بسم الله الرحمن الرحیم

Department of Physics

Morphological properties of Random fields

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The main goals:

1) **Morphology**
2) **Assessment of (Phase transition, Anisotropy, Non-Gaussianity, …);**
   Case studies: PTA, RSD, Density fields, Rough surface, …
3) **Morphological based analysis**
   - To put pristine constraints on the Physical parameters;
   - To use for early warning signal for phase transition;

For more details see:

[1] Mohammad Hossein Jalali and S. M. S. Movahed, "Probing the anisotropy and non-Gaussianity in redshift space through the derivative of excursion set moments", arXiv:2308.03086, submitted to The Astrophysical Journal.

[2] H. Masoomy, S. Tajik, and S. M. S. Movahed, "Homology groups of embedded fractional Brownian motion." *Physical Review E* 106.6 (2022): 064115.

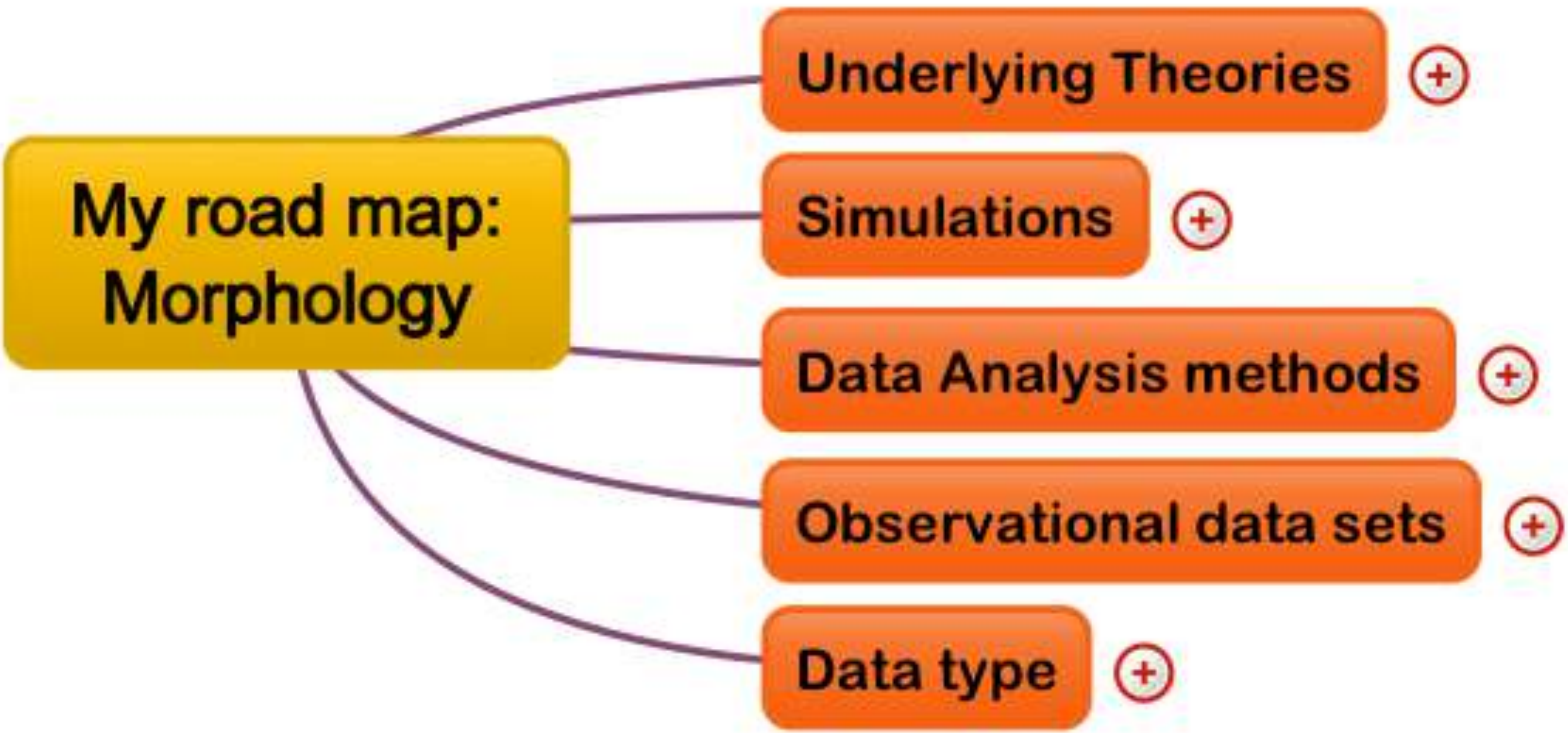
[3] H. Masoomy, B. Askari, M. N. Najafi and S. M. S. Movahed, “Persistent homology of fractional Gaussian noise”, *Physical Review E*, *104*.3 (2021):034116.

Outline

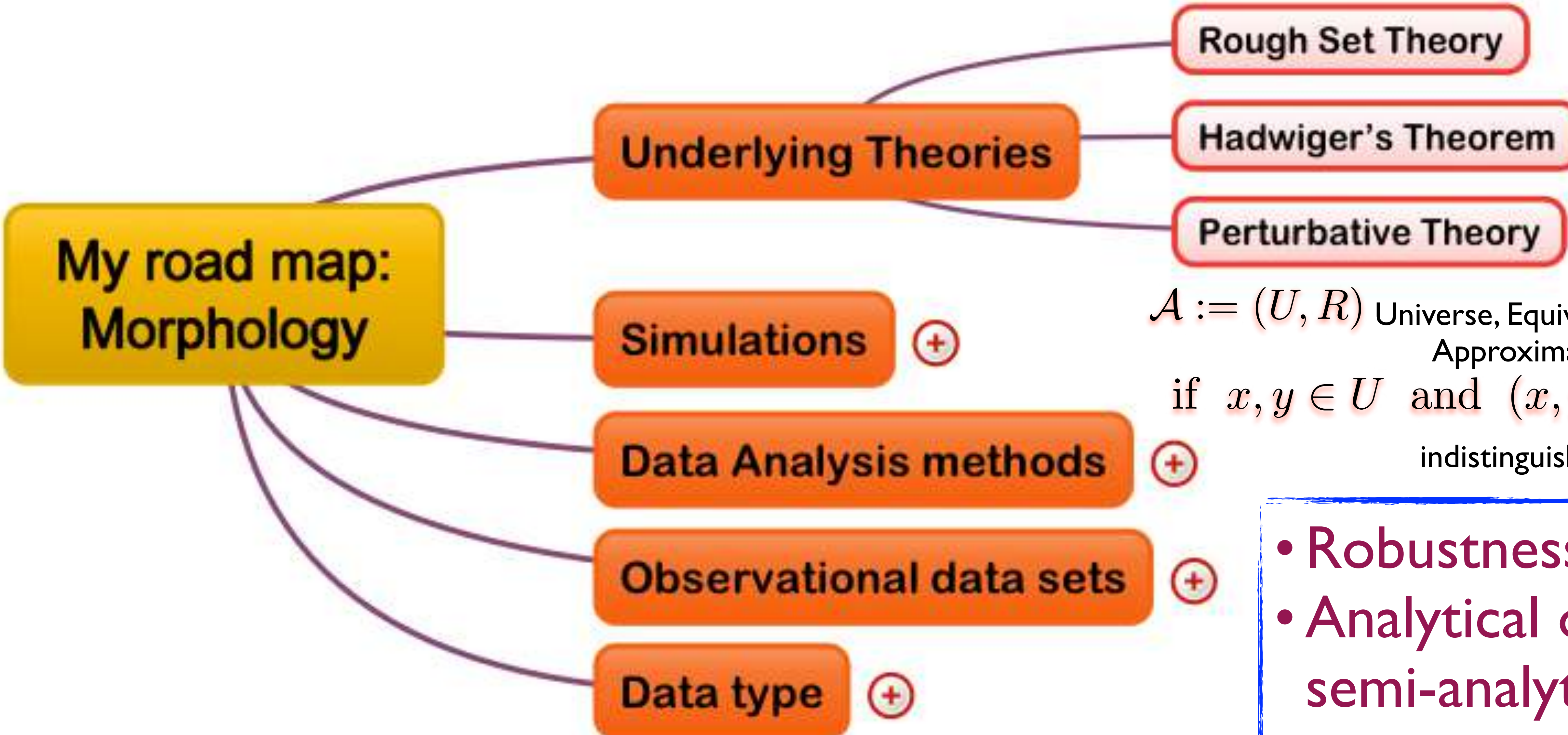
1) Morphological approaches to examine Stochastic fields
   - Geometrical and Topological measures
   - Probabilistic and Perturbation frameworks

2) Redshift Space (Anisotropy and Non-Gaussianity)

A glance at the roadmap (1)



A glance at the roadmap (2)



$\mathcal{A} := (U, R)$ Universe, Equivalence relation

Approximation space

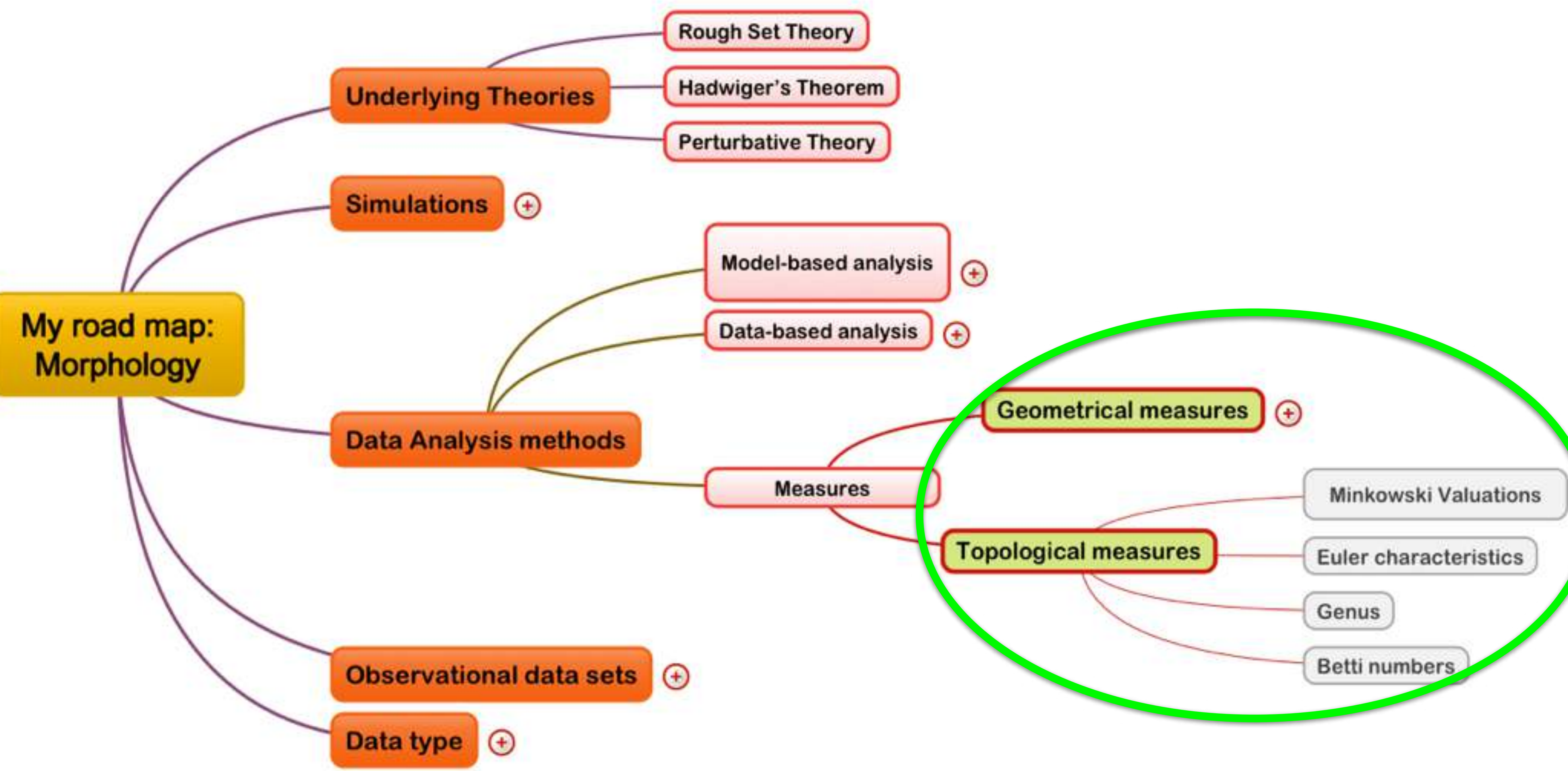
if $x, y \in U$ and $(x, y) \in R$

indistinguishable in A.S.

- Robustness
- Analytical or semi-analytical measures

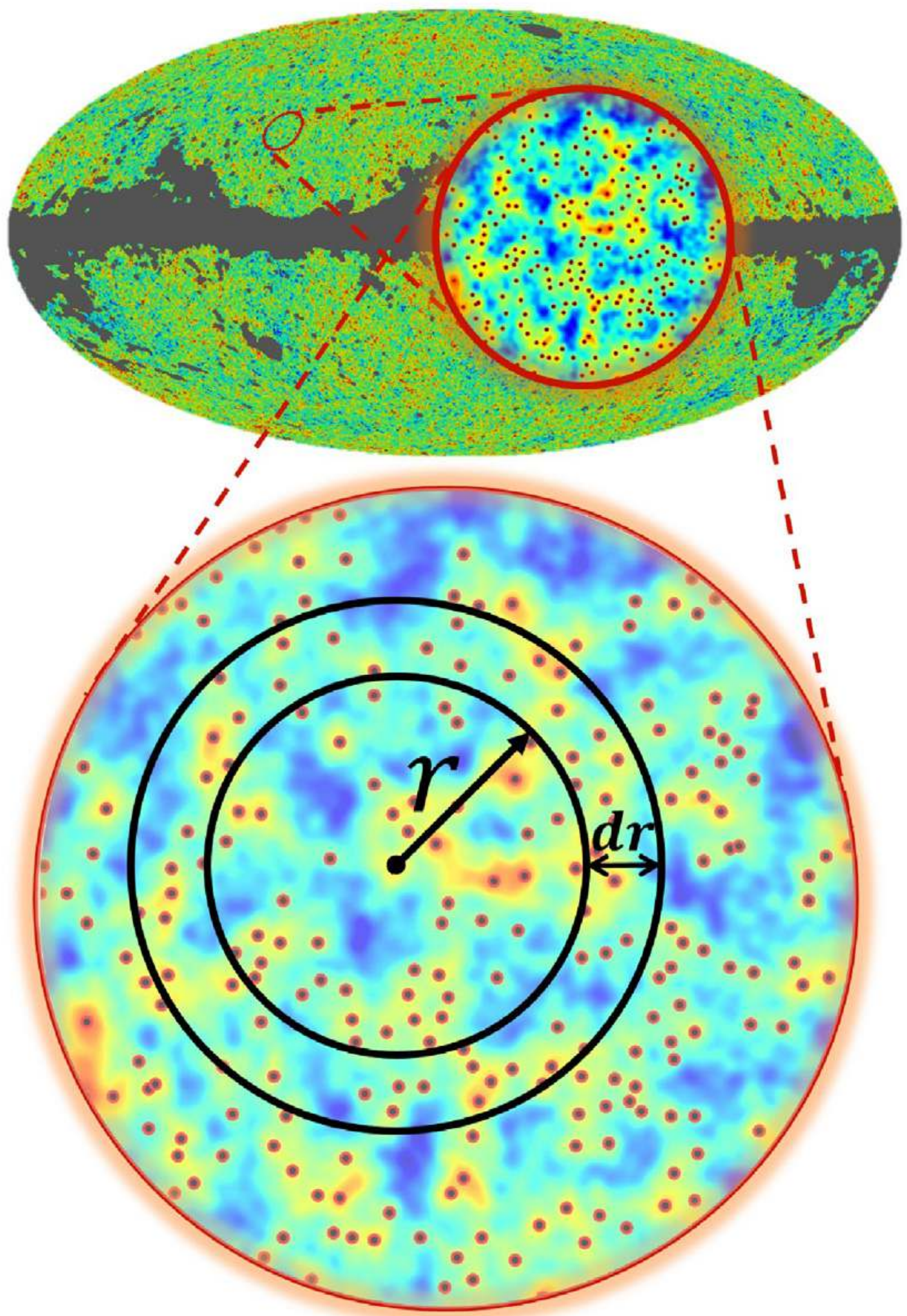
- Motion invariance $M(tB) = M(B) \quad \text{for any} \quad t \in T\,;B \in \mathcal{R}$
- Additivity $M(B_1 \cup B_2) = M(B_1) + M(B_2) - M(B_1 \cap B_2) \quad \text{for any} \quad B_1, B_2 \in \mathcal{R}$
- Conditional Continuity $M(K_i) \to M(K) \quad K_i \to K \quad \text{for} \quad K_i, K \in \mathcal{K}$

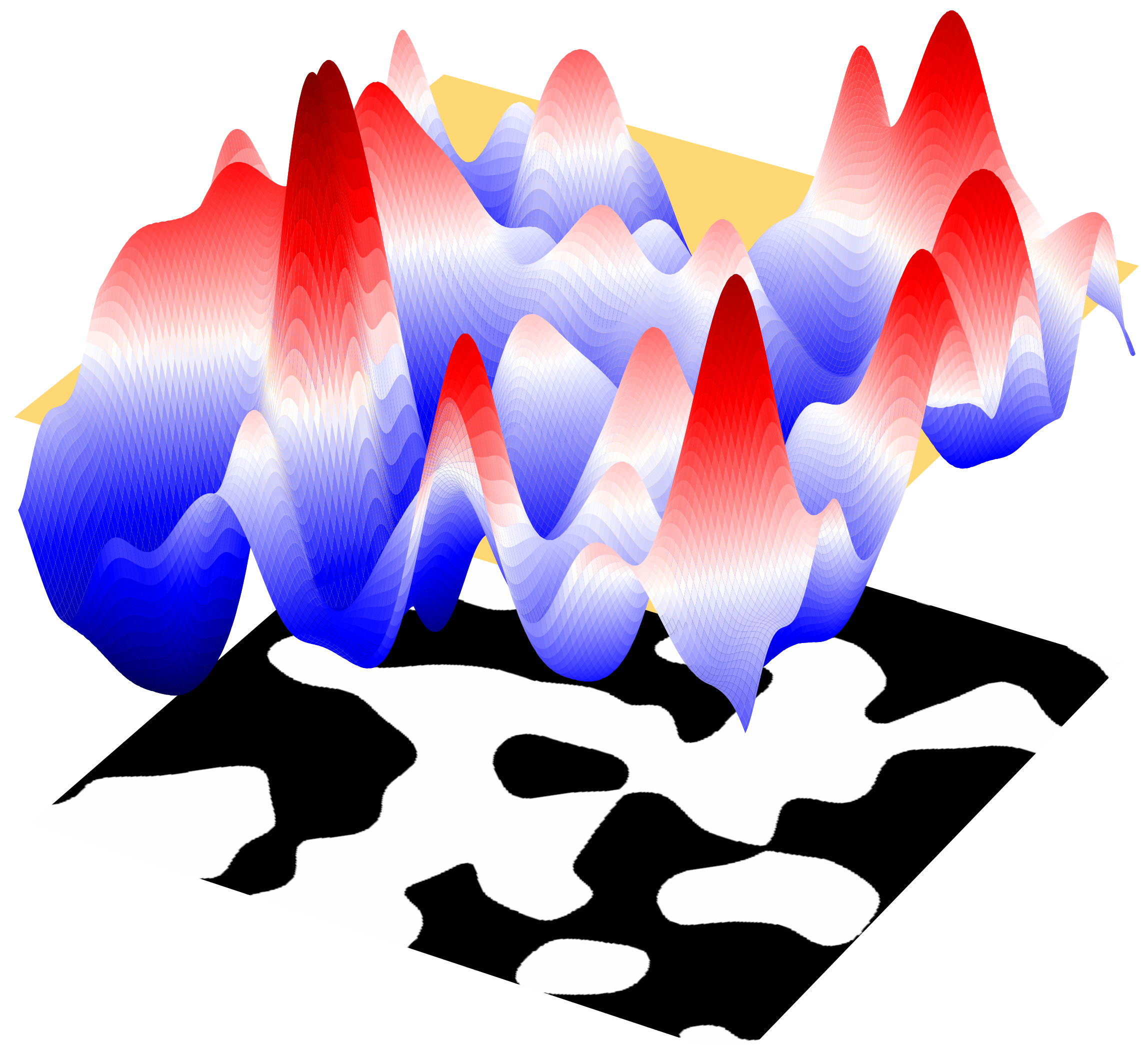
A glance at the roadmap (3)



1) Critical Sets

General features





$$\mathcal{A}_\vartheta(\mathcal{F}) \equiv \{X|\mathcal{F}(X) \geq \vartheta\}$$

2) Excursion Sets

Bardeen, James M., et al. , Astrophysical Journal, vol. 304, May 1, 1986, p. 15-61.
Vafaei Sadr, A., and SMSM, NMRAS (2021)
SMSM, B. Javanmardi and R. K. Sheth, MNRAS, (2013)

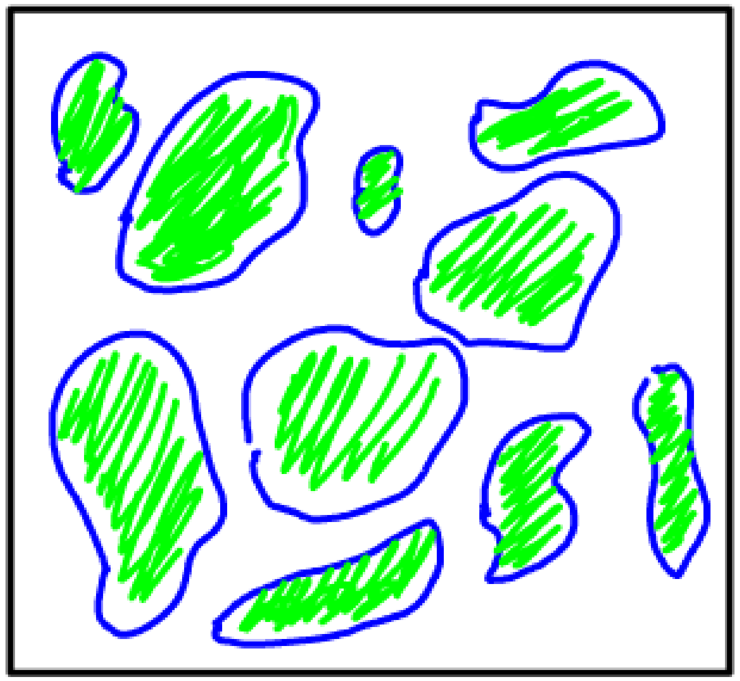
Mohammad Hossein Jalali and SMSM, arXiv:2308.03086,

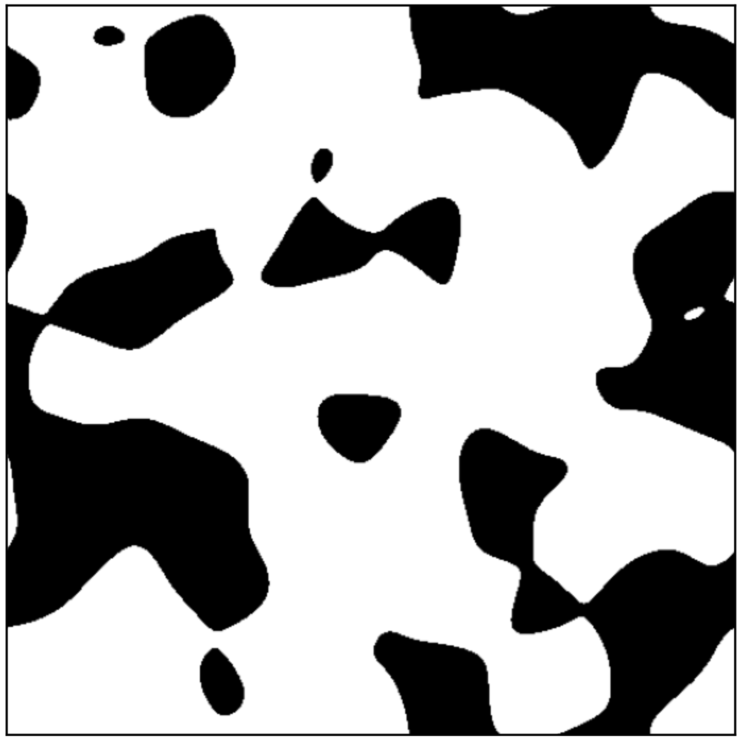
Minkowski Functionals (scalar)

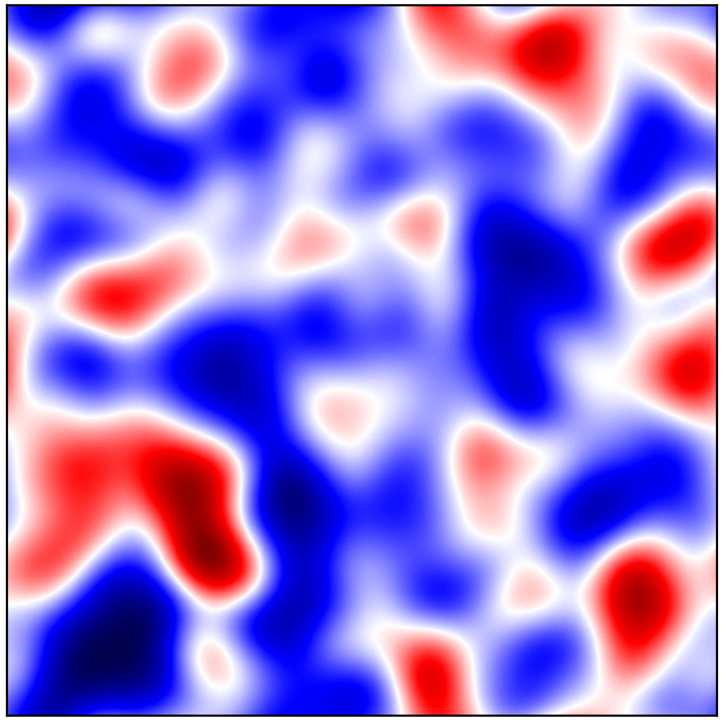
2D field

- $V_0(\nu) = \int_Q dA = \langle \Theta(\alpha - \nu) \rangle$
- $V_1(\nu) = \frac{1}{4} \int_{\partial Q} dl = \frac{\pi}{8} \langle \delta_D(\alpha - \nu) |\eta_1| \rangle \sim N_1(\nu)$

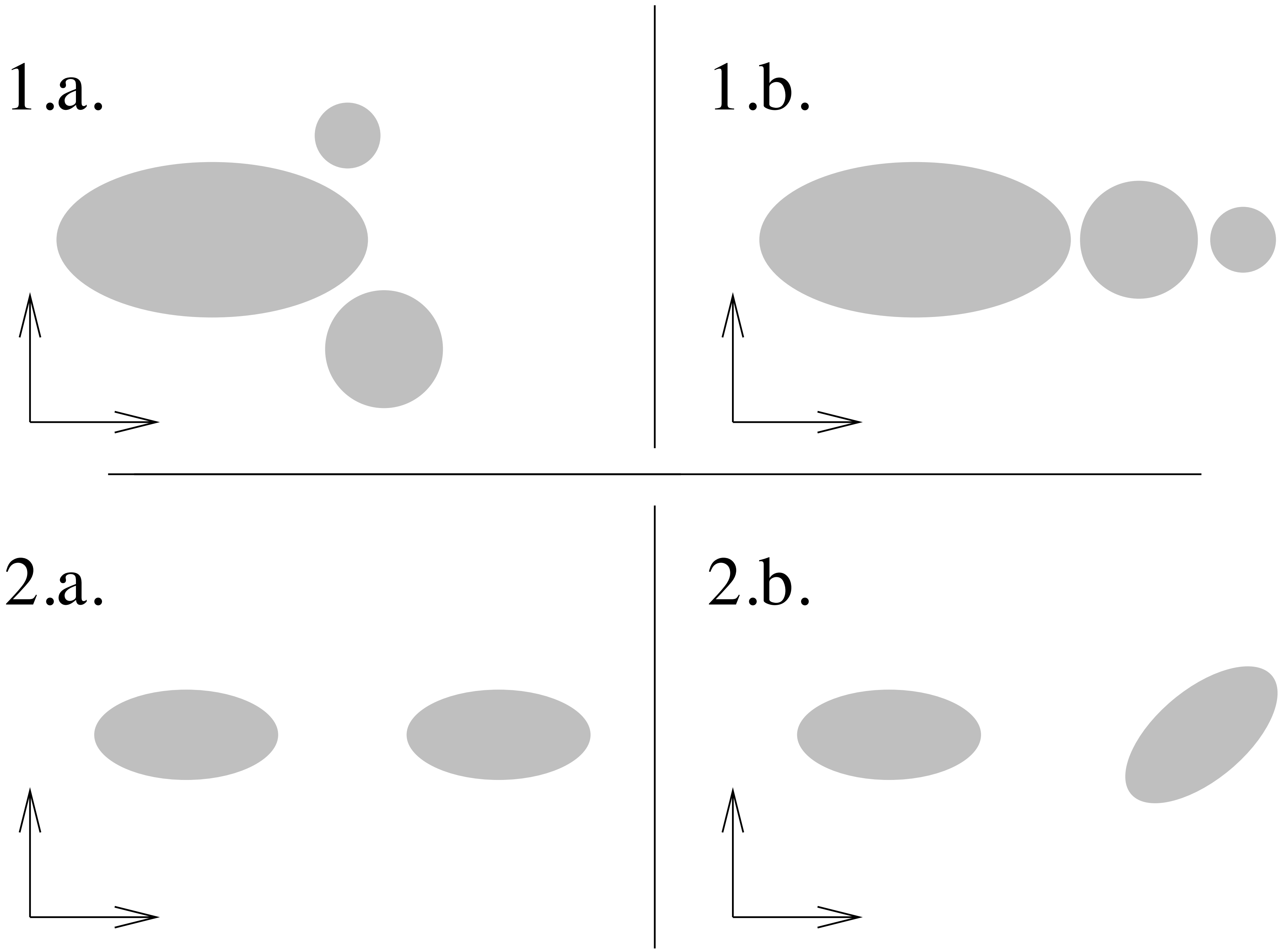
$$V_2(\nu) = \frac{1}{2\pi} \int_{\partial Q} \kappa \, dl = -\frac{1}{2} \langle \delta_D(\alpha - \nu) \delta_D(\eta_1) |\eta_2| \zeta_{11} \rangle$$







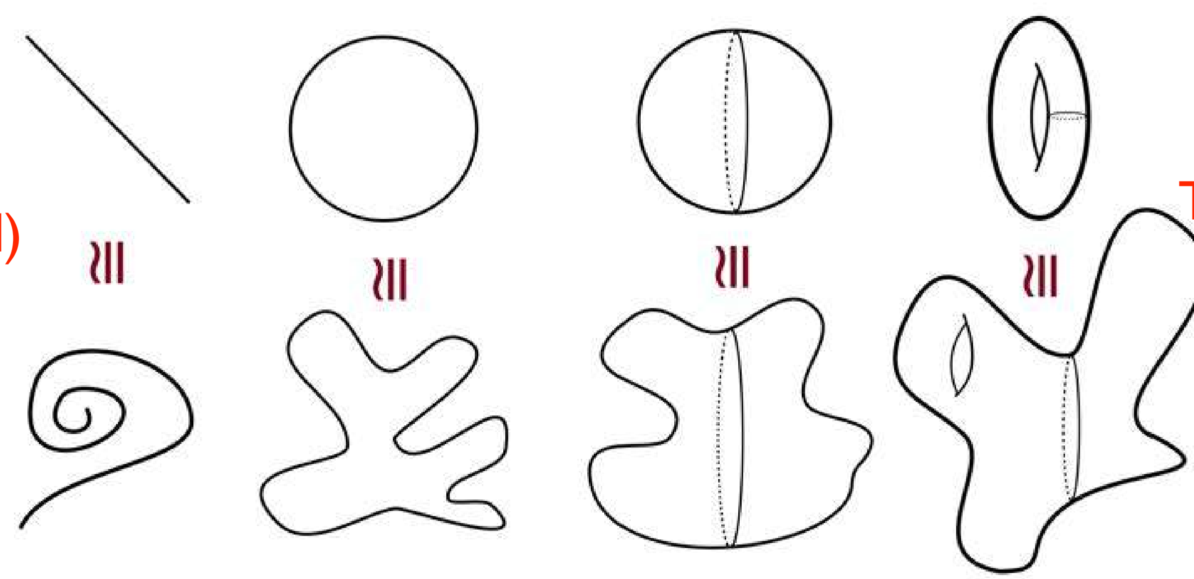
Generalization of Minkowski Functionals
Minkowski Valuations (MVs)



Beisbart, Claus, et al. "Vector-and tensor-valued descriptors for spatial patterns." Morphology of Condensed Matter: Physics and Geometry of Spatially Complex Systems (2002): 238-260.

Topology vs. Geometry

Topological equivalence:
Homeomorphism
(Elastic motion:
stretch, squeeze, twist, bend)

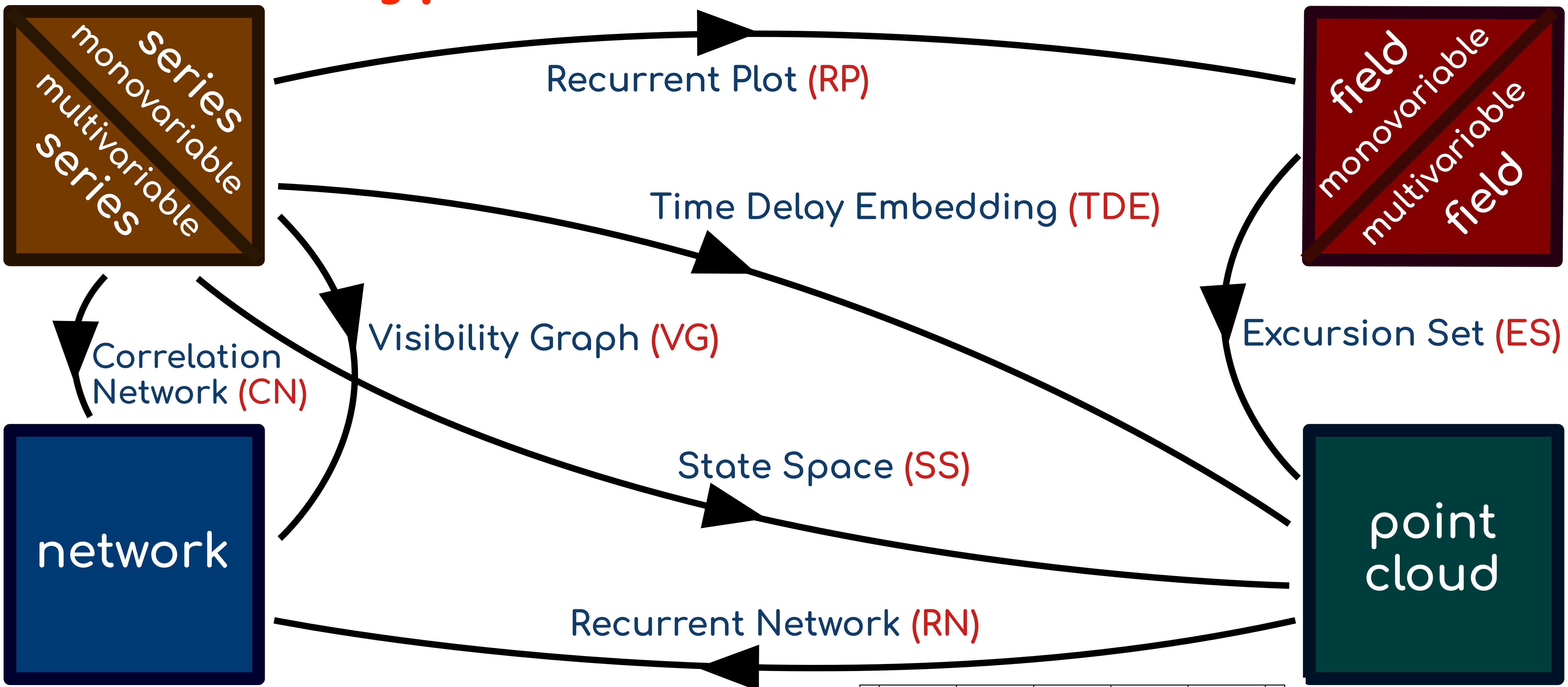


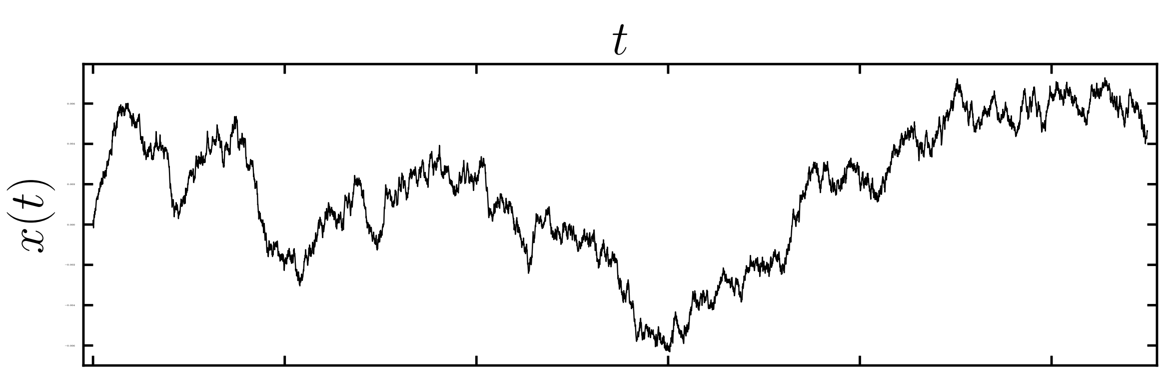
Geometrical equivalence:
congruence (Rigid motion:
Transformation, Rotation, Reflection)

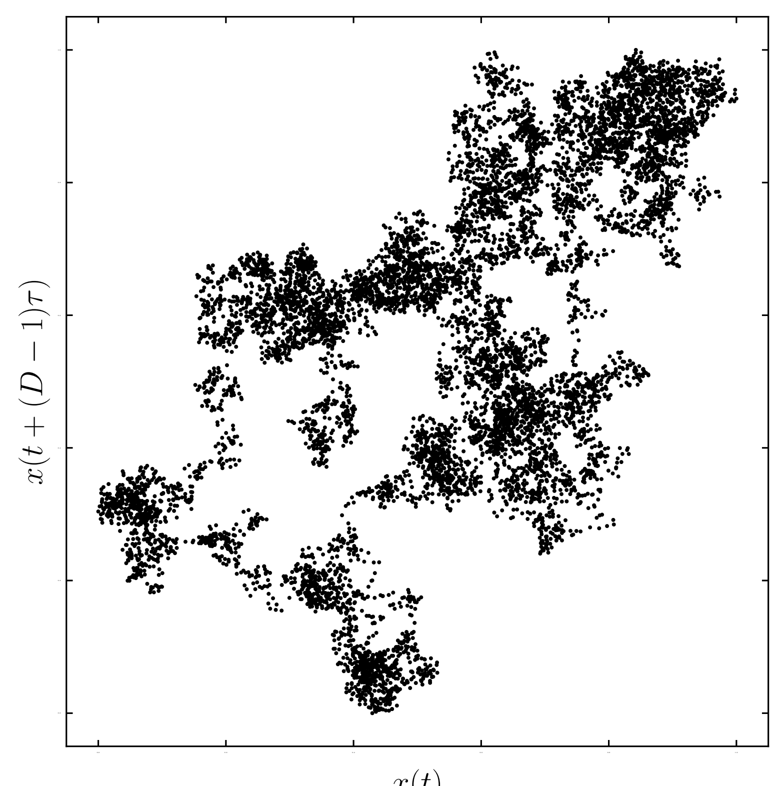
Any congruent spaces are topologically homeomorphic, but its inverse is not necessarily true

Arnold, Bradford Henry. Intuitive concepts in elementary topology. Courier Corporation, 2011.

Data types and Conversion methods







credit: Hossein Maasoumi, M.Sc. Thesis

To know more visit:
http://ccg.sbu.ac.ir/tdaw/

Probabilistic frameworks and Theoretical approach

One-point statistics

$$\langle f \rangle = \langle \text{Conditions correspond to feature} \rangle$$

$$= \langle f \rangle_{Gaussian} + \text{Perturbative Parts}\Big|_{NG + Anisotropy}$$

Two-point statistics

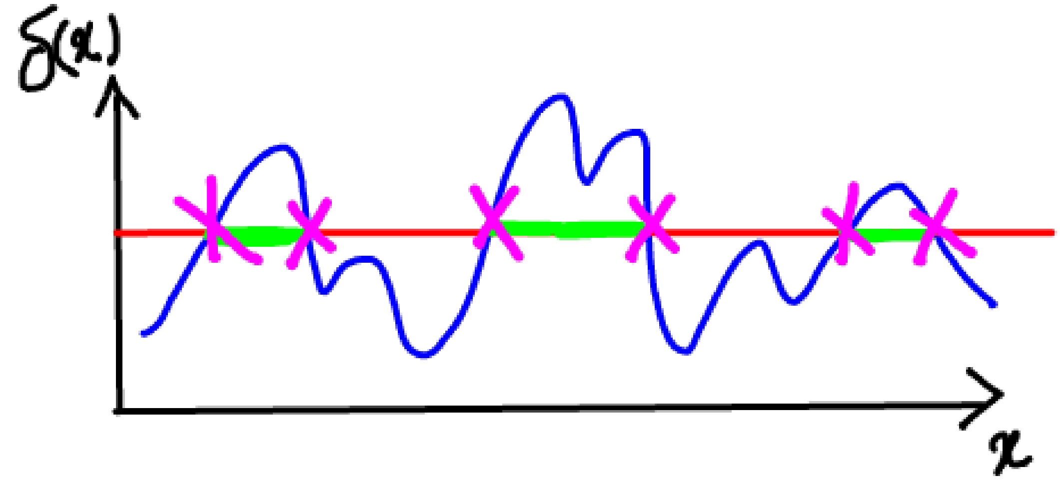
$$\langle f(r_1)\, g(r_2) \rangle = \int dA_1\, dA_2\, P(A_1, A_2)\, f(r_1)\, g(r_2)$$

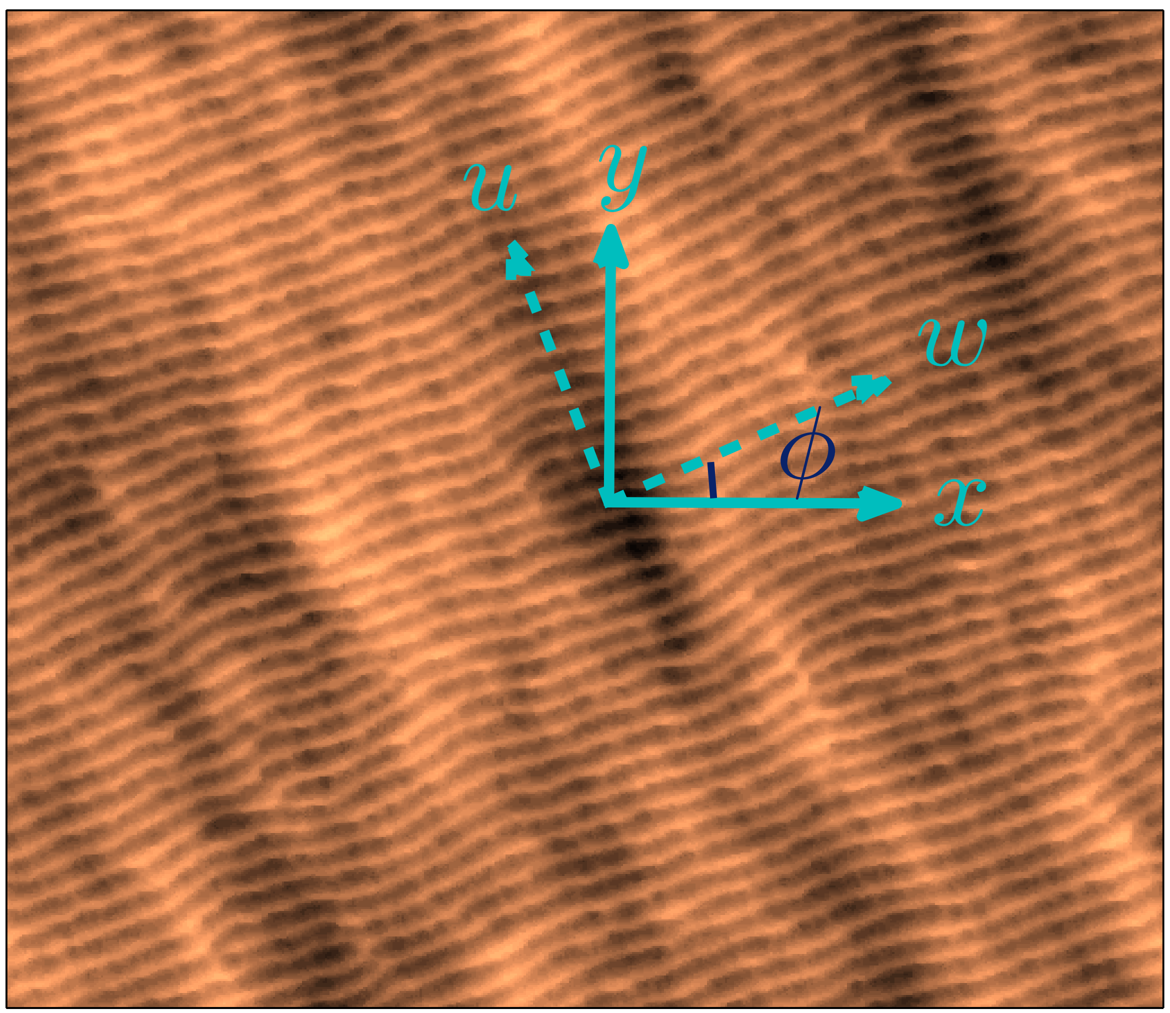
$$P(\vec{A}_1, \vec{A}_2) = \left[\frac{1}{2\pi^N \mathrm{Det}(K)}\right]^{1/2} \exp\left(-\frac{A_1^{\dagger} \cdot K^{-1} \cdot A_2}{2}\right)$$

T. Matsubara, APJ 2003; S. Codis et. al., 1305.7402, Christophe Gay et. al., PRD 2012

Proposed measure

$$N_{cr}^{(r,s)}(\vartheta, i) = \left\langle \delta_D\left(\delta^{(r,s)} - \vartheta\sigma_0^{(r,s)}\right) \left|\delta_{,i}^{(r,s)}\right| \right\rangle$$





Model 1: correlation length anisotropy

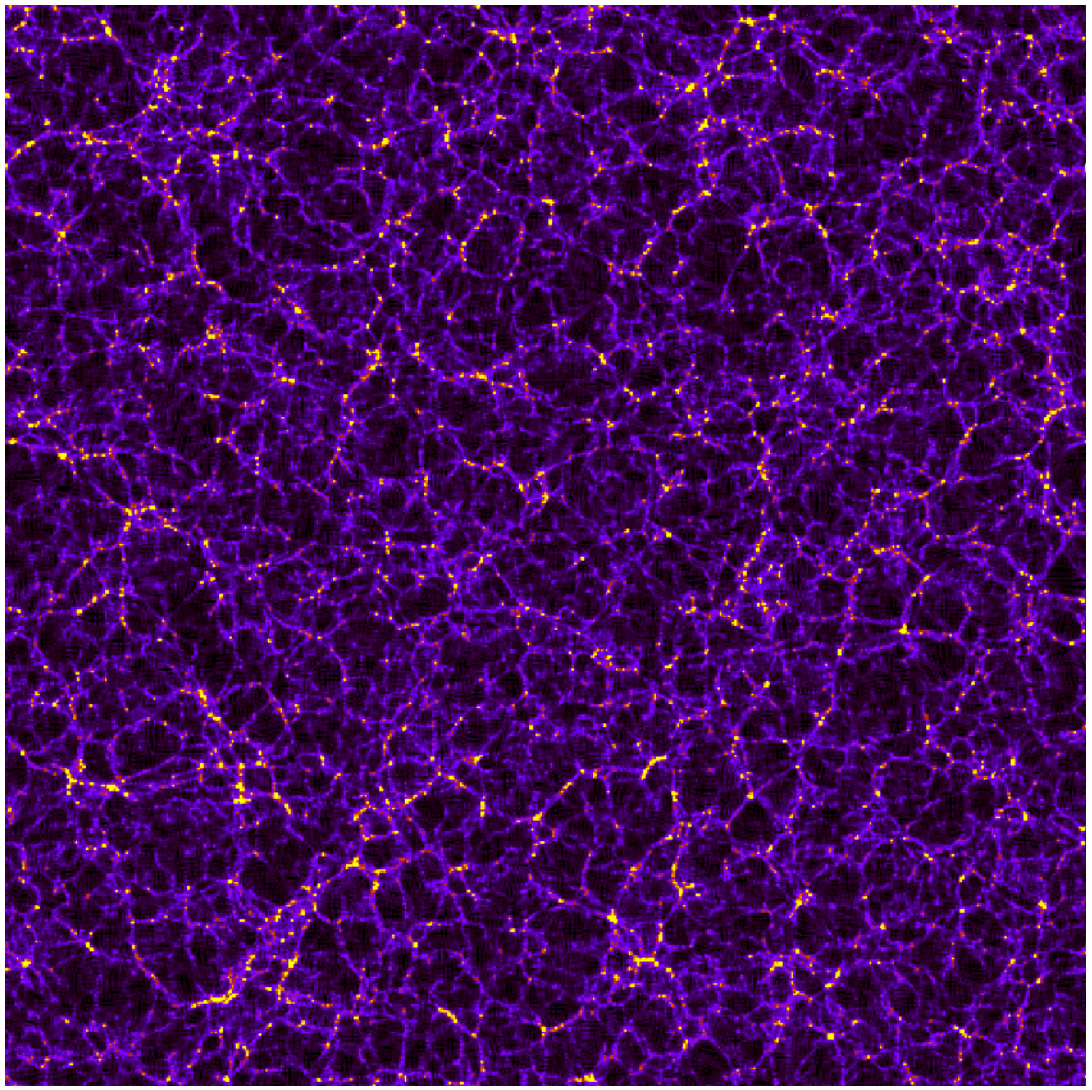
$$S^{(2-D)}(\mathbf{k}) = \frac{4\pi\gamma\sigma_0^2 k_c^{2\gamma}\xi_u\xi_w}{[k_c^2 + \xi_u^2 k_u^2 + \xi_w^2 k_w^2]^{\gamma+1}}$$

Model 2: Scaling anisotropy

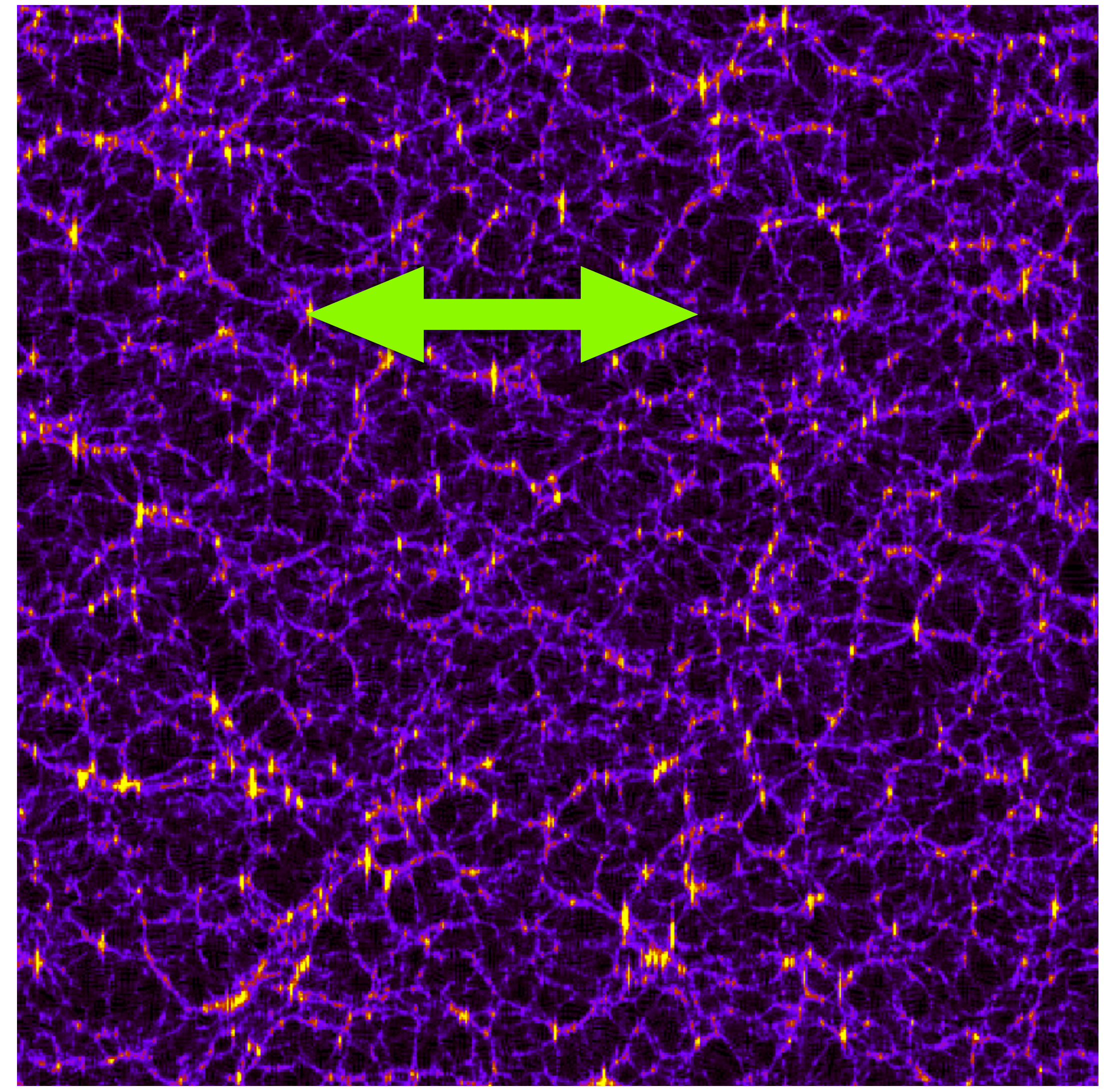
$$S^{(2-D)}(\mathbf{k}) = \frac{4\pi\sigma_0^2 k_c^{2(\gamma_u+\gamma_w)}\xi_u\xi_w \frac{\Gamma\left(\frac{1}{2}+\gamma_u\right)}{\Gamma(\gamma_u)} \frac{\Gamma\left(\frac{1}{2}+\gamma_w\right)}{\Gamma(\gamma_w)}}{[k_c^2 + \xi_u^2 k_u^2]^{\gamma_u+1/2}\,[k_c^2 + \xi_w^2 k_w^2]^{\gamma_w+1/2}}$$

M. Ghaseminezad, S.M.S.M. et al., (arXiv:1508.01409)

RSD: Linear Kaiser effect

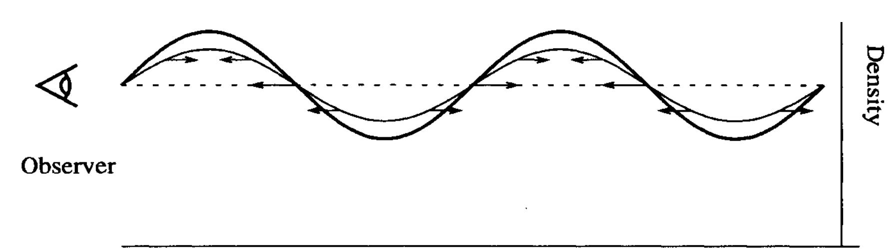


Without RSD



With RSD

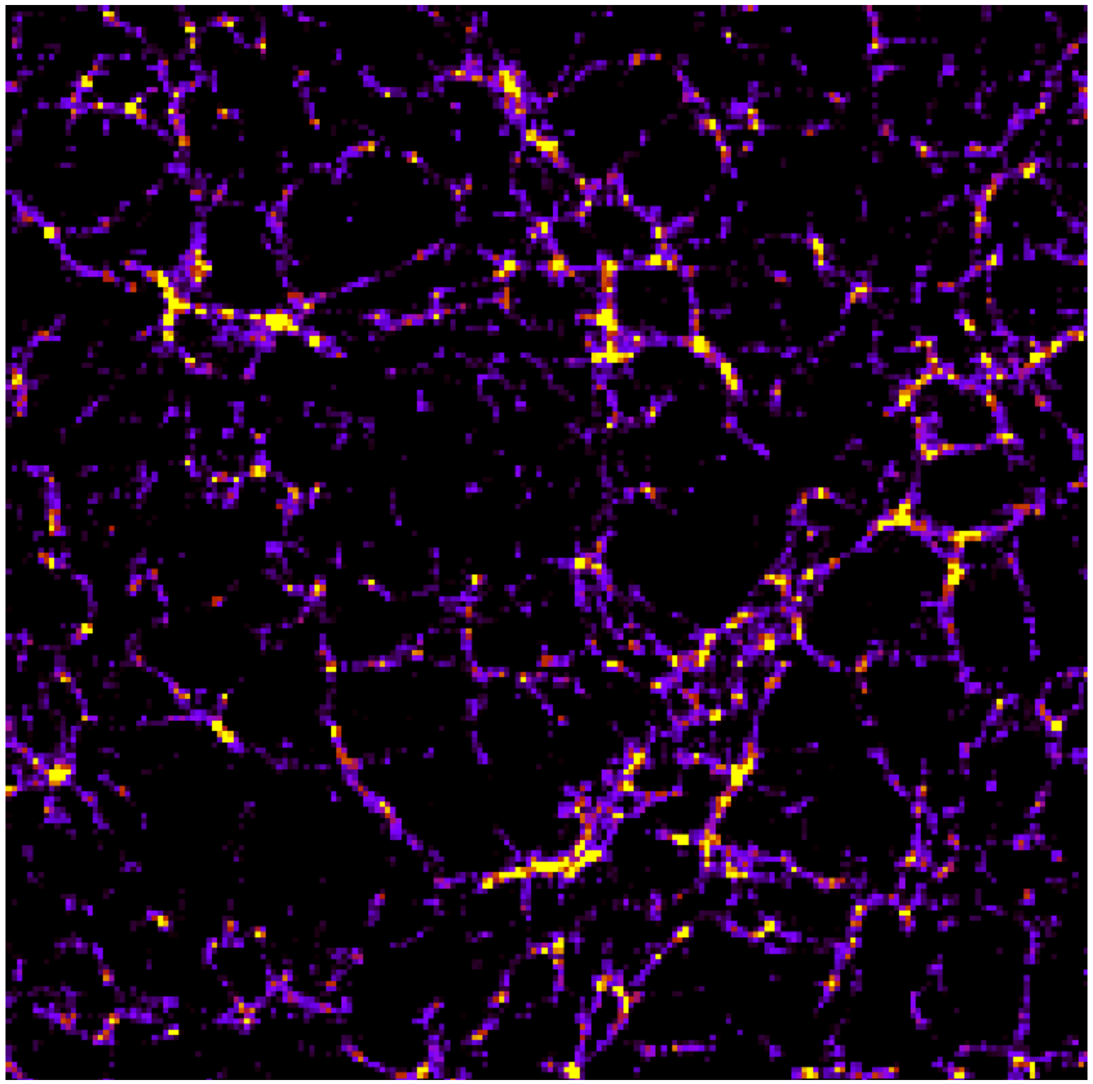
Line of sight



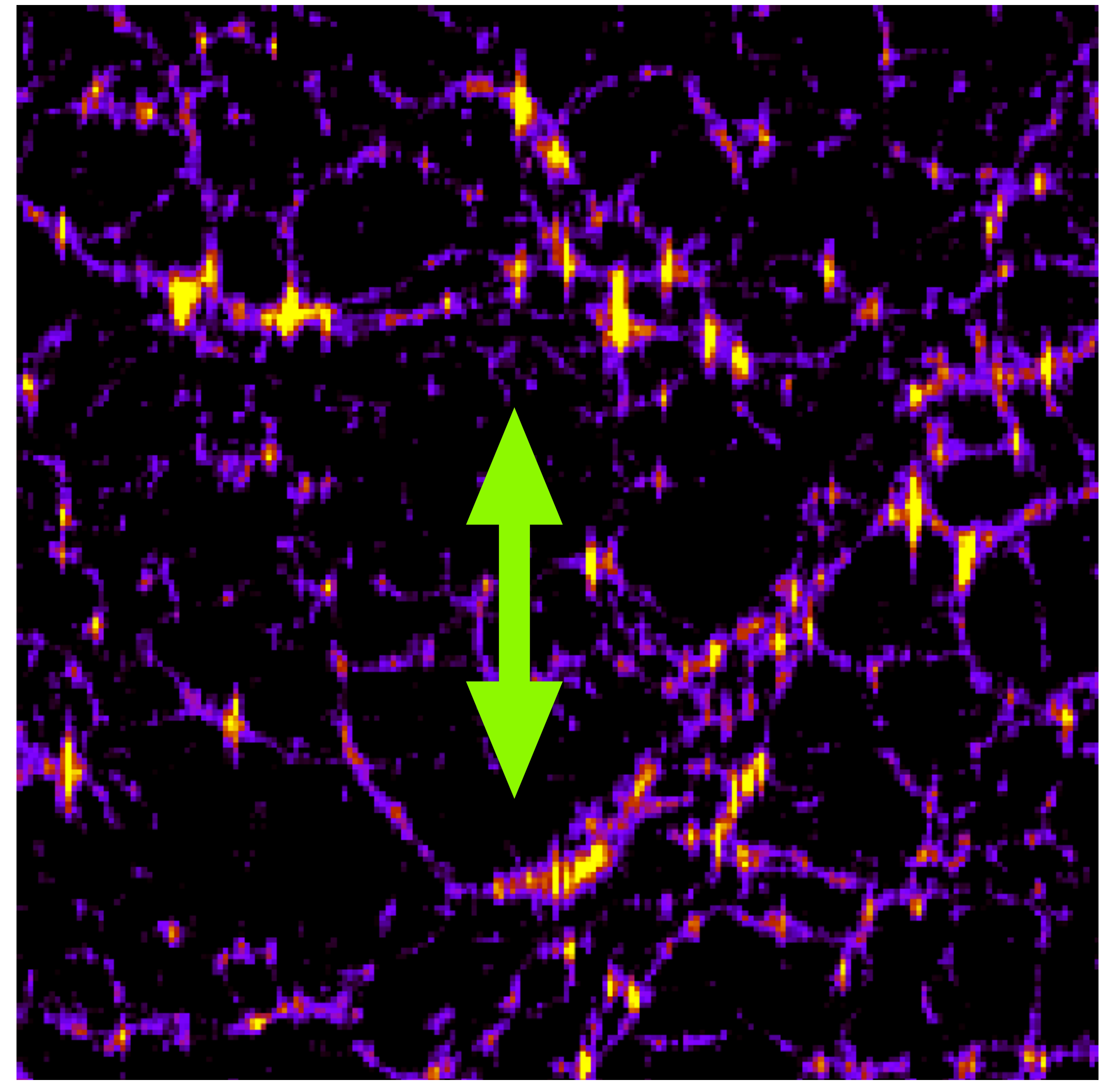
Mohammad Hossein Jalali and S. M. S. Movahed, ", arXiv:2308.03086,

Hamilton, A. J. S. 1998, Astrophysics and Space Science Library, Vol. 231, Linear Redshift Distortions: a Review, ed. D. Hamilton, 185,

FoG effect



Without RSD

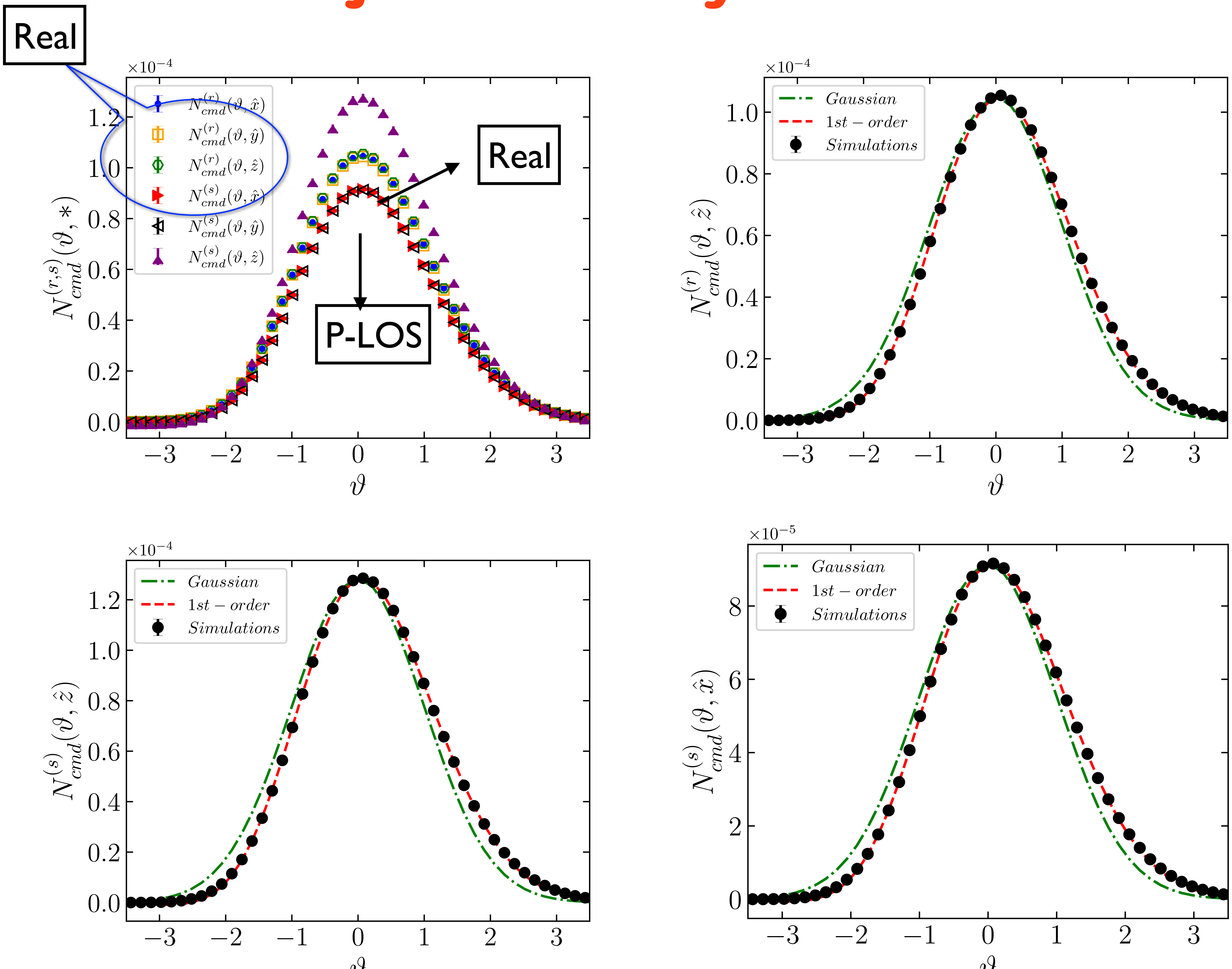


With RSD

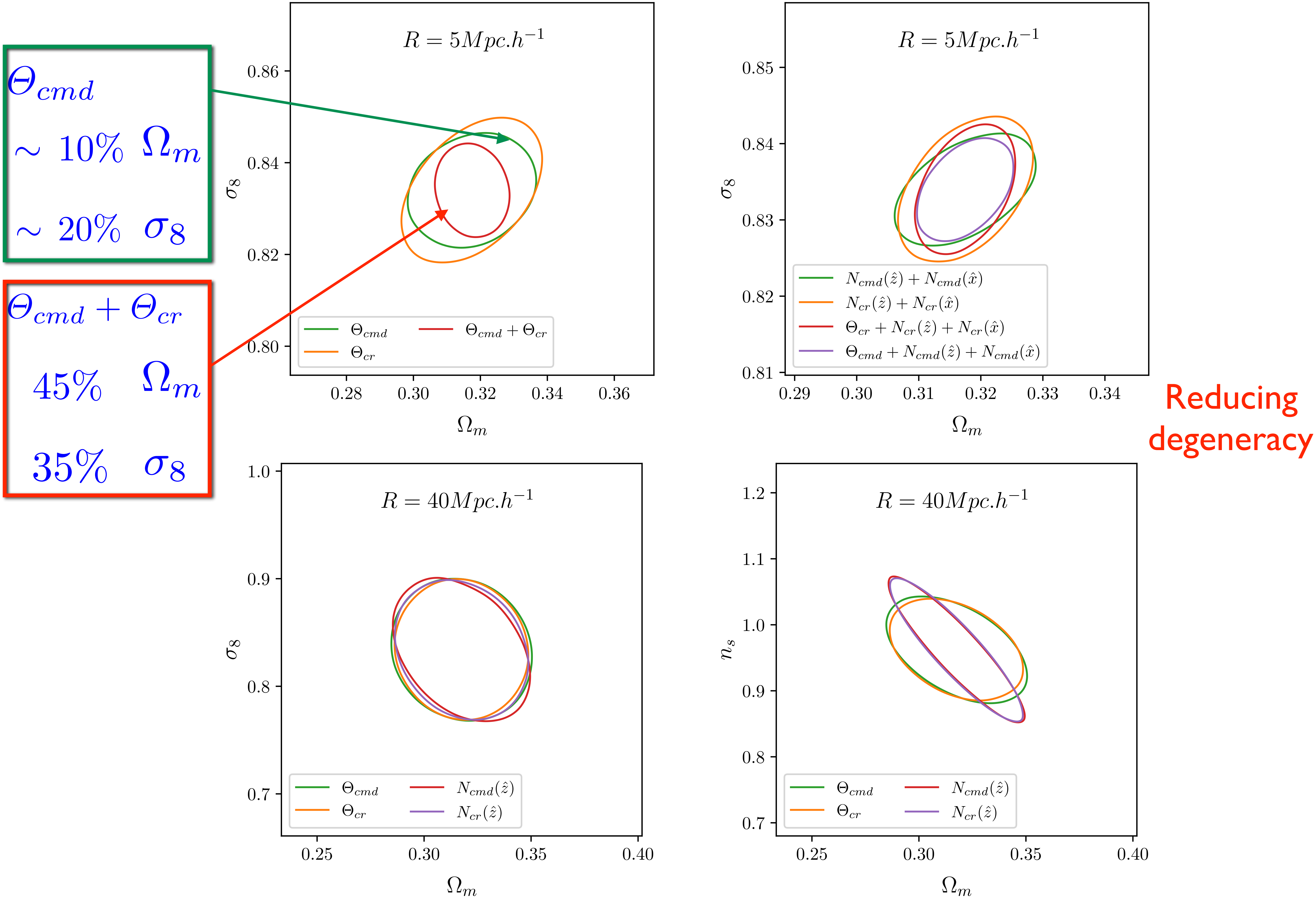
Line of sight

Mohammad Hossein Jalali and S. M. S. M., ", arXiv:2308.03086,

Quijote N-body simulation



Mohammad Hossein Jalali and S. M. S. M., ", arXiv:2308.03086,


Θ_{cmd}
$\sim$ 10% Ω_m
$\sim$ 20% σ_8
$\Theta_{cmd} + \Theta_{cr}$
45% Ω_m
35% σ_8
$R = 5Mpc.h^{-1}$
σ_8
Ω_m
Θ_{cmd}
Θ_{cr}
$\Theta_{cmd} + \Theta_{cr}$
$R = 5Mpc.h^{-1}$
$N_{cmd}(\hat{z}) + N_{cmd}(\hat{x})$
$N_{cr}(\hat{z}) + N_{cr}(\hat{x})$
$\Theta_{cr} + N_{cr}(\hat{z}) + N_{cr}(\hat{x})$
$\Theta_{cmd} + N_{cmd}(\hat{z}) + N_{cmd}(\hat{x})$
Reducing degeneracy
$R = 40Mpc.h^{-1}$
Θ_{cmd}
Θ_{cr}
$N_{cmd}(\hat{z})$
$N_{cr}(\hat{z})$
$R = 40Mpc.h^{-1}$
n_s

Concluding remarks and Take-Home massages

1) Morphologies contain valuable information, particularly in the high-precision data era;
2) Reducing the degeneracies via data-based modeling;
3) Put pristine constraints on relevant parameters;

What the next?

1) Construction new measures
2) Beyond Plane-Parallel approximation.
3) Various field (Finding preferred direction) + Moving window + Iterative Coarse Graining
4) Making a pipeline including various measures ranging from geometry to topology and utilizing Machine learning

از توجه شما سپاسگزارم

Stochastic fields, Stochastic processes, Random fields

1) Measure-theoretic definition (probability triple)

Probability space (probability triple) is represented by $(\Omega, \mathscr{F}, \mathbb{P})$
Ω is sample space. It contains all possible outcomes
$\mathscr{F}$ is event space (σ-algebra)
A probability function $(0 \leq \mathbb{P} \leq 1)$, assigns a probability to each event in the event space

Just one flip of Fair coin
$\Omega = \{Heads, Tails\} \equiv \{\mathbf{H}, \mathbf{T}\}$
$\mathscr{F} = \{\{\}, \{\mathbf{H}\}, \{\mathbf{T}\}, \{\mathbf{H}, \mathbf{T}\}\}$
$\mathbb{P}(\{\}) = 0$; $\mathbb{P}(\{\mathbf{H}\}) = 0.5$; $\mathbb{P}(\{\mathbf{T}\}) = 0.5$; $\mathbb{P}(\{\mathbf{H}\}, \{\mathbf{T}\}) = 1$

$$\{\Omega, \mathscr{F}, \mathbb{P}\}$$

$$f^d : \Omega \rightarrow \mathbb{R}^T$$

$$T \subset \mathbb{R}^N$$

f is a $(d+N)-$Dimensional Stochastic field

2) Probabilistic framework definition

$X(t, \omega)$ is stochastic variable

$$\mathbb{P}\{X(t_1), X(t_2), \ldots, X(t_m)\}$$

Probability space

Probability space (probability triple) is represented by $(\Omega, \mathscr{F}, \mathbb{P})$
Ω is sample space. It contains all possible outcomes
$\mathscr{F}$ is event space (σ-algebra)
A probability function ($0 \leq \mathbb{P} \leq 1$), assigns a probability to each event in the event space

Example 1:

Just one flip of Fair coin
$\Omega = \{Heads, Tails\} \equiv \{\mathbf{H}, \mathbf{T}\}$
$\mathscr{F} = \{\{\}, \{\mathbf{H}\}, \{\mathbf{T}\}, \{\mathbf{H}, \mathbf{T}\}\}$
$\mathbb{P}(\{\}) = 0$; $\mathbb{P}(\{\mathbf{H}\}) = 0.5$; $\mathbb{P}(\{\mathbf{T}\}) = 0.5$; $\mathbb{P}(\{\mathbf{H}\}, \{\mathbf{T}\}) = 1$

Example 2:

The fair coin is tossed three times.
$\Omega = \mathbf{HHH}, \mathbf{HHT}, \mathbf{HTH}, \mathbf{HTT}, \mathbf{THH}, \mathbf{THT}, \mathbf{TTH}, \mathbf{TTT}$
$\mathscr{F} = 2^{\|\Omega\|} = 2^8 = 256$