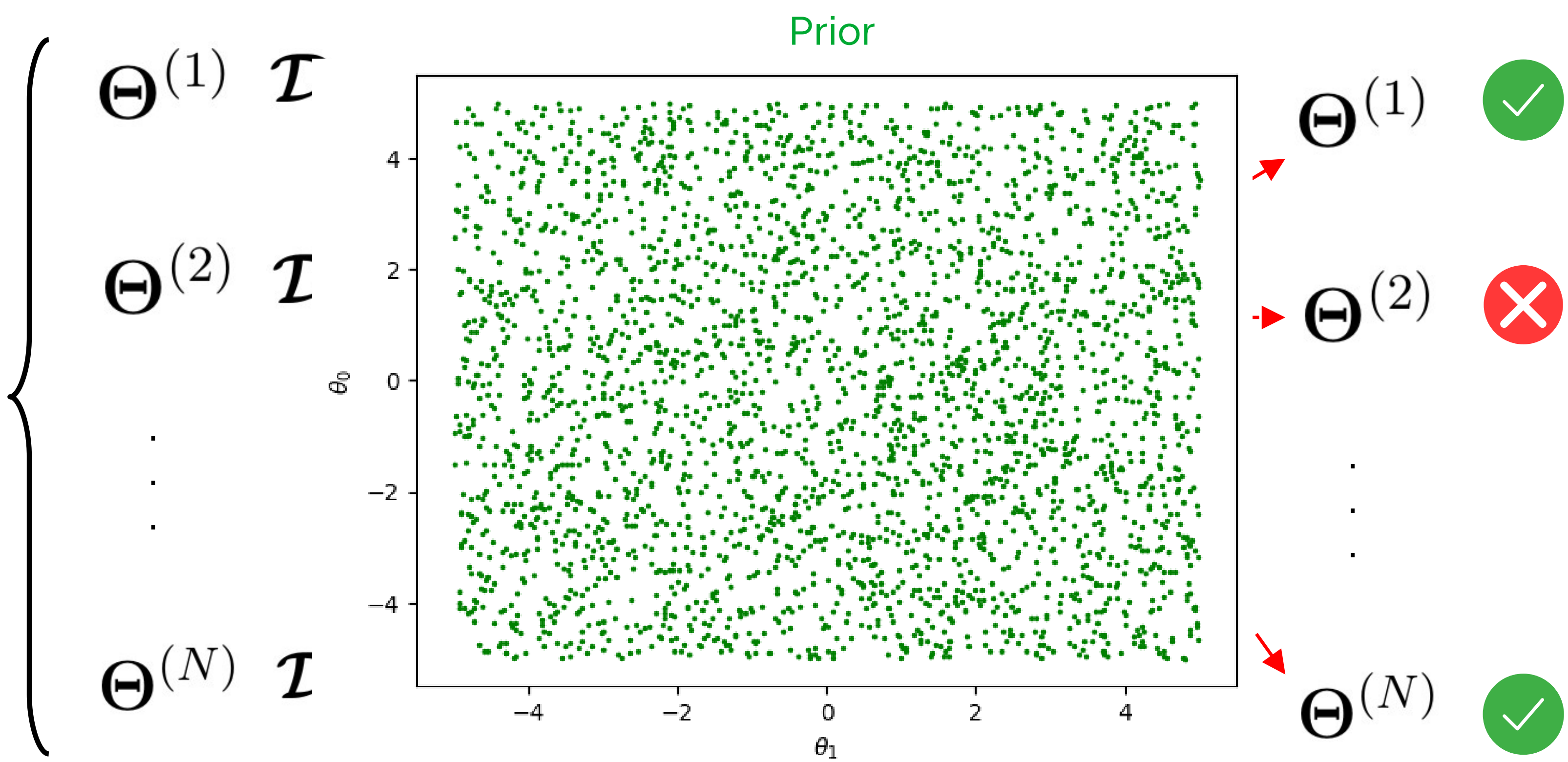
Starting point to SBI



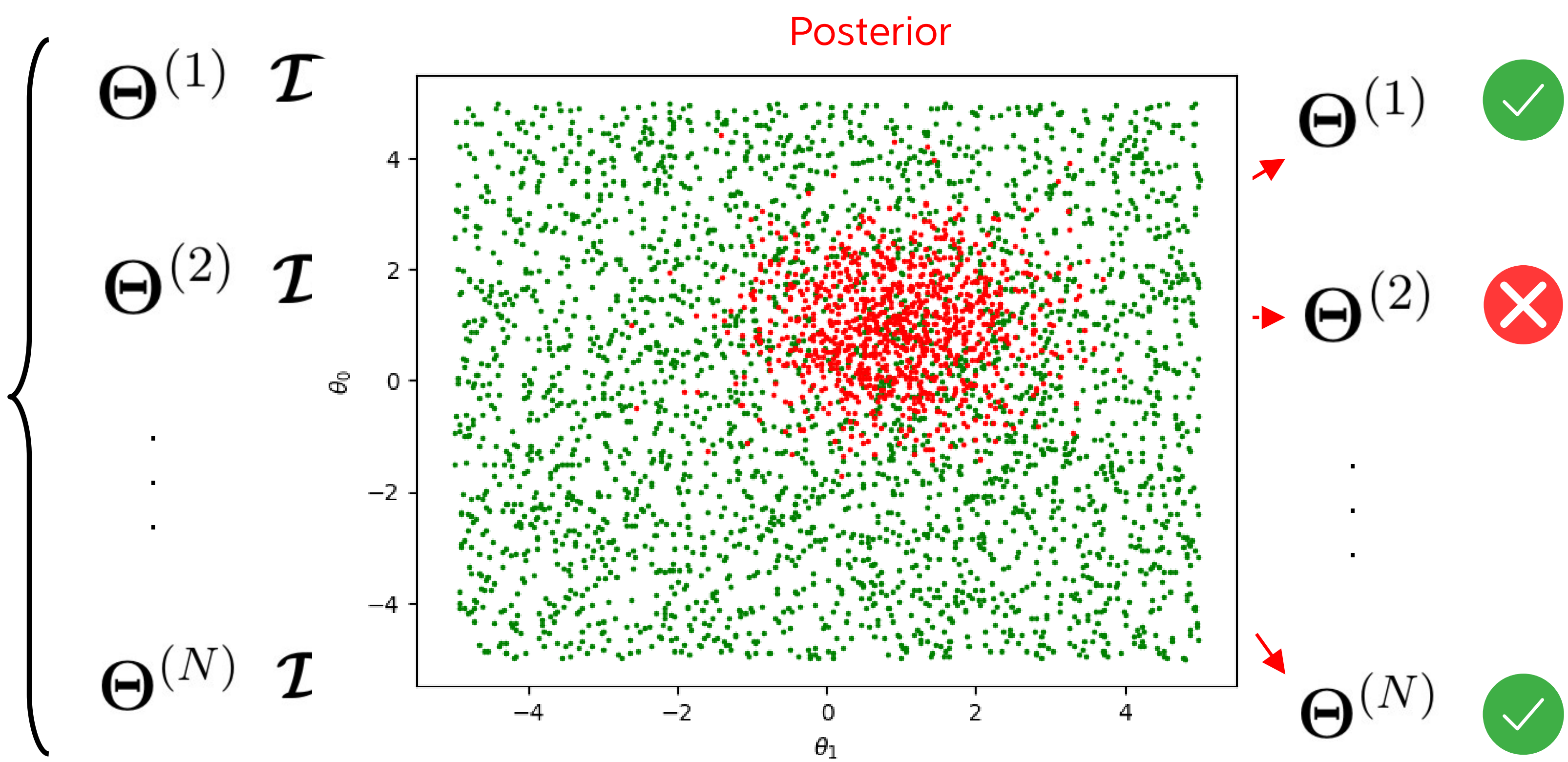
Approximate Bayesian Computation (ABC)



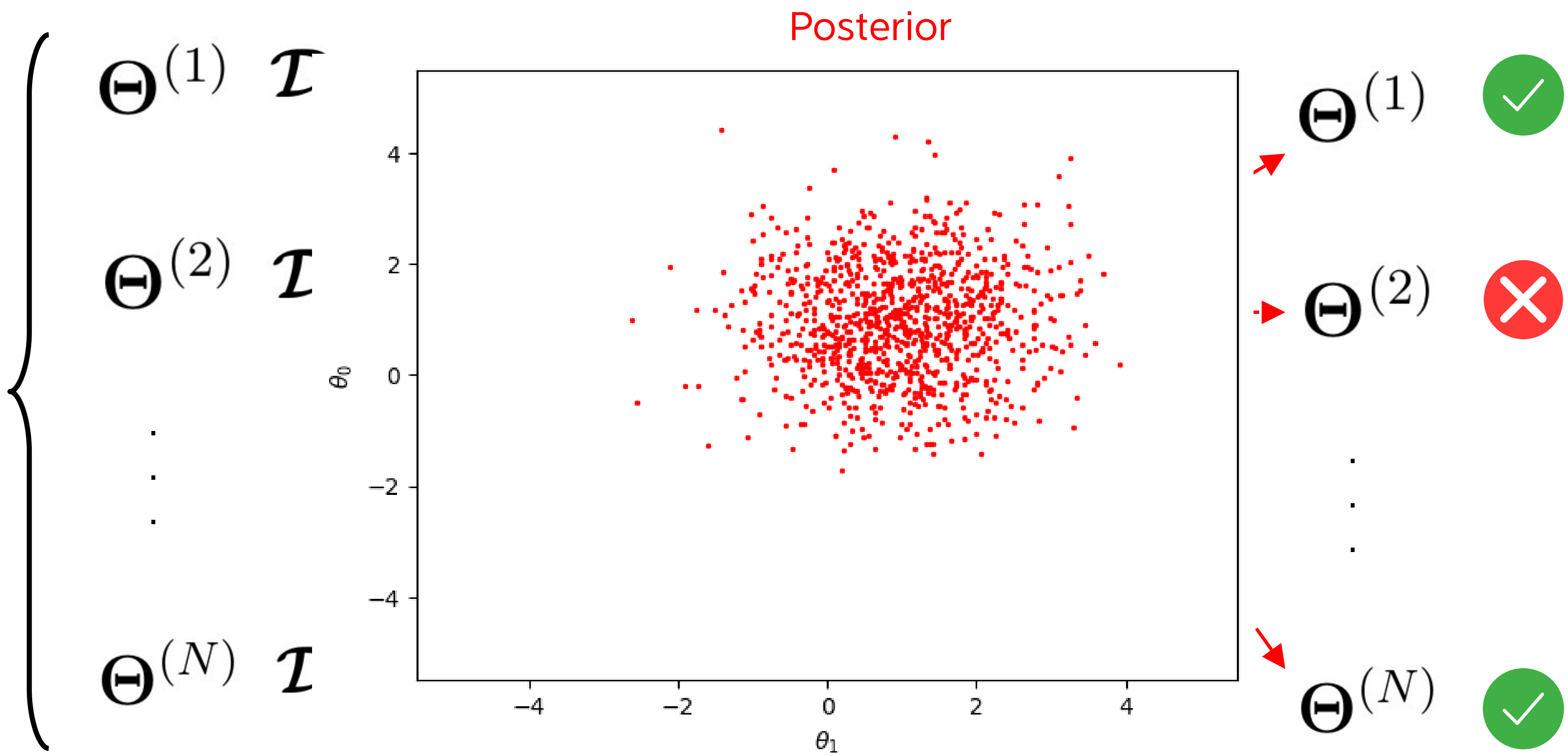
Approximate Bayesian Computation (ABC)



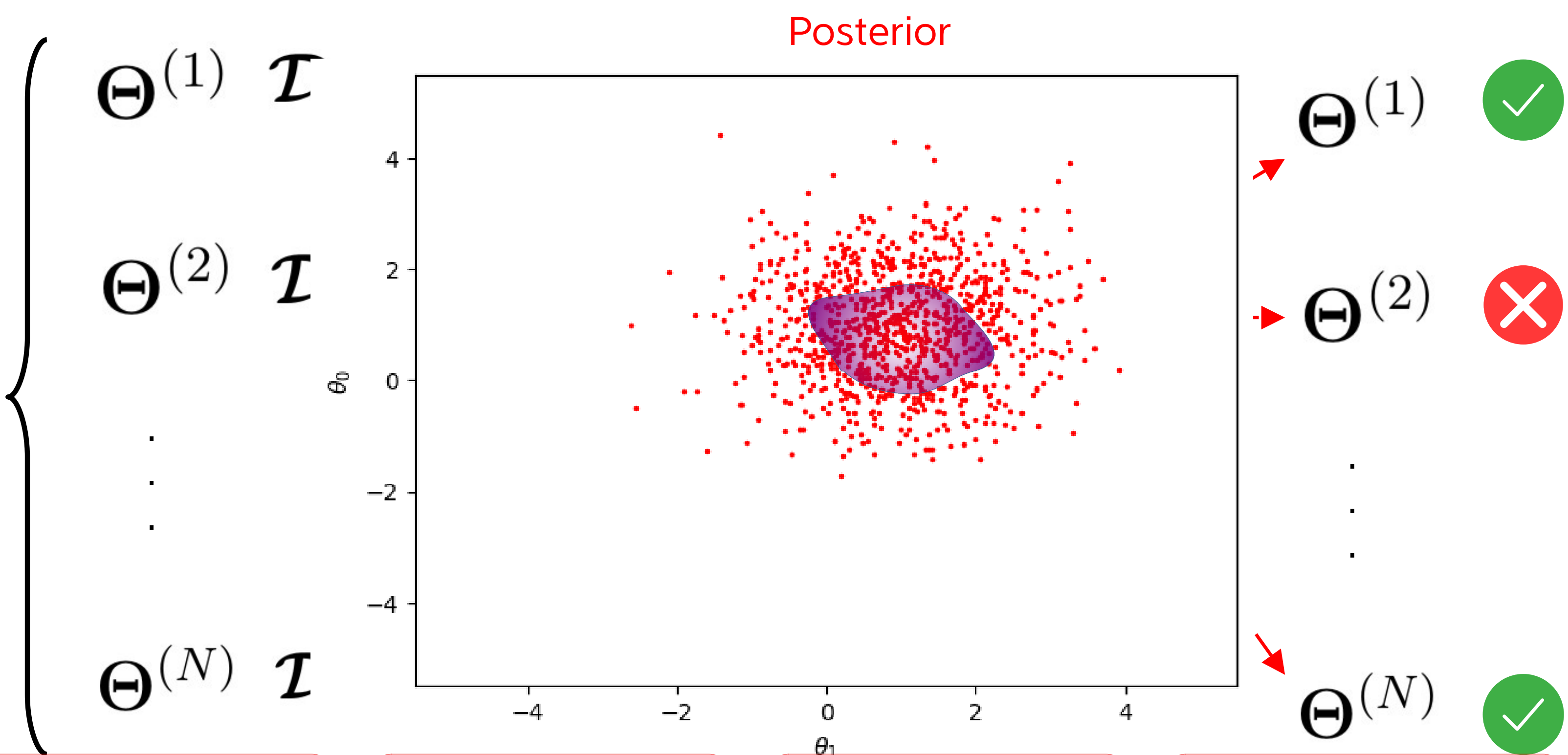
Approximate Bayesian Computation (ABC)



Approximate Bayesian Computation (ABC)



Approximate Bayesian Computation (ABC)



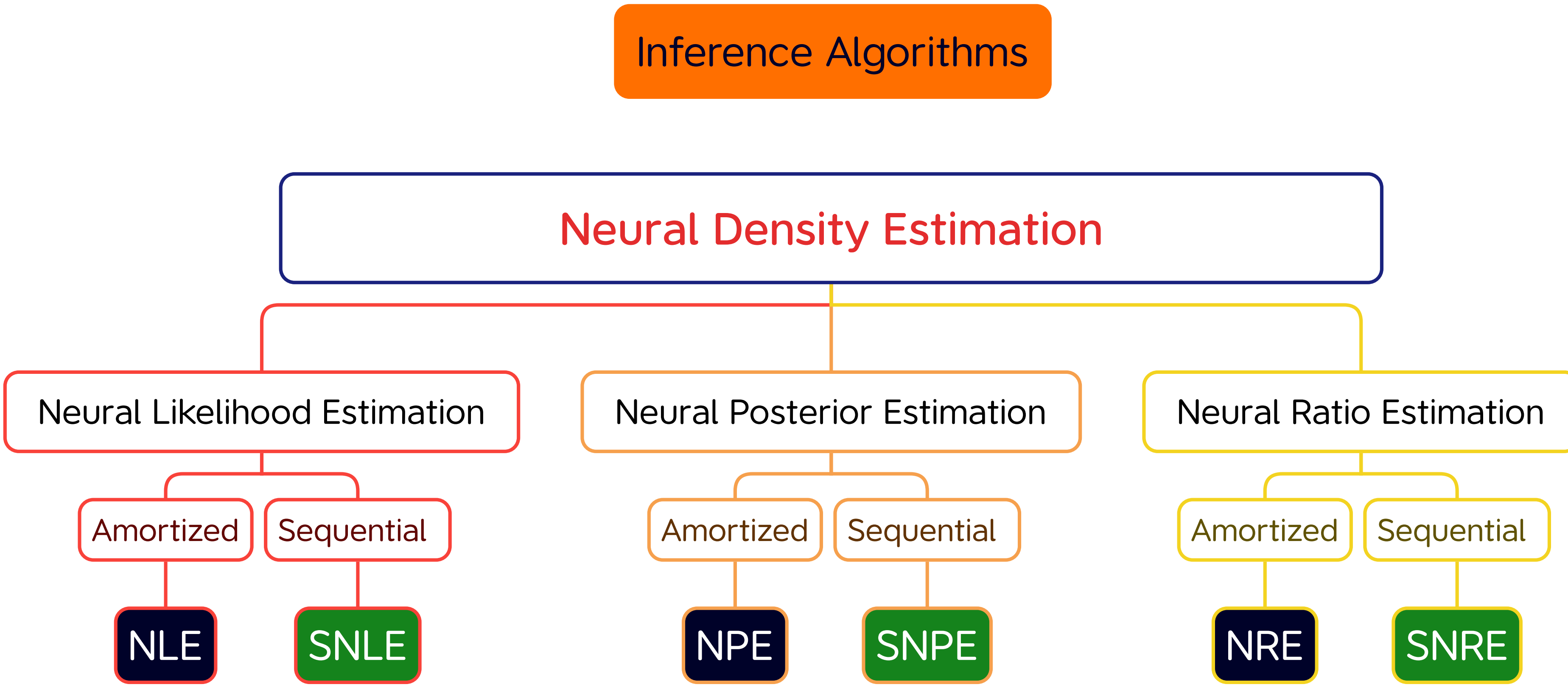
Depends on observation

Depends on epsilon

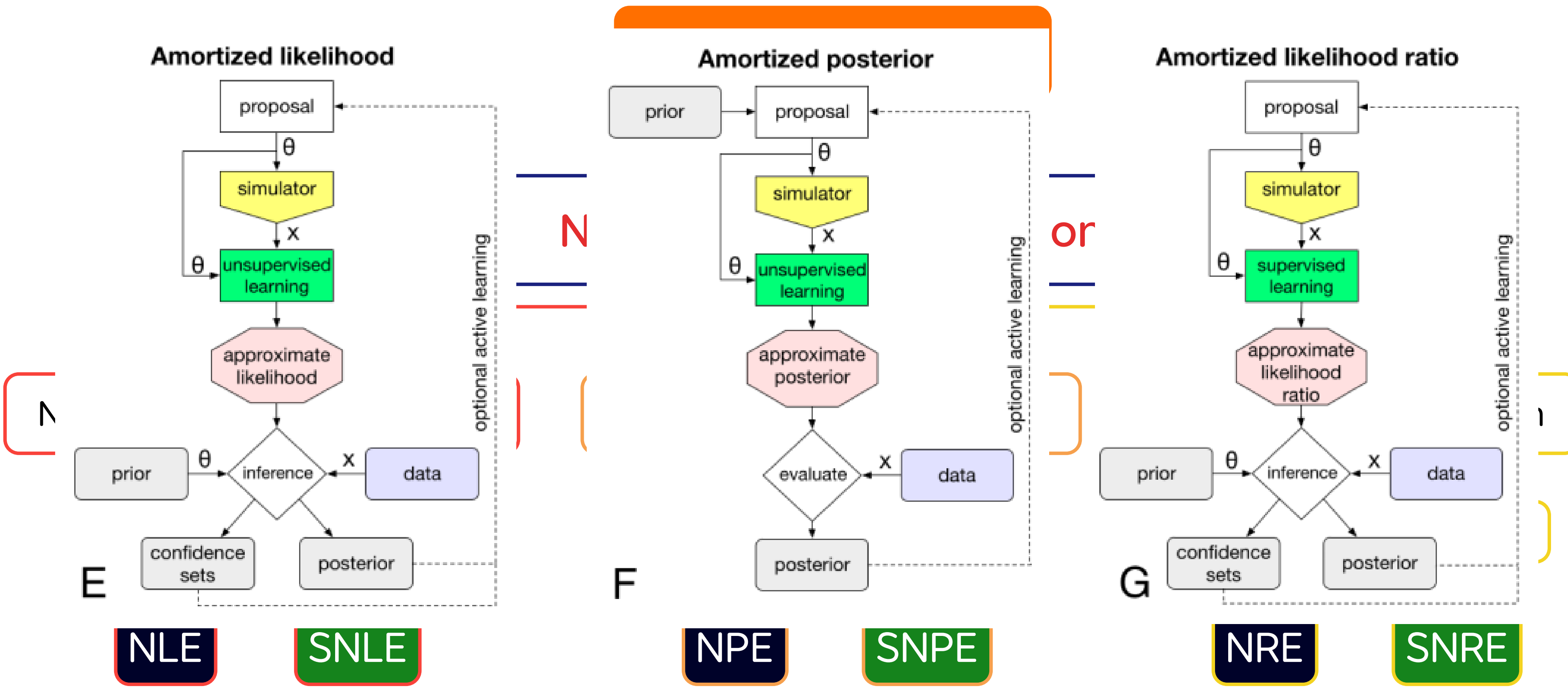
Require unfeasibly many simulations

Inefficient for high-dimensional data

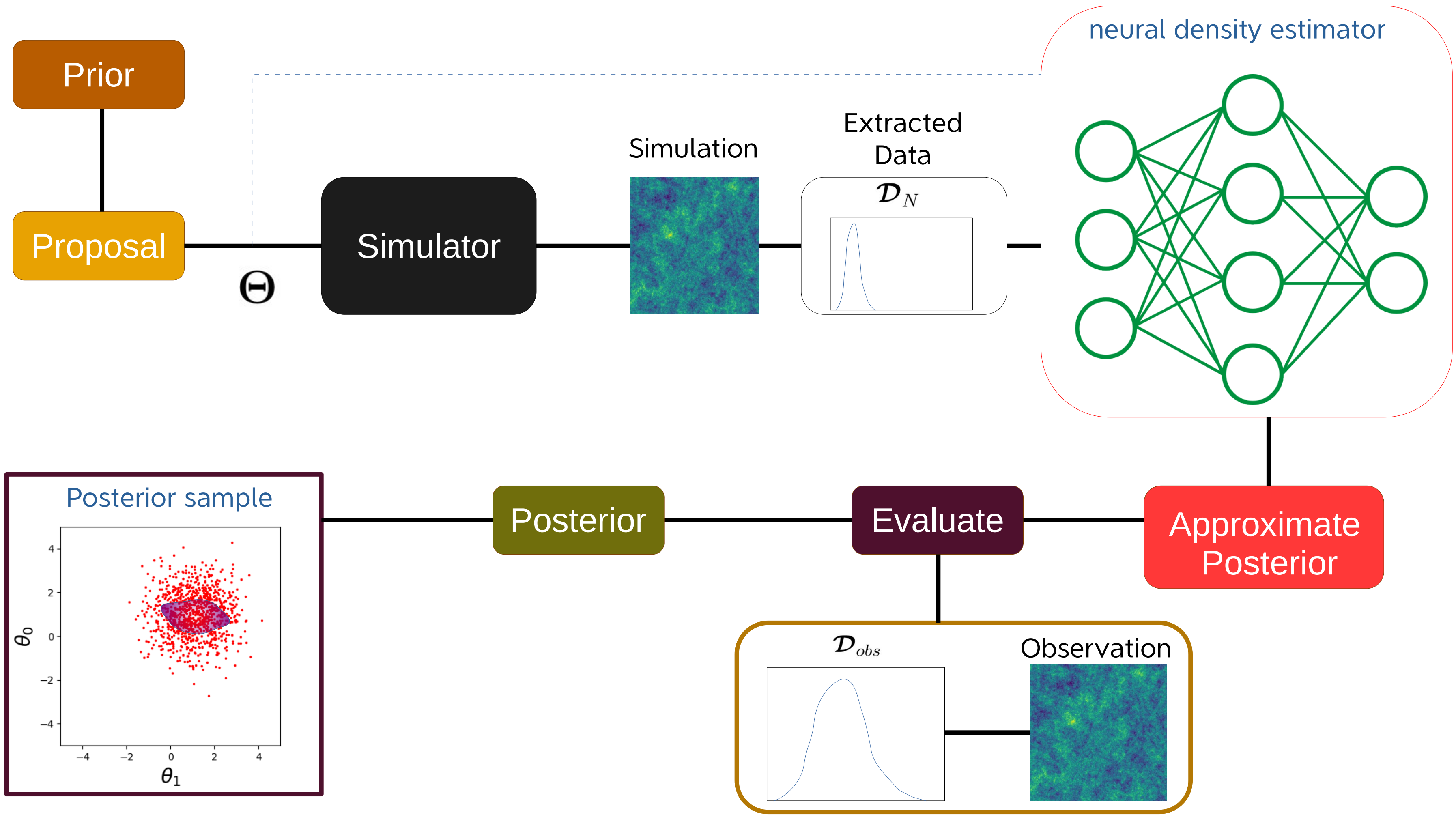
Neural Density Estimation



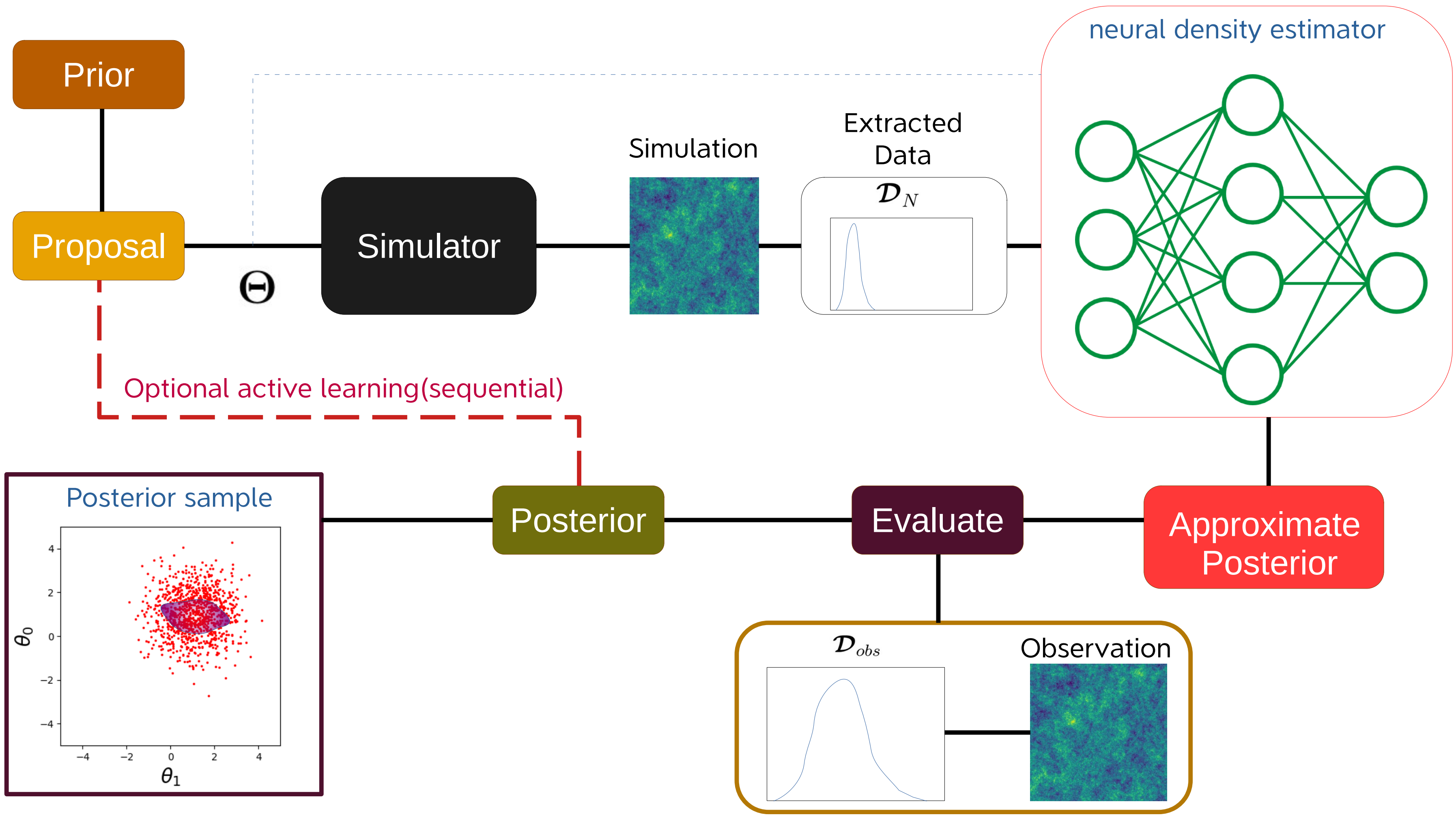
Neural Density Estimation



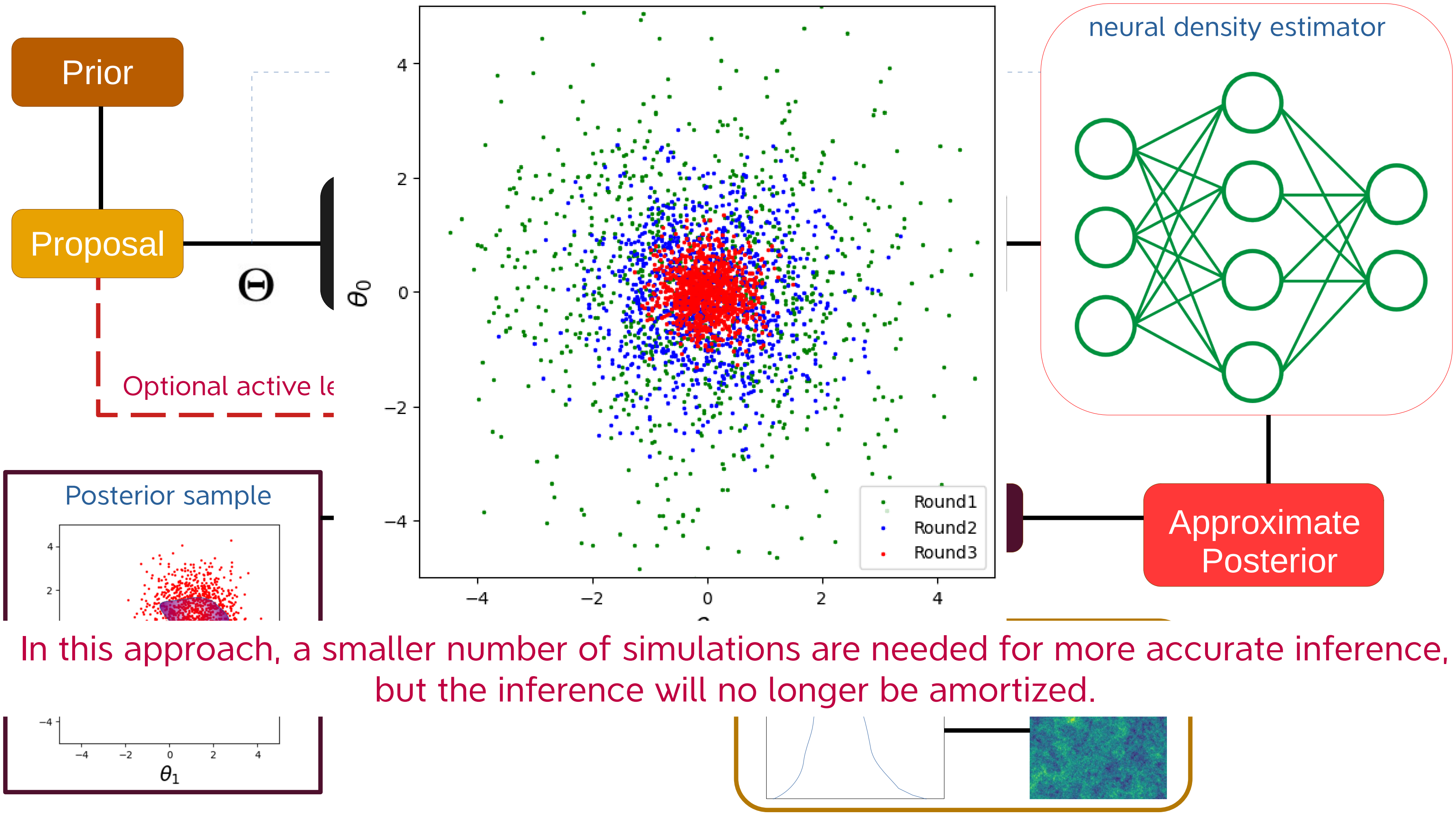
Neural Posterior Estimation(NPE)



Sequential Neural Posterior Estimation(SNPE)

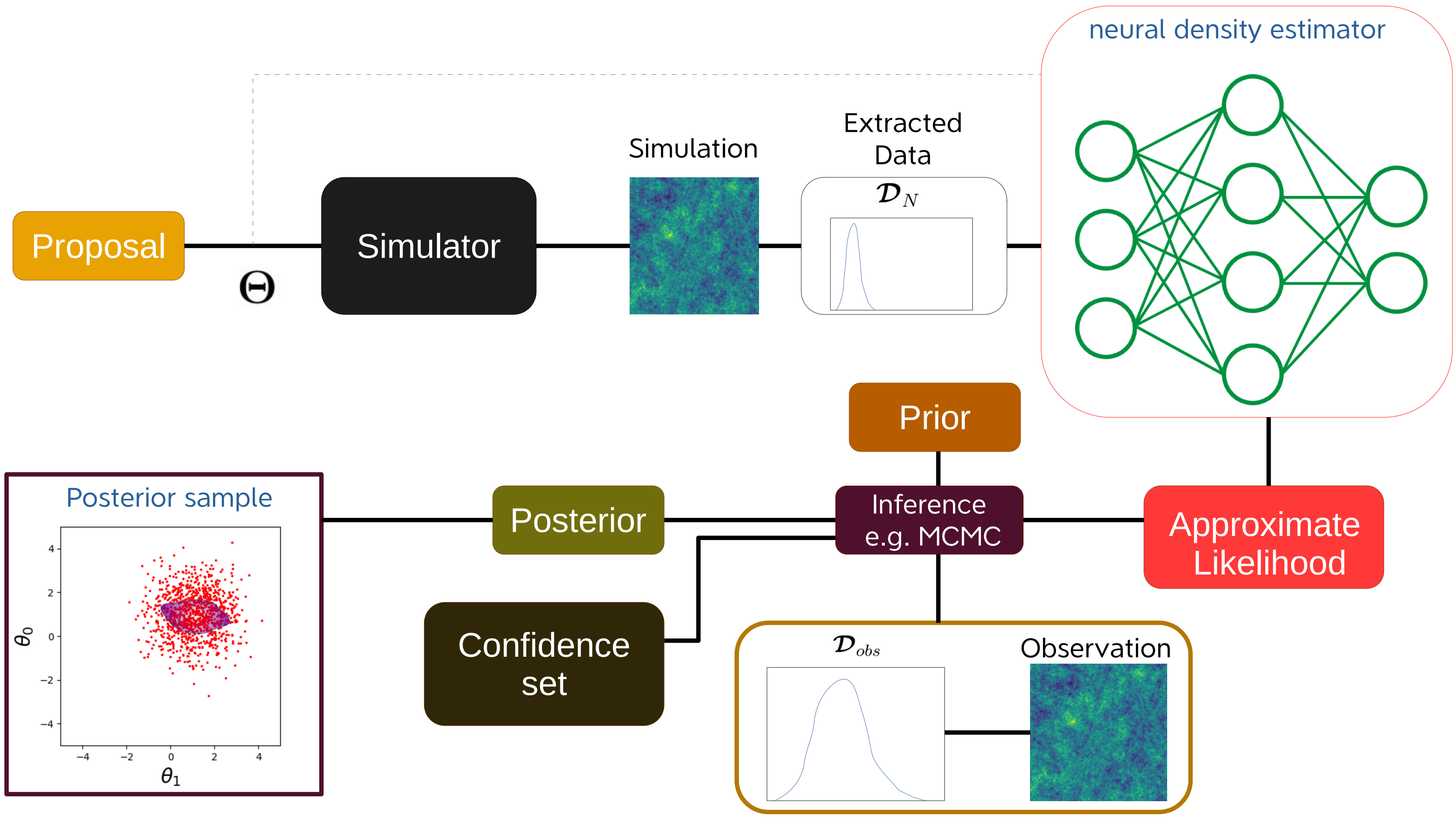


Sequential Neural Posterior Estimation(SNPE)

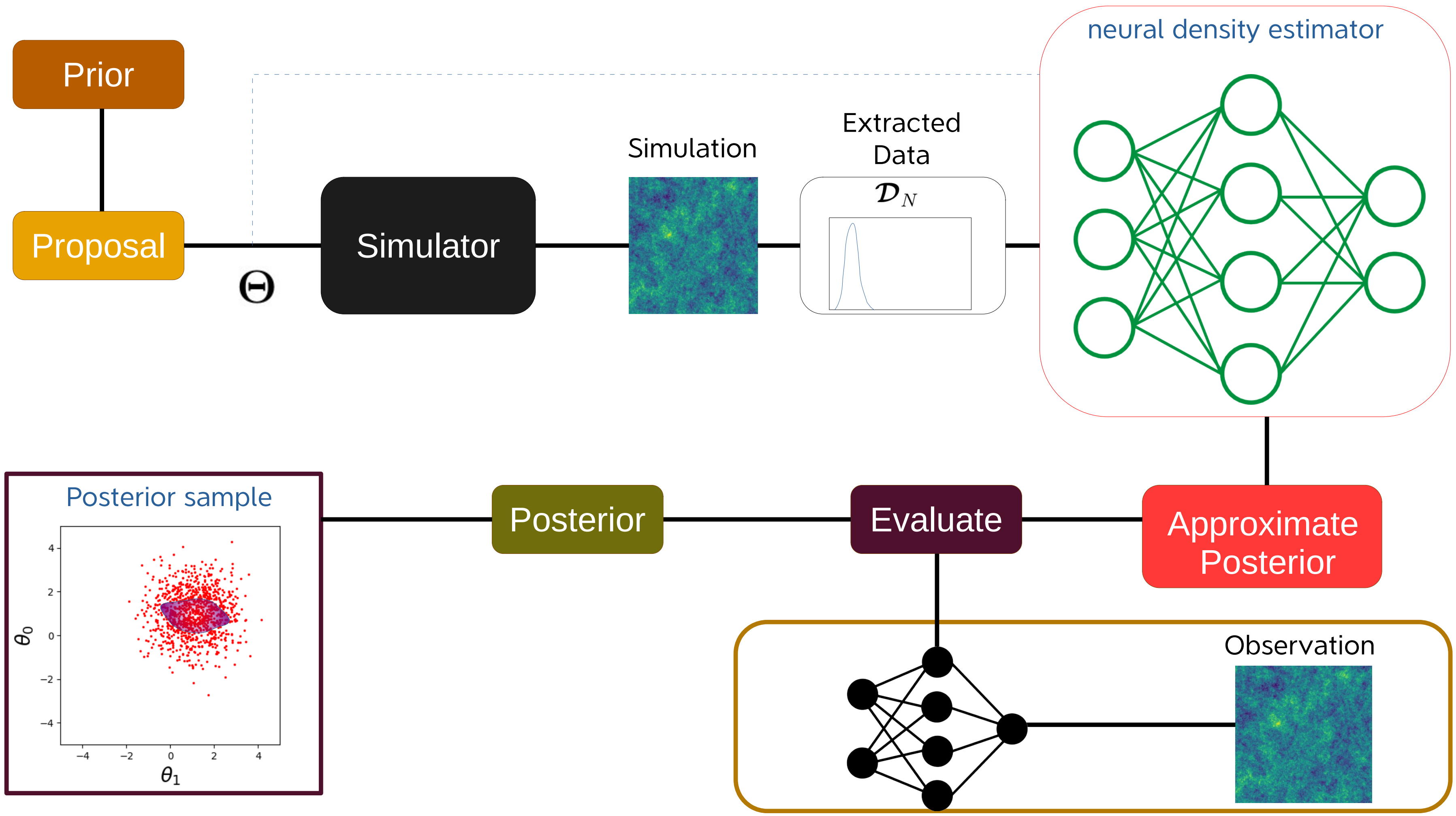


In this approach, a smaller number of simulations are needed for more accurate inference, but the inference will no longer be amortized.

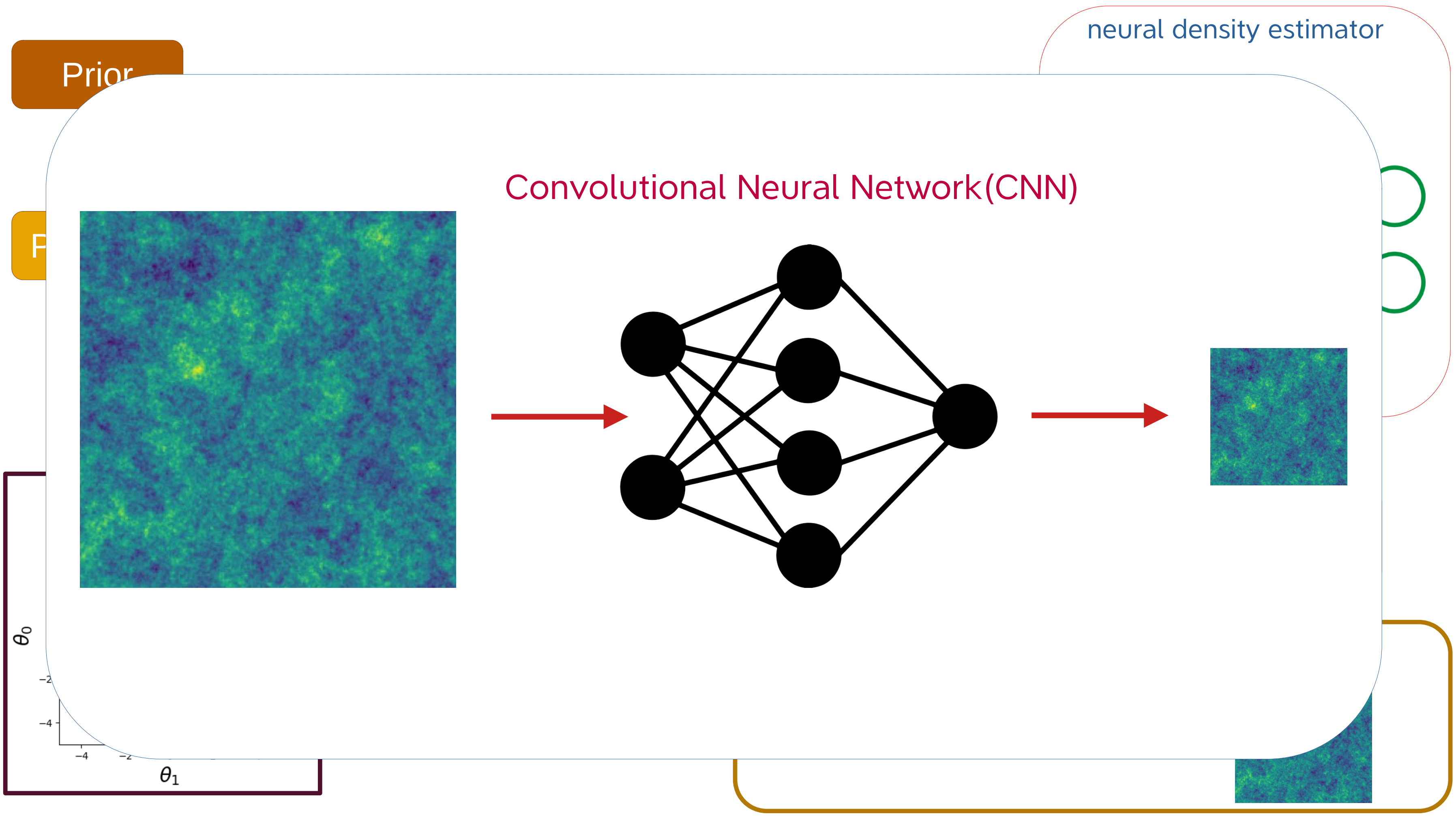
Neural Likelihood Estimation(NLE)



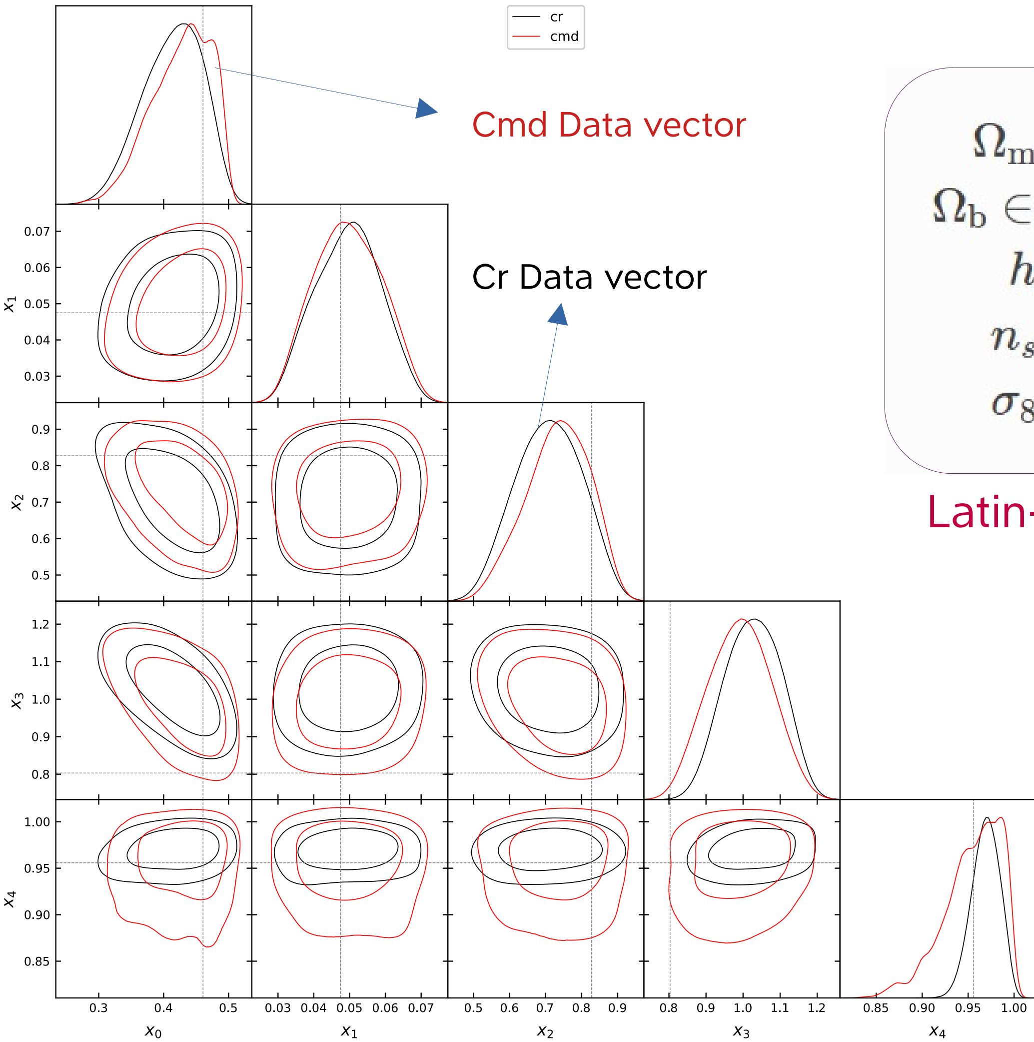
Embedding Network



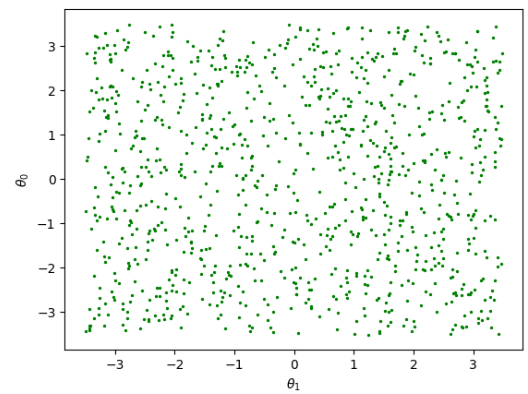
Embedding Network



Preliminary results



$$\Omega_m \in [0.1; 0.5]$$
$$\Omega_b \in [0.03; 0.07]$$
$$h \in [0.5; 0.9]$$
$$n_s \in [0.8; 1.2]$$
$$\sigma_8 \in [0.6; 1.0]$$



Latin-hypercubes

Includes 2000 simulations

The learning process has been done with 500 realizations of Quijote simulation

Using sbi python package

Some of my activities

1. Revision of the first paper: it was finally published in ApJ
2. Submitting the second paper to MNRAS journal : We are currently reviewing the article and replying to the referee
3. Submitting and presenting a paper on weighted morphology in Isfahan Computational Physics Conference
4. Holding a simulation-based inference workshop on the sidelines of the Isfahan Conference
5. Thesis writing: Progress $\sim$ 70 %

Future works

1. Completing the thesis writing
2. Preparing to dissertation defense
3. Developing the simulation-based approach to obtain results from galaxy catalogs and real data
4. writing a paper for Pakistan Conference
5. Completing the results related to Cosmic anisotropy (dipole)

Thank you for your attention!