

# \*\*\*\*\* Mathematica Tutorial:

## SBU\*\*\*\*\*

(Debug) In[ ]:=

**Today**

(Debug) Out[ ]:=

Tue 26 Nov 2024

## Removing all definitions and Setting Directory

Removing all definitions, values attributes, messages and default options that have been created in the Global` context and that don't have the Protected attribute (Protected attributes are those qualities, traits or characteristics that, by law, cannot be discriminated against)

(Debug) In[ ]:=

```
(*Quit Kernal*)  
Quit[]
```

(Debug) In[ ]:=

```
(*Keep the defined variables but will wash the assigned value*)  
ClearAll["Global`*"]
```

**aa**

(Debug) In[ ]:=

```
aa = 2
```

(Debug) Out[ ]:=

**2**

**aa**

(Debug) In[ ]:=

```
(*To remove a particular variable*)  
Clear[aa]
```

(Debug) In[ ]:=

```
SetDirectory[NotebookDirectory[]];
```

---

## Differential Equations

## Solve

(Debug) In[ ]:=

```
a1 = Solve[3 a3 - 2 b2 - c3 - h == 0, b] [[2, 1, 2]]
```

(Debug) Out[ ]:=

$$\frac{\sqrt{3 a^3 - c^3 - h}}{\sqrt{2}}$$

(Debug) In[ ]:=

```
N[a1 /. {a -> 2, c -> 1, h -> 2}]
```

(Debug) Out[ ]:=

```
3.24037
```

```
a11[a_, c_, h_] := Solve[3 a3 - 2 b2 - c3 - h == 0, b]
```

```
N[a11[2, 1, 2]]
```

```
Cc
```

## Magnetization

(Debug) In[ ]:=

```
Solve[M == Tanh[ $\frac{M * z}{T} + \frac{Bext}{T}$ ], M]
```

**...** Solve: This system cannot be solved with the methods available to Solve.

(Debug) Out[ ]:=

```
Solve[M == Tanh[ $\frac{Bext}{T} + \frac{M z}{T}$ ], M]
```

(Debug) In[ ]:=

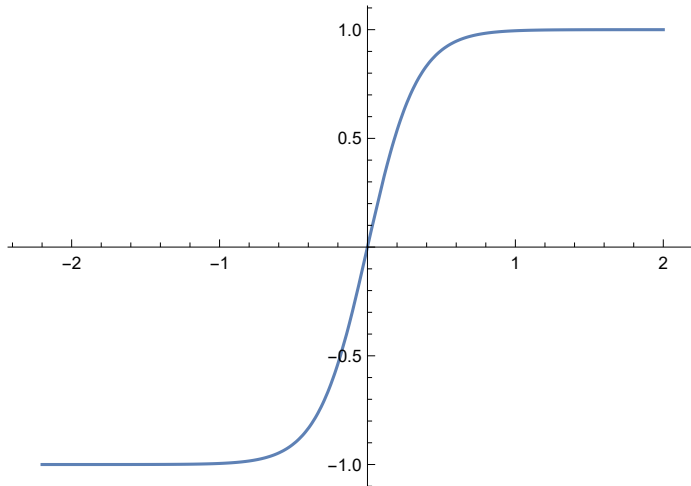
```
z = 3;
```

```
Bext = 0;
```

```
T = 1;
```

```
aa = ParametricPlot[{{M, Tanh[ $\frac{M * z}{T} + \frac{Bext}{T}$ ]}, {M, -2.2, 2}}, AspectRatio -> 0.7]
```

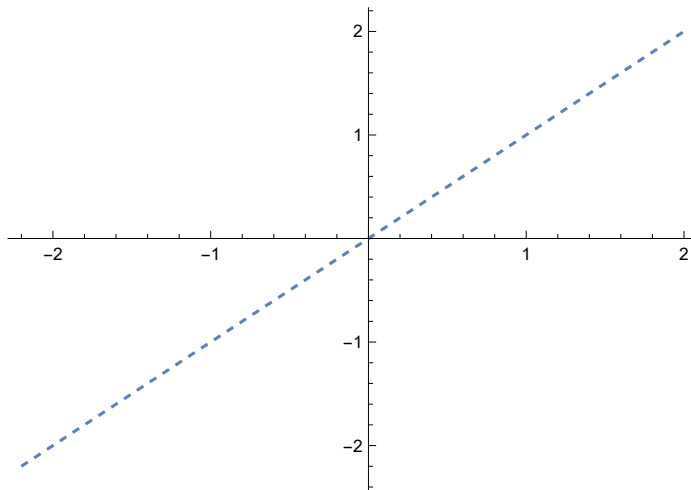
(Debug) Out[ ]:=



(Debug) In[ ]:=

```
bb = Plot[M, {M, -2.2, 2}, AspectRatio -> 0.7, PlotStyle -> Dashed]
```

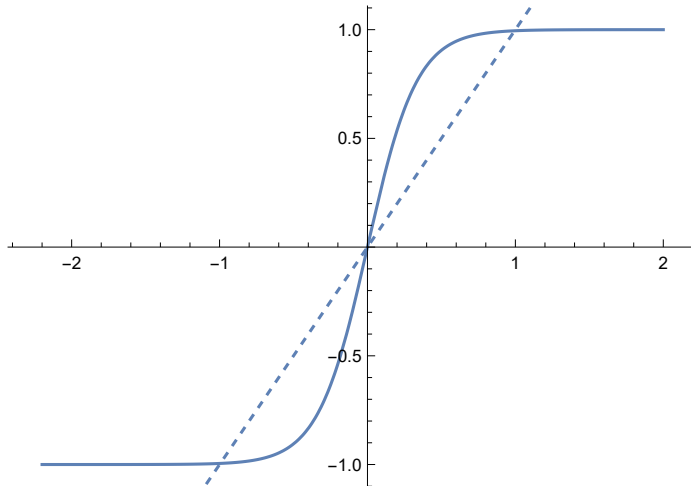
(Debug) Out[ ]:=



(Debug) In[ ]:=

```
Show[aa, bb]
```

(Debug) Out[ ]:=



## Differential Solve

(Debug) In[ ]:=

```
DSolve[b''[x] + b'[x] + b[x] == 0, b[x], x] [1, 1, 2]
```

(Debug) Out[ ]:=

$$e^{-x/2} c_2 \cos\left[\frac{\sqrt{3} x}{2}\right] + e^{-x/2} c_1 \sin\left[\frac{\sqrt{3} x}{2}\right]$$

```
D[b[x], {x, 9}]
```

## Numerical Differential Solve

## Simple Harmonic Oscillator

(Debug) In[ ]:=

```
α = 0;  
ω0 = 1;  
f = 0;  
ω = 0.666;  
ss1 =  
  NDSolve[{y''[t] + α * y'[t] + ω02 * y[t] == f * Cos[ω * t], y'[0] == 1, y[0] == 0}, y, {t, 0, 40}] [1, 1, 2]
```

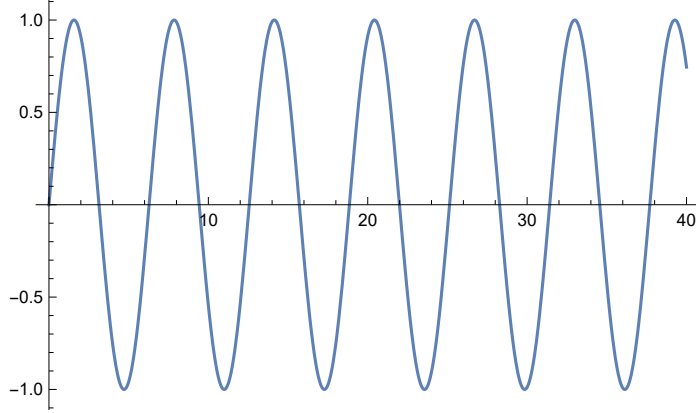
(Debug) Out[ ]:=

```
InterpolatingFunction[  Domain: {{0, 40}}  
Output: scalar ]
```

(Debug) In[ ]:=

```
Plot[ss1[t], {t, 0, 40}]
```

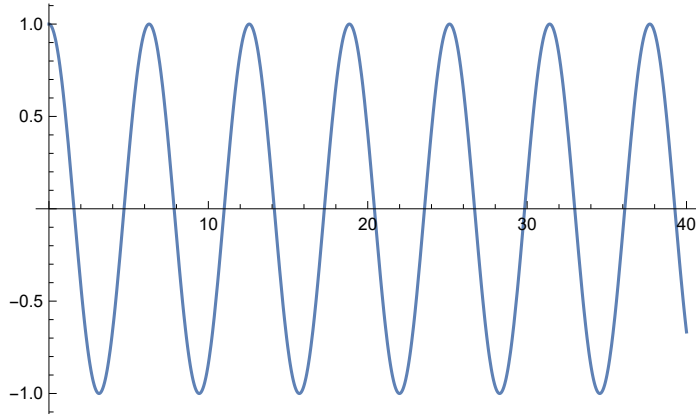
(Debug) Out[ ]:=



(Debug) In[ ]:=

```
Plot[ss1'[t], {t, 0, 40}]
```

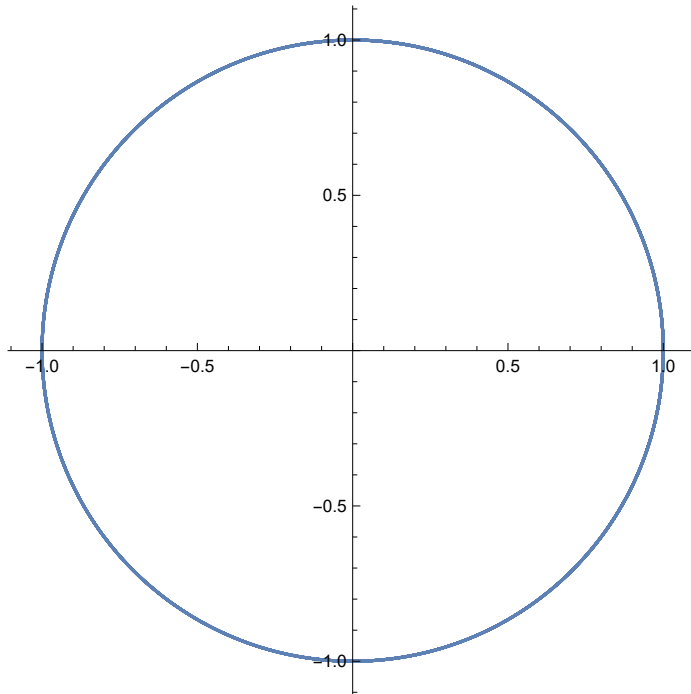
(Debug) Out[ ]:=



(Debug) In[ ]:=

```
ParametricPlot[{ss1[t], ss1'[t]}, {t, 0, 40}]
```

(Debug) Out[ ]:=



## Damped Harmonic Oscillator

(Debug) In[ ]:=

```

α = 0.2;
ω0 = 1;
f = 0.52;
ω = 0.666;
ss = NDSolve[{y''[t] + α * y'[t] + ω0^2 * Sin[y[t]] == f * Cos[ω * t], y'[0] == 1, y[0] == 0},
  y, {t, 0, 100}] [[1, 1, 2]]

```

(Debug) Out[ ]:=

InterpolatingFunction[  Domain: {{0, 100.}}  
Output: scalar ]

→

$\lambda \Delta \delta$

$\lambda$

$5 / 6$

$\frac{5}{6} 1^4$

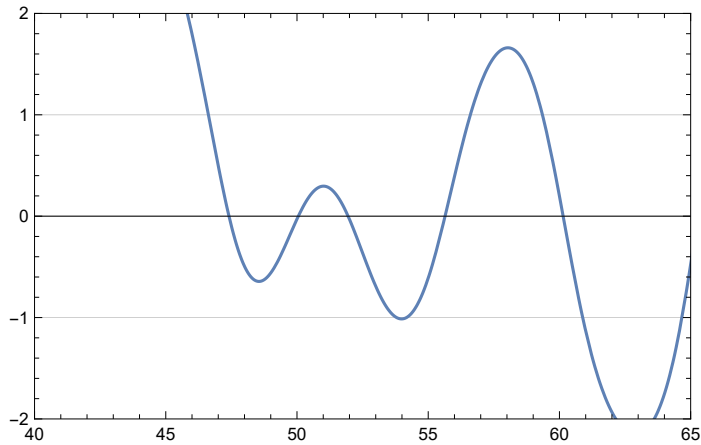
$s_3$

$g = 5 * 6 + 8 - 9$

(Debug) In[ ]:=

```
Plot[ss[t], {t, 0, 100}, PlotRange -> {{40, 65}, {-2, 2}},
     Frame -> True, GridLines -> {{-1, 0, 1}, {-1, 0, 1}}]
```

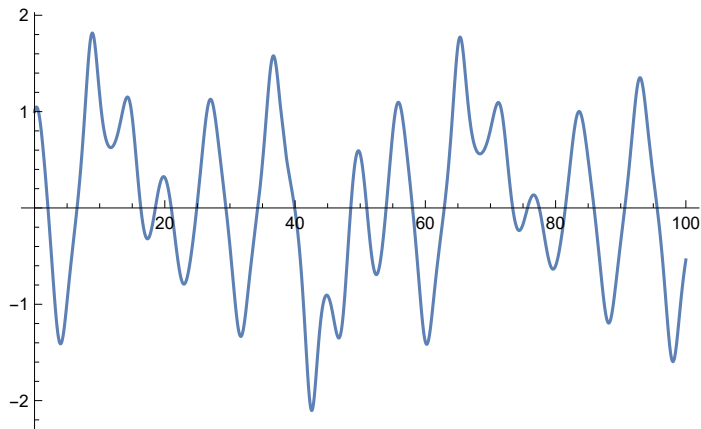
(Debug) Out[ ]:=



(Debug) In[ ]:=

```
Plot[ss'[t], {t, 0, 100}, PlotRange -> All]
```

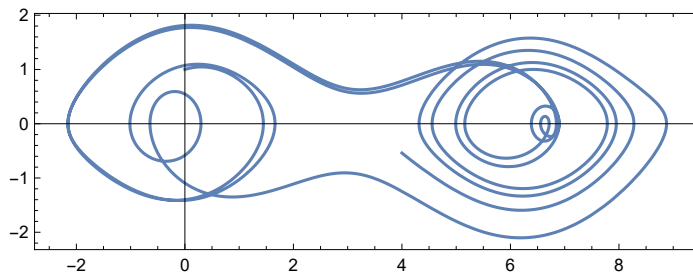
(Debug) Out[ ]:=



(Debug) In[ ]:=

```
ParametricPlot[{ss[t], ss'[t]}, {t, 0, 100}, PlotRange -> All, Frame -> True]
```

(Debug) Out[ ]:=



## Boundary Value Problems

Exp[ ]

(Debug) In[ ]:=

```
DSolve[{y''[x] + y[x] == E^x, y[0] == 1, y[1] == 1/2}, y, x]
```

(Debug) Out[ ]:=

```
{ { y -> Function[{x},  $\frac{1}{2} \text{Sin}[1] (\text{Cos}[x] \text{Csc}[1] + e^x \text{Cos}[x]^2 \text{Csc}[1] - e \text{Sin}[x] - e \text{Cot}[1]^2 \text{Sin}[x] - \text{Cot}[1] \text{Csc}[1] \text{Sin}[x] + \text{Csc}[1]^2 \text{Sin}[x] + e^x \text{Csc}[1] \text{Sin}[x]^2)$  ] ] }
```

## Partial Differential Equation

$$\frac{\partial P(x, t)}{\partial t} = d * \frac{\partial^2 P(x, t)}{\partial x^2}, \quad p(x, 0) = \text{Sin}[x]$$

$\partial$

$\frac{dt}{dx}$

(Debug) In[ ]:=

```
DSolve[{D[P[x, t], t] == d * D[P[x, t], {x, 2}], P[x, 0] == Sin[x]}, P[x, t], {x, t}]
```

(Debug) Out[ ]:=

```
{ { P[x, t] -> e-dt Sin[x] } }
```

(Debug) In[ ]:=

```
NDSolve[{D[u[x, t], t] == D[u[x, t], x, x], u[x, 0] == Sin[x], u[0, t] == 0, u[π, t] == 0}, u, {x, 0, π}, {t, 0, 1}]
```

(Debug) Out[ ]:=

```
{ { u -> InterpolatingFunction[  Domain: {{0., 3.14}, {0., 1.}} Output: scalar ] ] }
```

## Coupled Differential Equations

$$V = \Lambda \left( \left( 1 - \frac{\psi^2}{M^2} \right)^2 + \frac{\phi - \phi_c}{m_1} - \frac{(\phi - \phi_c)^2}{m_2^2} + \frac{\phi^4 \psi^2}{M^2 \phi_c^4} + \frac{\psi}{b} \right).$$

The Klien-Gordon classical background equations of motion in the number of e-fold times are given by [32],

$$\phi'' + \left( \frac{H'}{H} + 3 \right) \phi' + \frac{V_\phi}{H^2} = 0, \quad \psi'' + \left( \frac{H'}{H} + 3 \right) \psi' + \frac{V_\psi}{H^2} = 0. \quad (3.1)$$

Here,  $V_X = dV/dX$  where,  $X = \{\phi, \psi\}$ , prime is the derivative with respect to the number of e-folds and the Hubble rate  $H$  is defined as  $H^2 = 2V/(6 - \phi'^2 - \psi'^2)$ . The evolution of the field



(Debug) In[ ]:=

```

ClearAll["Global`*"]
mp = 2.435515 * 1018; RM = Sqrt[2.137 * 10-9];
kc = (6.394970897 * 10-39) / mp; k0 = 0.05 * kc;
M = 0.1;
φc = M;
m1 = 3 * 105;
m2 = 11;
Λ =  $\frac{2.21 * 10^{-9}}{3.429508708250553} * \frac{12 \pi^2}{m1^2}$ ;
ψ0 =  $\sqrt{\frac{\Lambda \sqrt{2 \phi c m1 M}}{96 \pi^{3/2}}}$ ;
b = -8 * 109;
V[n_] = Λ  $\left( \left( 1 - \frac{\psi[n]^2}{M^2} \right)^2 + \frac{(\phi[n] - \phi c)}{m1} - \frac{(\phi[n] - \phi c)^2}{m2^2} + \frac{2 \phi[n]^4 \psi[n]^2}{M^4 \phi c^2} + \frac{\psi[n]}{b} \right)$ ;
SV[n_] = V[n] /. {ψ[n] → sψi[n], φ[n] → sφi[n]};
φi = φc (1 + 0.0011);
ψi = ψ0;
dψi = (-D[V[n], ψ[n]]) / V[n] /. {ψ[n] → ψi, φ[n] → φi};
dφi = (-D[V[n], φ[n]]) / V[n] /. {ψ[n] → ψi, φ[n] → φi};
ni = Log[(k0 / (Hi * 1 * 106))];
nf = -54;

Hi =  $\sqrt{(V[n] / (3 (1 - d\psi i^2 / 6 - d\phi i^2 / 6)))}$  /. {ψ[n] → ψi, φ[n] → φi};

HPH[n_] = (1 / (2 V[n])) (6 - φ'[n]2 - ψ'[n]2) (V'[n] / (6 - φ'[n]2 - ψ'[n]2) -
(V[n] (-2 φ'[n] × φ''[n] - 2 ψ'[n] × ψ''[n])) / (6 - φ'[n]2 - ψ'[n]2))2;

sHPH[n_] := HPH[n] /. {φ → sφi, ψ → sψi};

H[n_] =  $\sqrt{(2 * V[n]) / (6 - ((\phi'[n])^2 + (\psi'[n])^2))}$ ;


sH1[n_] := H[n] /. {φ → sφi, ψ → sψi};


varsI = {φ[n], ψ[n]};
initsI = {φ[ni] == φi, ψ[ni] == ψi, φ'[ni] == dφi, ψ'[ni] == dψi};
eqnsI = {φ''[n] + (HPH[n] + 3) * φ'[n] + D[V[n], φ[n]] / (H[n])2 == 0,
ψ''[n] + (HPH[n] + 3) * ψ'[n] + D[V[n], ψ[n]] / (H[n])2 == 0};


{sφi[n_], sψi[n_]} =
varsI /. NDSolve[Join[eqnsI, initsI], varsI, {n, ni, nf}, MaxSteps → Infinity,
MaxStepSize → 0.001, Method → {"ExplicitRungeKutta"}][[1]]


```

(Debug) Out[ ]:=

{InterpolatingFunction [  Domain: {{-126., -54.}} Output: scalar ] [n],

Data not in notebook. Store now 

InterpolatingFunction [  Domain: {{-126., -54.}} Output: scalar ] [n]

Data not in notebook. Store now 

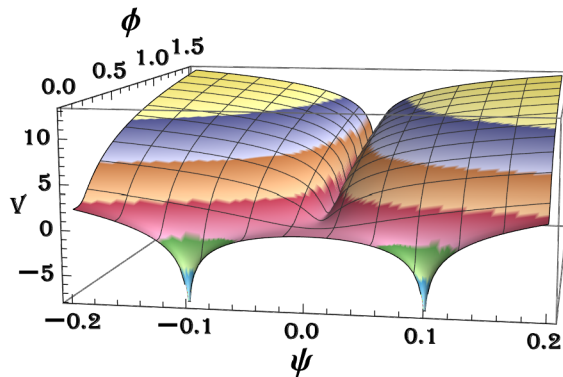
a[h\_, c\_] = h + c

a[h, c]

(Debug) In[ ]:=

```
Plot3D[Log[ $\frac{V[n]}{\Delta}$ ], { $\psi[n]$ , -0.2, 0.2}, { $\phi[n]$ , 0, 1.5}, Boxed → True, PlotPoints → 100,
Mesh → 10, ColorFunction → "DarkBands", Axes → True, AxesLabel → {" $\psi$ ", " $\phi$ ", "V"},
AxesStyle → Directive[Black, 12, FontFamily → "Algerian", Bold], ImageSize → 490]
```

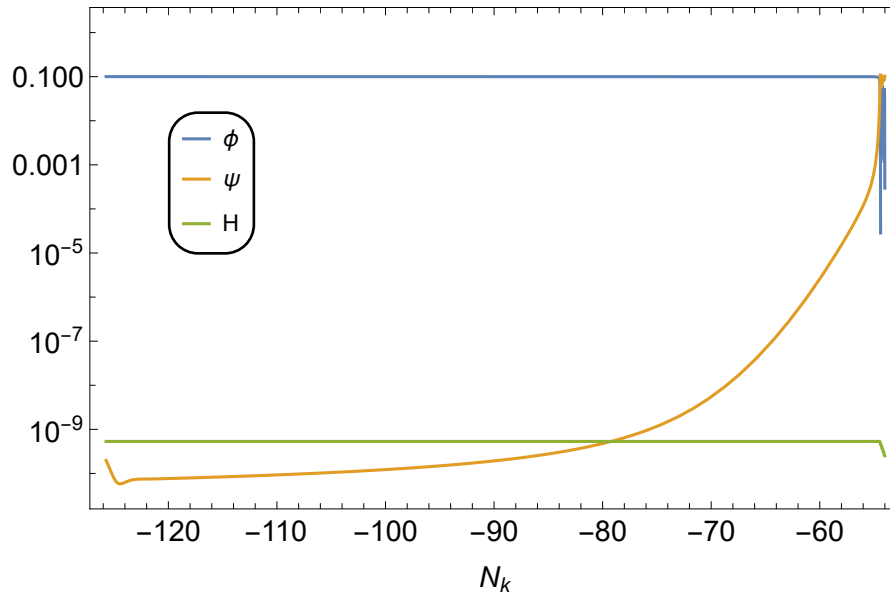
(Debug) Out[ ]:=



(Debug) In[ ]:=

```
LogPlot[{sphi[n], Abs[spsi[n]], sH1[n]}, {n, ni, nf},
Frame → True, FrameLabel → {"Nk", ""}, FrameStyle → Directive[Black, 15],
PlotLegends → Placed[LineLegend[{" $\phi$ ", " $\psi$ ", "H"}, LabelStyle → 12,
LegendMarkerSize → {{15, 10}}, LegendFunction → (Framed[#, RoundingRadius → 15] &),
LegendMargins → 2], {0.15, 0.65}], ImageSize → 490]
```

```
(Debug) Out[ ]:=
```



## (Numerical) Integration

### Definite

```
(Debug) In[ ]:=
```

$$\mathbf{aa[x\_]} = \mathbf{Integrate}\left[\frac{1}{x^3 + 1}, x\right]$$

```
(Debug) Out[ ]:=
```

$$\frac{\text{ArcTan}\left[\frac{-1+2x}{\sqrt{3}}\right]}{\sqrt{3}} + \frac{1}{3} \text{Log}[1+x] - \frac{1}{6} \text{Log}[1-x+x^2]$$

```
(Debug) In[ ]:=
```

```
N[aa[2] - aa[1]]
```

```
(Debug) Out[ ]:=
```

```
0.254353
```

```
(Debug) In[ ]:=
```

$$\mathbf{NIntegrate}\left[\frac{1}{x^3 + 1}, \{x, 1, 2\}\right]$$

```
(Debug) Out[ ]:=
```

```
0.254353
```

## Code for nested integrals

$$\Omega_{\text{gw}}^r(f) = \frac{16\pi}{3} \left( \frac{G\mu}{H_0} \right)^2 \frac{\Gamma}{f a_0} \int_{a_{\text{min}}}^{a_{\text{eq}}} \left( \frac{a(t)}{a_0} \right)^4 \frac{da}{H(a)} n_r(l, t),$$

$$\Omega_{\text{gw}}(f, q, n_*) = \sum_j^{n_*} \frac{j^{-q}}{\mathcal{E}} \Omega_{\text{gw}}^j(f), \quad (12)$$

(Debug) In[ ]:=

```

ClearAll["Global`*"]
Gμ = 1 * 10-10;
a0 = 1;
h = 0.6736;
H0 = ((100 * 103 * h) / (3.08568 * 1022));
ξr = √(1 - Y) * √k (k + ct);
ct = 0.23;
k =  $\frac{2 \sqrt{2}}{\pi} \left( \frac{1 - 8 * \nu r^6}{1 + 8 * \nu r^6} \right) (1 - \nu r^2) (1 + 2 \sqrt{2} \nu r^3)$ ;
Y = 0.8;
νr = 0.662 * √(1 - Y);
Γ0 = 50;
Γ = Γ0 * (1 - Fppm)BcΓ;
Fppm = √Y;
(*For Chiral Current*)
BcΓ =  $\frac{BsΓ}{\sqrt{2}}$ ;
(*1.5 for kinks, (2 for cusps)*)
BsΓ = 2.6;
α = 0.3688566917981026;
Ωr = 9.2188 * 10-5;
Ωm = 0.308;
ct = 0.23;
(*G1=Import [
  "C:\\Users\\adeel\\OneDrive\\Desktop\\Current_Carrying_CSs\\Data_points\\Gvsa.DAT"];
G=Interpolation[G1, InterpolationOrder→1];*)
G[a_] = 1;
F = 0.1;
εr =  $\frac{\alpha * \xi r}{\Gamma * G\mu}$ ;
tpl = 5.39 * 10-44;

```

$$\text{aminr} = \frac{a\theta * \sqrt{2 * H\theta * \sqrt{\Omega r} * (\epsilon r + 1) * \text{tp1}}}{G\mu};$$

$$\text{aeq} = a\theta * \frac{\Omega r}{\Omega m};$$

$$l = \frac{2 * a * j}{f * a\theta};$$

$$\text{Ar} = \mathcal{F} * \frac{ct * \nu r}{\sqrt{2} * \xi r^3};$$

$$\text{Cr} = \frac{\text{Ar}}{\alpha * \xi r} (\alpha * \xi r + \Gamma * G\mu)^{3/2};$$

$$\text{Hr} = H\theta * \sqrt{\mathcal{G}[a] * \Omega r} \left(\frac{a\theta}{a}\right)^2;$$

$$\text{tr} = \frac{a^2}{2 a\theta^2 H\theta \sqrt{\mathcal{G}[a] * \Omega r}};$$

$$\text{nr}[f\_ ] = \frac{\text{Cr}}{\text{tr}^{3/2} (1 + \Gamma * G\mu * \text{tr})^{5/2}};$$

$$q = \frac{5}{3} (*\text{Kinks}*);$$

$$\text{Pj} = \frac{\Gamma * j^{-q}}{\text{Zeta}\left[\frac{4}{3}\right]};$$

$$\text{ss} = 100;$$

$$\text{ss1} = 101;$$

$$\text{NRs}[f\_?NumericQ] := \text{Sum}\left[\text{NIntegrate}\left[\left(\frac{a}{a\theta}\right)^4 j * \text{Pj} * \frac{\text{nr}[f]}{\text{Hr}}, \{a, \text{aminr}, \text{aeq}\}, \text{MaxRecursion} \rightarrow 20, \text{WorkingPrecision} \rightarrow 16\right], \{j, 1, \text{ss}\}\right];$$

$$\text{NRi}[f\_?NumericQ] := \text{NIntegrate}\left[\left(\frac{a}{a\theta}\right)^4 j * \text{Pj} * \frac{\text{nr}[f]}{\text{Hr}}, \{a, \text{aminr}, \text{aeq}\}, \{j, \text{ss1}, 10^5\}, \text{MaxRecursion} \rightarrow 20, \text{WorkingPrecision} \rightarrow 16\right];$$

$$\text{NR}[f\_ ] = \text{NRs}[f] + \text{NRi}[f];$$

$$\Omega r\text{GW}[f\_ ] = h^2 * ((16 * \pi) / 3) (G\mu / H\theta)^2 (1 / (a\theta * f)) (\text{NR}[f]);$$

(Debug) In[ ]:=

```
f = 10f1; kk = 0;
Do[
  Clear[ΩGWTotSum];
  ΩGWTotSum[f_] = ΩrGW[f];
  kk = kk + 1;
  ddf[kk] = {f, ΩGWTotSum[f]};
  Print[ddf[kk]];
  ddd = Table[ddf[jj], {jj, 1, kk}];
  , {f1, Log10[0.31622776601683794` ], Log10[103], 0.2}]
```

```
{0.316228, 2.82403 × 10-10}
```

```
{0.501187, 2.8248 × 10-10}
```

```
{0.794328, 2.82534 × 10-10}
```

•• NIntegrate: Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration is 0, highly oscillatory integrand, or WorkingPrecision too small.

```
{1.25893, 2.82571 × 10-10}
```

•• NIntegrate: Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration is 0, highly oscillatory integrand, or WorkingPrecision too small.

```
{1.99526, 2.82595 × 10-10}
```

•• NIntegrate: Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration is 0, highly oscillatory integrand, or WorkingPrecision too small.

•• General: Further output of NIntegrate::slwcon will be suppressed during this calculation.

```
{3.16228, 2.82612 × 10-10}
```

```
{5.01187, 2.82622 × 10-10}
```

```
{7.94328, 2.82629 × 10-10}
```

```
{12.5893, 2.82633 × 10-10}
```

```
{19.9526, 2.82636 × 10-10}
```

```
{31.6228, 2.82638 × 10-10}
```

```
{50.1187, 2.82639 × 10-10}
```

```
{79.4328, 2.8264 × 10-10}
```

```
{125.893, 2.8264 × 10-10}
```

```
{199.526, 2.8264 × 10-10}
```

```
{316.228, 2.82641 × 10-10}
```

```
{501.187, 2.82641 × 10-10}
```

```
{794.328, 2.82641 × 10-10}
```

```
(Debug) In[ ]:=
```

```
ddd
```

```
(Debug) In[ ]:=
```

```

cc = { {1.`*^-12, 2.387992815460308`*^-20},
  {3.1622776601683794`*^-12, 1.3428659051720894`*^-19},
  {1.`*^-11, 7.551469668241913`*^-19}, {3.1622776601683794`*^-11,
  4.246467435574246`*^-18}, {1.`*^-10, 2.3879000649252278`*^-17},
  {3.1622776601683795`*^-10, 1.3427009812130997`*^-16},
  {1.`*^-9, 7.548537556973553`*^-16}, {3.1622776601683795`*^-9, 4.24125724135352`*^-15},
  {1.`*^-8, 2.3786568563816526`*^-14},
  {3.162277660168379`*^-8, 1.3263863917068893`*^-13},
  {1.`*^-7, 7.265105498643671`*^-13}, {3.162277660168379`*^-7, 3.771645611509287`*^-12},
  {1.`*^-6, 1.693651380688711`*^-11}, {3.162277660168379`*^-6, 5.596343752145578`*^-11},
  {0.00001`, 1.2143170387412448`*^-10}, {0.000031622776601683795`,
  1.8545473320185802`*^-10}, {0.0001`, 2.2990103790072095`*^-10},
  {0.00031622776601683794`, 2.5572136241355057`*^-10},
  {0.001`, 2.6940085293627206`*^-10}, {0.0031622776601683794`, 2.7629192262618254`*^-10},
  {0.01`, 2.796631122065546`*^-10}, {0.03162277660168379`, 2.812823078810208`*^-10},
  {0.1`, 2.8204891564907916`*^-10}, {0.31622776601683794`, 2.824025096002165`*^-10},
  {0.5011872336272722`, 2.824798743528717`*^-10},
  {0.7943282347242815`, 2.82533884505459`*^-10},
  {1.2589254117941675`, 2.8257078900715945`*^-10},
  {1.9952623149688797`, 2.825954968534554`*^-10},
  {3.1622776601683795`, 2.826117568128351`*^-10},
  {5.011872336272725`, 2.826223150019687`*^-10}, {7.943282347242818`,
  2.826291056068594`*^-10}, {12.589254117941675`, 2.826334429579627`*^-10},
  {19.952623149688797`, 2.826362064886411`*^-10}, {31.622776601683793`,
  2.826379570299117`*^-10}, {50.11872336272725`, 2.82639065229726`*^-10},
  {79.43282347242821`, 2.8263976582007043`*^-10}, {125.89254117941675`,
  2.8264020817557667`*^-10}, {199.52623149688807`, 2.826404871483842`*^-10},
  {316.22776601683796`, 2.826406623222553`*^-10}, {501.18723362727246`,
  2.826407707412721`*^-10}, {794.3282347242822`, 2.826408361580744`*^-10} };
cc1 = { {1.`*^-12, 2.387992815460308`*^-20},
  {3.1622776601683794`*^-12, 1.3428659051720894`*^-19},
  {1.`*^-11, 7.551469668241913`*^-19}, {3.1622776601683794`*^-11,
  4.246467435574246`*^-18}, {1.`*^-10, 2.3879000649252278`*^-17},
  {3.1622776601683795`*^-10, 1.3427009812130997`*^-16},
  {1.`*^-9, 7.548537556973553`*^-16}, {3.1622776601683795`*^-9, 4.24125724135352`*^-15},
  {1.`*^-8, 2.3786568563816526`*^-14},
  {3.162277660168379`*^-8, 1.3263863917068893`*^-13},
  {1.`*^-7, 7.265105498643671`*^-13}, {3.162277660168379`*^-7, 3.771645611509287`*^-12},
  {1.`*^-6, 1.693651380688711`*^-11}, {3.162277660168379`*^-6, 5.596343752145578`*^-11},
  {0.00001`, 1.2143170387412448`*^-10}, {0.000031622776601683795`,
  1.8545473320185802`*^-10}, {0.0001`, 2.2990103790072095`*^-10},
  {0.00031622776601683794`, 2.5572136241355057`*^-10},
  {0.001`, 2.6940085293627206`*^-10}, {0.0031622776601683794`, 2.7629192262618254`*^-10},
  {0.01`, 2.796631122065546`*^-10}, {0.03162277660168379`, 2.812823078810208`*^-10},

```

```
{0.1`, 2.8204891564907916`*^-10}, {0.31622776601683794`, 2.824025096002165`*^-10},
{0.5011872336272722`, 2.824798743528717`*^-10},
{0.7943282347242815`, 2.82533884505459`*^-10},
{1.2589254117941675`, 2.8257078900715945`*^-10},
{1.9952623149688797`, 2.825954968534554`*^-10},
{3.1622776601683795`, 2.826117568128351`*^-10},
{5.011872336272725`, 2.826223150019687`*^-10}, {7.943282347242818`,
2.826291056068594`*^-10}, {12.589254117941675`, 2.826334429579627`*^-10},
{19.952623149688797`, 2.826362064886411`*^-10}, {31.622776601683793`,
2.826379570299117`*^-10}, {50.11872336272725`, 2.82639065229726`*^-10},
{79.43282347242821`, 2.8263976582007043`*^-10}, {125.89254117941675`,
2.8264020817557667`*^-10}, {199.52623149688807`, 2.826404871483842`*^-10},
{316.22776601683796`, 2.826406623222553`*^-10}, {501.18723362727246`,
2.826407707412721`*^-10}, {794.3282347242822`, 2.826408361580744`*^-10}};
```

---

## Stop

---

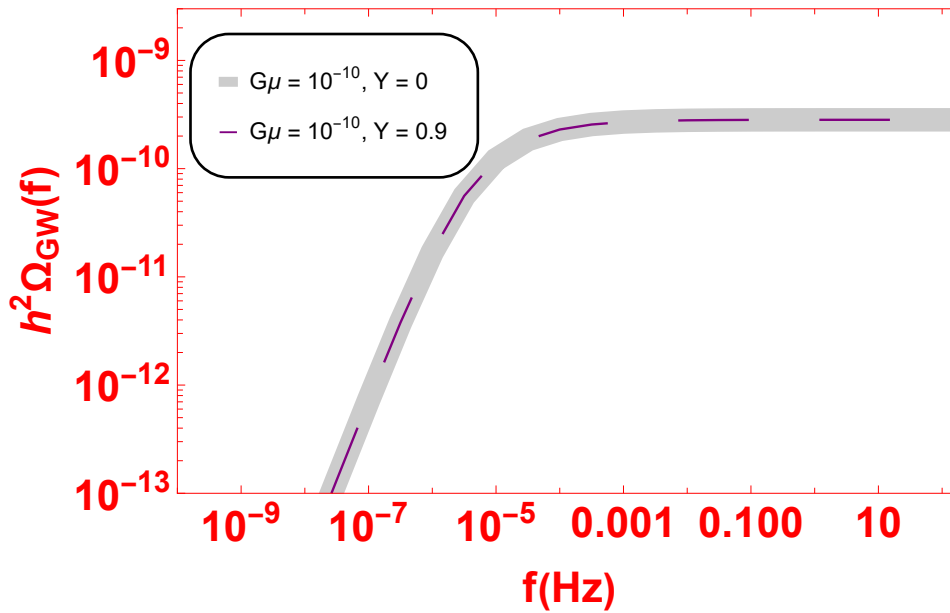
## Part-2

(Debug) In[ ]:=

```
V1 = ListLogLogPlot[{cc, cc1}, Joined → True,
Frame → True, FrameLabel → {{h2ΩGW(f)", None}, {"f (Hz)", None}},
PlotRange → {{10-10, 2 * 102}, {1 * 10-13, 3 * 10-9}}, FrameStyle → Directive[Red, 22, Bold],
ImageSize → 500, PlotStyle → {{Black, Thickness[0.03], Opacity[0.2]},
{Purple, Thickness[0.003], Dashing[0.09]}, {Orange, Thickness[0.003]},
{Darker[Green], Thickness[0.003], Dashing[0.006]}, {Darker[Red], Thickness[0.003]}},
LabelStyle → Directive[Black, 15], PlotLegends →
Placed[LineLegend[{"Gμ = 10-10, Y = 0", "Gμ = 10-10, Y = 0.9"},
LabelStyle → {12, Background → White}, LegendMarkerSize → {{10, 10}},
LegendFunction → (Framed[#, RoundingRadius → 20] &), LegendMargins → 10], {0.2, 0.8}]]
```



(Debug) Out[ ]:=



## Some unanswered Questions

**Use Wolfram Cloud**: If you are using Wolfram Cloud, you can share a notebook with multiple users . While users can edit their copies, real - time collaboration is still limited . Coordination between users is required .

---

## Loop, Data Import, Export, Table Formation

### Nested loop

(Debug) In[ ]:=

```
function[x_, y_] = x3 + y4 + 4 y3 + x2 + y2 + y;
```

(Debug) In[ ]:=

```

x = x1;
kk = 0;
xmin = -4;
xmax = 4;
ymin = -4;
ymax = 4;
y = y1;
Do[
  Clear[function];
  Do[function[x_, y_] = x3 + y4 + 4 y + x2;
    kk = kk + 1;
    ddf[kk] = {x, y, function[x, y]};
    Print[ddf[kk]];
    ddd = Table[ddf[jj], {jj, 1, kk}];, {y1, ymin, ymax, 1}];, {x1, xmin, xmax, 1}]

{-4, -4, 192}
{-4, -3, 21}
{-4, -2, -40}
{-4, -1, -51}
{-4, 0, -48}
{-4, 1, -43}
{-4, 2, -24}
{-4, 3, 45}
{-4, 4, 224}
{-3, -4, 222}
{-3, -3, 51}
{-3, -2, -10}
{-3, -1, -21}
{-3, 0, -18}
{-3, 1, -13}
{-3, 2, 6}
{-3, 3, 75}
{-3, 4, 254}
{-2, -4, 236}
{-2, -3, 65}
{-2, -2, 4}
{-2, -1, -7}
{-2, 0, -4}
{-2, 1, 1}

```

$\{-2, 2, 20\}$   
 $\{-2, 3, 89\}$   
 $\{-2, 4, 268\}$   
 $\{-1, -4, 240\}$   
 $\{-1, -3, 69\}$   
 $\{-1, -2, 8\}$   
 $\{-1, -1, -3\}$   
 $\{-1, 0, 0\}$   
 $\{-1, 1, 5\}$   
 $\{-1, 2, 24\}$   
 $\{-1, 3, 93\}$   
 $\{-1, 4, 272\}$   
 $\{0, -4, 240\}$   
 $\{0, -3, 69\}$   
 $\{0, -2, 8\}$   
 $\{0, -1, -3\}$   
 $\{0, 0, 0\}$   
 $\{0, 1, 5\}$   
 $\{0, 2, 24\}$   
 $\{0, 3, 93\}$   
 $\{0, 4, 272\}$   
 $\{1, -4, 242\}$   
 $\{1, -3, 71\}$   
 $\{1, -2, 10\}$   
 $\{1, -1, -1\}$   
 $\{1, 0, 2\}$   
 $\{1, 1, 7\}$   
 $\{1, 2, 26\}$   
 $\{1, 3, 95\}$   
 $\{1, 4, 274\}$   
 $\{2, -4, 252\}$   
 $\{2, -3, 81\}$   
 $\{2, -2, 20\}$   
 $\{2, -1, 9\}$   
 $\{2, 0, 12\}$   
 $\{2, 1, 17\}$

```

{2, 2, 36}
{2, 3, 105}
{2, 4, 284}
{3, -4, 276}
{3, -3, 105}
{3, -2, 44}
{3, -1, 33}
{3, 0, 36}
{3, 1, 41}
{3, 2, 60}
{3, 3, 129}
{3, 4, 308}
{4, -4, 320}
{4, -3, 149}
{4, -2, 88}
{4, -1, 77}
{4, 0, 80}
{4, 1, 85}
{4, 2, 104}
{4, 3, 173}
{4, 4, 352}

```

```
(Debug) In[ ]:=
```

```
ddd
```

```
(Debug) Out[ ]:=
```

```

{{-4, -4, 192}, {-4, -3, 21}, {-4, -2, -40}, {-4, -1, -51}, {-4, 0, -48}, {-4, 1, -43},
{-4, 2, -24}, {-4, 3, 45}, {-4, 4, 224}, {-3, -4, 222}, {-3, -3, 51}, {-3, -2, -10},
{-3, -1, -21}, {-3, 0, -18}, {-3, 1, -13}, {-3, 2, 6}, {-3, 3, 75}, {-3, 4, 254},
{-2, -4, 236}, {-2, -3, 65}, {-2, -2, 4}, {-2, -1, -7}, {-2, 0, -4}, {-2, 1, 1},
{-2, 2, 20}, {-2, 3, 89}, {-2, 4, 268}, {-1, -4, 240}, {-1, -3, 69}, {-1, -2, 8},
{-1, -1, -3}, {-1, 0, 0}, {-1, 1, 5}, {-1, 2, 24}, {-1, 3, 93}, {-1, 4, 272},
{0, -4, 240}, {0, -3, 69}, {0, -2, 8}, {0, -1, -3}, {0, 0, 0}, {0, 1, 5}, {0, 2, 24},
{0, 3, 93}, {0, 4, 272}, {1, -4, 242}, {1, -3, 71}, {1, -2, 10}, {1, -1, -1},
{1, 0, 2}, {1, 1, 7}, {1, 2, 26}, {1, 3, 95}, {1, 4, 274}, {2, -4, 252}, {2, -3, 81},
{2, -2, 20}, {2, -1, 9}, {2, 0, 12}, {2, 1, 17}, {2, 2, 36}, {2, 3, 105}, {2, 4, 284},
{3, -4, 276}, {3, -3, 105}, {3, -2, 44}, {3, -1, 33}, {3, 0, 36}, {3, 1, 41},
{3, 2, 60}, {3, 3, 129}, {3, 4, 308}, {4, -4, 320}, {4, -3, 149}, {4, -2, 88},
{4, -1, 77}, {4, 0, 80}, {4, 1, 85}, {4, 2, 104}, {4, 3, 173}, {4, 4, 352}}

```

```
(*Table helps in: Multiplying a pre-factor, shuffle the axis, Log,*)
```

```
(Debug) In[ ]:=
```

```
ddd[[All, 3]]
```

```
(Debug) Out[ ]:=
```

```
{192, 21, -40, -51, -48, -43, -24, 45, 224, 222, 51, -10, -21, -18, -13, 6, 75, 254,  
236, 65, 4, -7, -4, 1, 20, 89, 268, 240, 69, 8, -3, 0, 5, 24, 93, 272, 240, 69, 8, -3,  
0, 5, 24, 93, 272, 242, 71, 10, -1, 2, 7, 26, 95, 274, 252, 81, 20, 9, 12, 17, 36, 105,  
284, 276, 105, 44, 33, 36, 41, 60, 129, 308, 320, 149, 88, 77, 80, 85, 104, 173, 352}
```

```
(Debug) In[ ]:=
```

```
Length[ddd]
```

```
(Debug) Out[ ]:=
```

```
81
```

```
(Debug) In[ ]:=
```

```
gg = Table[{ddd[[jj, 1]], ddd[[jj, 2]], ddd[[jj, 3]]}, {jj, 1, Length[ddd]}]
```

```
(Debug) Out[ ]:=
```

```
{{-4, -4, 192}, {-4, -3, 21}, {-4, -2, -40}, {-4, -1, -51}, {-4, 0, -48}, {-4, 1, -43},  
{-4, 2, -24}, {-4, 3, 45}, {-4, 4, 224}, {-3, -4, 222}, {-3, -3, 51}, {-3, -2, -10},  
{-3, -1, -21}, {-3, 0, -18}, {-3, 1, -13}, {-3, 2, 6}, {-3, 3, 75}, {-3, 4, 254},  
{-2, -4, 236}, {-2, -3, 65}, {-2, -2, 4}, {-2, -1, -7}, {-2, 0, -4}, {-2, 1, 1},  
{-2, 2, 20}, {-2, 3, 89}, {-2, 4, 268}, {-1, -4, 240}, {-1, -3, 69}, {-1, -2, 8},  
{-1, -1, -3}, {-1, 0, 0}, {-1, 1, 5}, {-1, 2, 24}, {-1, 3, 93}, {-1, 4, 272},  
{0, -4, 240}, {0, -3, 69}, {0, -2, 8}, {0, -1, -3}, {0, 0, 0}, {0, 1, 5}, {0, 2, 24},  
{0, 3, 93}, {0, 4, 272}, {1, -4, 242}, {1, -3, 71}, {1, -2, 10}, {1, -1, -1},  
{1, 0, 2}, {1, 1, 7}, {1, 2, 26}, {1, 3, 95}, {1, 4, 274}, {2, -4, 252}, {2, -3, 81},  
{2, -2, 20}, {2, -1, 9}, {2, 0, 12}, {2, 1, 17}, {2, 2, 36}, {2, 3, 105}, {2, 4, 284},  
{3, -4, 276}, {3, -3, 105}, {3, -2, 44}, {3, -1, 33}, {3, 0, 36}, {3, 1, 41},  
{3, 2, 60}, {3, 3, 129}, {3, 4, 308}, {4, -4, 320}, {4, -3, 149}, {4, -2, 88},  
{4, -1, 77}, {4, 0, 80}, {4, 1, 85}, {4, 2, 104}, {4, 3, 173}, {4, 4, 352}}
```

```
(Debug) In[ ]:=
```

```
gg[[All, 3]]
```

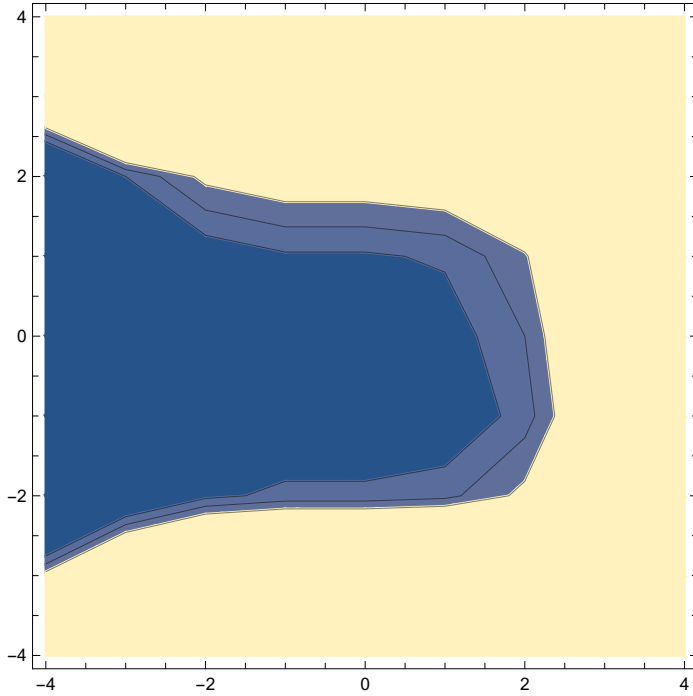
```
(Debug) Out[ ]:=
```

```
{192, 21, -40, -51, -48, -43, -24, 45, 224, 222, 51, -10, -21, -18, -13, 6, 75, 254,  
236, 65, 4, -7, -4, 1, 20, 89, 268, 240, 69, 8, -3, 0, 5, 24, 93, 272, 240, 69, 8, -3,  
0, 5, 24, 93, 272, 242, 71, 10, -1, 2, 7, 26, 95, 274, 252, 81, 20, 9, 12, 17, 36, 105,  
284, 276, 105, 44, 33, 36, 41, 60, 129, 308, 320, 149, 88, 77, 80, 85, 104, 173, 352}
```

```
(Debug) In[ ]:=
```

```
ListContourPlot[gg, Contours -> {6, 12, 18}]
```

(Debug) Out[ ]:=



(Debug) In[ ]:=

```
Clear[x, y, function]
```

```
(*%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%*)
```

## Professor Movahed's Question

(Debug) In[ ]:=

```
function1[x_, y_] = x3 + y4 + 4 y + x2 + x4;
```

(Debug) In[ ]:=

```
ss = Table[function1[x, y], {y, -4, 4, 1}, {x, -4, 4, 1}];
```

(Debug) In[ ]:=

```
ss // MatrixForm
```

(Debug) Out[ ]//MatrixForm=

448	303	252	241	240	243	268	357	576
277	132	81	70	69	72	97	186	405
216	71	20	9	8	11	36	125	344
205	60	9	-2	-3	0	25	114	333
208	63	12	1	0	3	28	117	336
213	68	17	6	5	8	33	122	341
232	87	36	25	24	27	52	141	360
301	156	105	94	93	96	121	210	429
480	335	284	273	272	275	300	389	608

(Debug) In[ ]:=

```
ss
```

```
tt = {{448, 303, 252, 241, 240, 243, 268, 357, 576},
      {277, 132, 81, 70, 69, 72, 97, 186, 405}, {216, 71, 20, 9, 8, 11, 36, 125, 344},
      {205, 60, 9, -2, -3, 0, 25, 114, 333}, {208, 63, 12, 1, 0, 3, 28, 117, 336},
      {213, 68, 17, 6, 5, 8, 33, 122, 341}, {232, 87, 36, 25, 24, 27, 52, 141, 360},
      {301, 156, 105, 94, 93, 96, 121, 210, 429}, {480, 335, 284, 273, 272, 275, 300, 389, 608}}
```

```
(Debug) In[ ]:=
Dimensions[%88]
```

```
(Debug) Out[ ]:=
{9, 9}
```

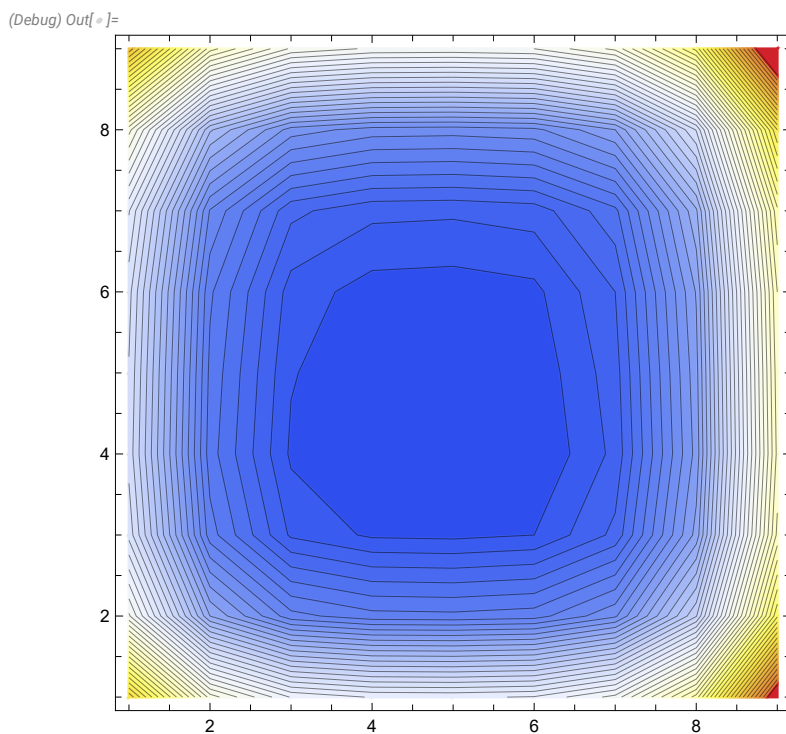
```
(Debug) In[ ]:=
Table[function1[x, -4], {x, -4, 4, 1}]
```

```
(Debug) Out[ ]:=
{448, 303, 252, 241, 240, 243, 268, 357, 576}
```

```
(Debug) In[ ]:=
Dimensions[ss]
```

```
(Debug) Out[ ]:=
{9, 9}
```

```
(Debug) In[ ]:=
ListContourPlot[ss, Contours -> 50, ColorFunction -> "TemperatureMap"]
```



```
(*%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%*)
```

```
(Debug) In[ ]:=
function[x_, y_] = x3 + y4 + 4 y3 + x2 + y2 + y;
```

(Debug) In[ ]:=

```
ss1 = Flatten[Table[{x, y, function[x, y]}, {y, -2, 2, 0.1}, {x, -4, 4, 0.1}], 1]
```

(Debug) Out[ ]:=

```
{
  {-4., -2., -62.}, {-3.9, -2., -58.109}, {-3.8, -2., -54.432}, {-3.7, -2., -50.963},
  {-3.6, -2., -47.696}, {-3.5, -2., -44.625}, {-3.4, -2., -41.744},
  {-3.3, -2., -39.047}, {-3.2, -2., -36.528}, {-3.1, -2., -34.181}, {-3., -2., -32.},
  {-2.9, -2., -29.979}, {-2.8, -2., -28.112}, {-2.7, -2., -26.393}, ... 3294 ...,
  {2.8, 2., 83.792}, {2.9, 2., 86.799}, {3., 2., 90.}, {3.1, 2., 93.401}, {3.2, 2., 97.008},
  {3.3, 2., 100.827}, {3.4, 2., 104.864}, {3.5, 2., 109.125}, {3.6, 2., 113.616},
  {3.7, 2., 118.343}, {3.8, 2., 123.312}, {3.9, 2., 128.529}, {4., 2., 134.}
}
```

Full expression not available (original memory size: 0.4 MB)



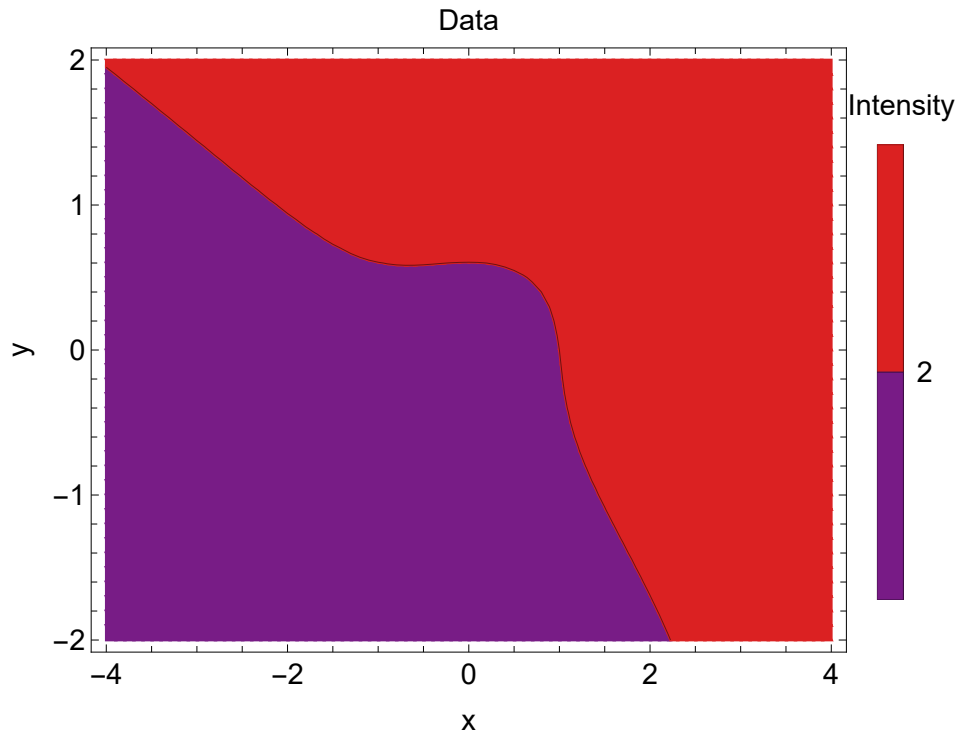
(Debug) In[ ]:=

```
Table[{ss1[[jj, 2]], ss1[[jj, 3]]}, {jj, 1, Length[ss1]}
```

(Debug) In[ ]:=

```
ListContourPlot[ss1, ColorFunction -> "Rainbow", AspectRatio -> 0.8,
  Frame -> True, FrameLabel -> {"x", "y", "Data"}, FrameStyle -> Directive[Black, 15],
  ImageSize -> 440, PlotLegends -> Placed[BarLegend[Automatic,
  LegendLabel -> "Intensity", LabelStyle -> Directive[FontSize -> 15, Black],
  LegendMargins -> {{-5, 5}, {10, 5}}, LegendMarkerSize -> 260], Right], Contours -> {2}]
```

(Debug) Out[ ]:=



(Debug) In[ ]:=

```
ddd
```



```
(Debug) Out[ ]:=
```

```
{{-4, -4, 192}, {-4, -3, 21}, {-4, -2, -40}, {-4, -1, -51}, {-4, 0, -48}, {-4, 1, -43},
{-4, 2, -24}, {-4, 3, 45}, {-4, 4, 224}, {-3, -4, 222}, {-3, -3, 51}, {-3, -2, -10},
{-3, -1, -21}, {-3, 0, -18}, {-3, 1, -13}, {-3, 2, 6}, {-3, 3, 75}, {-3, 4, 254},
{-2, -4, 236}, {-2, -3, 65}, {-2, -2, 4}, {-2, -1, -7}, {-2, 0, -4}, {-2, 1, 1},
{-2, 2, 20}, {-2, 3, 89}, {-2, 4, 268}, {-1, -4, 240}, {-1, -3, 69}, {-1, -2, 8},
{-1, -1, -3}, {-1, 0, 0}, {-1, 1, 5}, {-1, 2, 24}, {-1, 3, 93}, {-1, 4, 272},
{0, -4, 240}, {0, -3, 69}, {0, -2, 8}, {0, -1, -3}, {0, 0, 0}, {0, 1, 5}, {0, 2, 24},
{0, 3, 93}, {0, 4, 272}, {1, -4, 242}, {1, -3, 71}, {1, -2, 10}, {1, -1, -1},
{1, 0, 2}, {1, 1, 7}, {1, 2, 26}, {1, 3, 95}, {1, 4, 274}, {2, -4, 252}, {2, -3, 81},
{2, -2, 20}, {2, -1, 9}, {2, 0, 12}, {2, 1, 17}, {2, 2, 36}, {2, 3, 105}, {2, 4, 284},
{3, -4, 276}, {3, -3, 105}, {3, -2, 44}, {3, -1, 33}, {3, 0, 36}, {3, 1, 41},
{3, 2, 60}, {3, 3, 129}, {3, 4, 308}, {4, -4, 320}, {4, -3, 149}, {4, -2, 88},
{4, -1, 77}, {4, 0, 80}, {4, 1, 85}, {4, 2, 104}, {4, 3, 173}, {4, 4, 352}}
```

```
(Debug) In[ ]:=
```

```
SetDirectory[NotebookDirectory[]];
```

```
Export["function_1.DAT", ddd];
```

```
(Debug) In[ ]:=
```

```
a1 = Import["function_1.DAT"]
```

```
(Debug) Out[ ]:=
```

```
{{-4, -4, 192}, {-4, -3, 21}, {-4, -2, -40}, {-4, -1, -51}, {-4, 0, -48}, {-4, 1, -43},
{-4, 2, -24}, {-4, 3, 45}, {-4, 4, 224}, {-3, -4, 222}, {-3, -3, 51}, {-3, -2, -10},
{-3, -1, -21}, {-3, 0, -18}, {-3, 1, -13}, {-3, 2, 6}, {-3, 3, 75}, {-3, 4, 254},
{-2, -4, 236}, {-2, -3, 65}, {-2, -2, 4}, {-2, -1, -7}, {-2, 0, -4}, {-2, 1, 1},
{-2, 2, 20}, {-2, 3, 89}, {-2, 4, 268}, {-1, -4, 240}, {-1, -3, 69}, {-1, -2, 8},
{-1, -1, -3}, {-1, 0, 0}, {-1, 1, 5}, {-1, 2, 24}, {-1, 3, 93}, {-1, 4, 272},
{0, -4, 240}, {0, -3, 69}, {0, -2, 8}, {0, -1, -3}, {0, 0, 0}, {0, 1, 5}, {0, 2, 24},
{0, 3, 93}, {0, 4, 272}, {1, -4, 242}, {1, -3, 71}, {1, -2, 10}, {1, -1, -1},
{1, 0, 2}, {1, 1, 7}, {1, 2, 26}, {1, 3, 95}, {1, 4, 274}, {2, -4, 252}, {2, -3, 81},
{2, -2, 20}, {2, -1, 9}, {2, 0, 12}, {2, 1, 17}, {2, 2, 36}, {2, 3, 105}, {2, 4, 284},
{3, -4, 276}, {3, -3, 105}, {3, -2, 44}, {3, -1, 33}, {3, 0, 36}, {3, 1, 41},
{3, 2, 60}, {3, 3, 129}, {3, 4, 308}, {4, -4, 320}, {4, -3, 149}, {4, -2, 88},
{4, -1, 77}, {4, 0, 80}, {4, 1, 85}, {4, 2, 104}, {4, 3, 173}, {4, 4, 352}}
```

---

## Conversion to FORTran and L<sup>A</sup>T<sub>E</sub>X

```
(Debug) In[ ]:=
```

```
TeXForm[xx2 + xx4 - xx6 + 1]
```

```
(Debug) Out[ ]//TeXForm=
```

```
- \text{xx}^6 + \text{xx}^4 + \text{xx}^2 + 1
```

```
(Debug) In[ ]:=
```

```
(xx2 + xx4 - xx6 + 1) // TeXForm
```

```
(Debug) Out[ ]//TeXForm=
```

```
- \text{xx}^6 + \text{xx}^4 + \text{xx}^2 + 1
```

```
(Debug) In[ ]:=
```

```
FortranForm[xx2 + xx4 - xx6 + 1]
```

```
(Debug) Out[ ]//FortranForm=
```

```
1 + xx**2 + xx**4 - xx**6
```

## Extras

```
(Debug) In[ ]:=
```

```

 $\mathcal{A} = \{\{\delta\}, \{\delta x\}, \{\delta y\}\};$ 
 $\mathcal{A}^{\text{transpose}} = \text{Transpose}[\mathcal{A}];$ 
 $\mathcal{K} = \{\{\sigma\theta^2, \theta, \theta\}, \{\theta, \sigma 1x^2, \theta\}, \{\theta, \theta, \sigma 1y^2\}\};$ 
 $\text{det}\mathcal{K} = \text{Det}[\mathcal{K}];$ 
 $\mathcal{K}^{\text{2inverse}} = \text{Inverse}[\mathcal{K}];$ 

```

```
(Debug) In[ ]:=
```

```
{ $\mathcal{K}$  // MatrixForm, MatrixForm[ $\mathcal{K}^{\text{2inverse}}$ ]}
```

```
(Debug) Out[ ]:=
```

$$\left\{ \begin{pmatrix} \sigma\theta^2 & \theta & \theta \\ \theta & \sigma 1x^2 & \theta \\ \theta & \theta & \sigma 1y^2 \end{pmatrix}, \begin{pmatrix} \sigma\theta^2 & \theta & \theta \\ \theta & \sigma 1x^2 & \theta \\ \theta & \theta & \sigma 1y^2 \end{pmatrix} \right\}$$

```
(Debug) In[ ]:=
```

$$\text{Pdf} = \frac{1}{\sqrt{(2\pi)^3} \sqrt{\text{det}\mathcal{K}}} \text{Exp}\left[\frac{-1}{2} \mathcal{A}^{\text{transpose}} \cdot \mathcal{K}^{\text{2inverse}} \cdot \mathcal{A}\right][[1, 1]]$$

```
(Debug) Out[ ]:=
```

$$\frac{e^{\frac{1}{2} \left( -\frac{\delta^2}{\sigma\theta^2} - \frac{\delta x^2}{\sigma 1x^2} - \frac{\delta y^2}{\sigma 1y^2} \right)}}{2 \sqrt{2} \pi^{3/2} \sqrt{\sigma\theta^2 \sigma 1x^2 \sigma 1y^2}}$$

```
(Debug) In[ ]:=
```

```

normalrsd = Integrate[Pdf, {\delta, -Infinity, Infinity}, {\delta x, -Infinity, Infinity},
  {\delta y, -Infinity, Infinity}, Assumptions -> {\sigma\theta^2 > \theta, \sigma 1x^2 > \theta, \sigma 1y^2 > \theta}]

```

```
(Debug) Out[ ]:=
```

$$\sqrt{2} \pi$$

```
(Debug) Out[ ]:=
```

$$\frac{1}{\sqrt{\frac{1}{\sigma\theta^2}} \sqrt{\frac{1}{\sigma 1x^2}} \sqrt{\frac{1}{\sigma 1y^2}} \sqrt{\sigma\theta^2 \sigma 1x^2 \sigma 1y^2}} \text{ if } \text{Re}[\sigma\theta^2] > \theta$$

## Crossing Statistics in (1+2)D

(Debug) In[ ]:=

```
Ncr1 = Abs[δx];
Ncr1Gauss = Integrate[Pdf * Ncr1 * DiracDelta[δ - σθ * θ],
  {δ, -Infinity, Infinity}, {δx, -Infinity, Infinity},
  {δy, -Infinity, Infinity}, Assumptions → {σθ > 0, σ1x > 0, σ1y > 0, θ > 0}]
```

(Debug) Out[ ]:=

$$\frac{e^{-\frac{\sigma^2}{2}} \sigma 1x}{\pi \sigma \theta}$$

(Debug) Out[ ]:=

$$\frac{1}{2 \sqrt{2} \pi^{3/2} \sqrt{\sigma \theta^2 \sigma 1x^2 \sigma 1y^2}}$$

$$\text{Integrate}\left[\text{DiracDelta}[\delta - \theta \sigma \theta] \left\{ \begin{array}{l} \frac{2 e^{-\frac{\sigma^2}{2}} \sqrt{2} \pi \sigma 1x^2}{\sqrt{\frac{1}{\sigma 1y^2}}} \\ \text{Integrate}\left[\frac{e^{-\frac{\sigma^2}{2} - \frac{\delta x^2}{2 \sigma 1x^2}} \sqrt{2} \pi \text{Abs}[\delta x]}{\sqrt{\frac{1}{\sigma 1y^2}}}, \{\delta x, -\infty, \infty\}, \right. \\ \text{Assumptions} \rightarrow \text{Re}[\sigma 1y^2] > 0 \ \&\& \ \text{Re}[\sigma 1x^2] \leq 0, \\ \left. \text{GenerateConditions} \rightarrow \text{True}, \text{PrincipalValue} \rightarrow \text{False} \right] \\ \text{GenerateConditions} \rightarrow \text{Automatic}, \text{PrincipalValue} \rightarrow \text{False} \Big] \text{ if } \text{Re}[\sigma 1y^2] > 0 \end{array} \right.$$

(Debug) In[ ]:=

```
a = 1;
Which[a == 1, x, a == 2, b]
```

(Debug) Out[ ]:=

x

(Debug) In[ ]:=

```
cc = 100;
Which[cc > 200, θ, True, Exp[-cc]]
```

(Debug) Out[ ]:=

$$\frac{1}{e^{100}}$$

(Debug) In[ ]:=

```
Normal[Series[Exp[x1], {x1, 0, 6}]]
```

(Debug) Out[ ]:=

$$1 + x1 + \frac{x1^2}{2} + \frac{x1^3}{6} + \frac{x1^4}{24} + \frac{x1^5}{120} + \frac{x1^6}{720}$$

(Debug) In[ ]:=

**Expand**[(1 + x)<sup>3</sup> (5 x<sup>2</sup> + 1) (x<sup>5</sup> + 1)]

(Debug) Out[ ]:=

1 + 3 x + 8 x<sup>2</sup> + 16 x<sup>3</sup> + 15 x<sup>4</sup> + 6 x<sup>5</sup> + 3 x<sup>6</sup> + 8 x<sup>7</sup> + 16 x<sup>8</sup> + 15 x<sup>9</sup> + 5 x<sup>10</sup>

(Debug) In[ ]:=

**ExpToTrig**[e<sup>x1</sup>]

(Debug) Out[ ]:=

Cosh[x1] + Sinh[x1]

(Debug) In[ ]:=

**TrigToExp**[Cosh[x1] + Sinh[x1]]

(Debug) Out[ ]:=

e<sup>x1</sup>

## Ctrl + 4

(Debug) In[ ]:=

a \times b 10<sup>{-3}</sup>

(Debug) Out[ ]:=

$\frac{a \times b}{1000}$



(Debug) In[ ]:=

```
sd=1;
d=2;
sd**d*2
```

(Debug) Out[ ]:=

2

(Debug) In[ ]:=

**Sinh**

Assuming "Sinh" is referring to a mathematical definition | Use as [a math function](#) instead

Input interpretation:

hyperbolic sine

Alternate name:

sh

Definition:

The hyperbolic sine is defined as

$\sinh(x) = \frac{e^x - e^{-x}}{2}$

[More details](#)

$$\sinh z \equiv \frac{1}{2} (e^z - e^{-z}).$$

The notation  $\operatorname{sh} z$  is sometimes also used (Gradshteyn and Ryzhik 2000, p. xxix). It is implemented in the Wolfram Language as `Sinh[z]`.

Special values include

$$\sinh 0 = 0$$

$$\sinh(\ln \phi) = \frac{1}{2},$$

where  $\phi$  is the golden ratio.

[More information »](#)

Related terms:

beta exponential function | bipolar coordinates | bipolar cylindrical coordinates | bispherical coordinates | catenary | catenoid | conical function | cubic equation | de Moivre's identity | Dixon-Ferrar formula | elliptic cylindrical coordinates | Elsasser function | Gudermannian | helicoid | Helmholtz differential equation—elliptic cylindrical coordinates | hyperbolic cosecant | hyperbolic functions | inverse hyperbolic sine | Laplace's equation—bispherical coordinates | Laplace's equation—toroidal coordinates | Lebesgue constants | Lorentz group | Mercator projection | Miller cylindrical projection | modified Bessel function of the second kind | modified spherical Bessel function of the first kind | modified Struve function | Nicholson's formula | oblate spheroidal coordinates | parabola involute | partition function P | Poincot's spirals | prolate spheroidal coordinates | Schläfli's formula | Shi | sine | sine-Gordon equation | surface of revolution | tau function | toroidal coordinates | toroidal function | tractrix | Watson's formula

Related Wolfram Language symbol:

`Sinh`

Subject classifications:

[Show details](#)

MathWorld:

**hyperbolic functions**

MSC 2010:

33B10

WolframAlpha



Wolfram|Alpha doesn't know how to interpret your input. ?

WolframAlpha

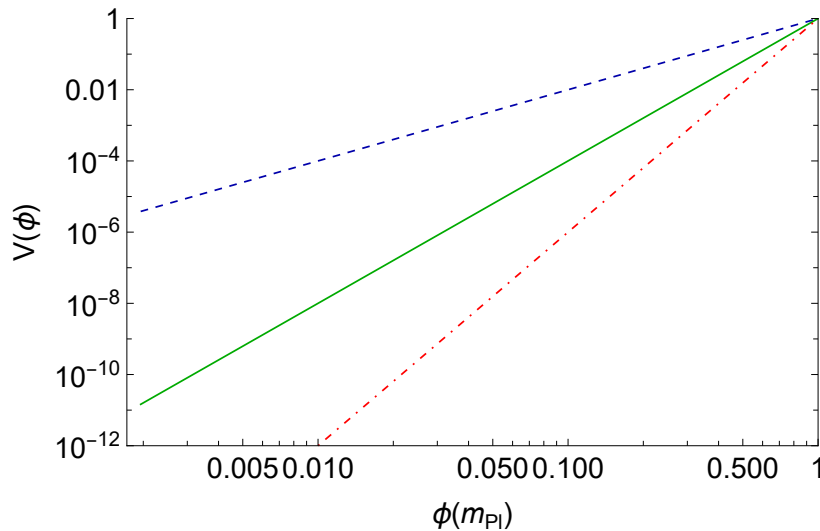
# Plotting

## Plotting With Padding

(Debug) In[ ]:=

```
PP = LogLogPlot[{xx2, xx4, xx6}, {xx, -1, 1},
  Frame → {{True, True}, {True, False}}, PlotRange → {{-1, 1}, {10-12, 100}},
  PlotStyle → {{Darker[Blue], Thickness[0.0025], Dashed},
    {Darker[Green], Thickness[0.0025]}, {Red, Thickness[0.0025], DotDashed},
    {Blue, Dashing[0.006]}, {Red}, {Red, Dashing[0.006]}}, FrameLabel → {"ϕ (mPl)", "V(ϕ)"},
  FrameStyle → Directive[Black, 15], ImagePadding → 65, ImageSize → 490]
```

(Debug) Out[ ]:=

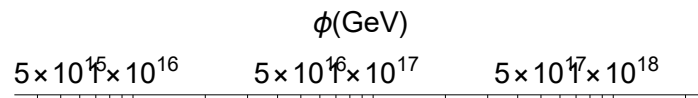


```
mpl = 2.435515 * 10-18 (*GeV*)
```

(Debug) In[ ]:=

```
a22 = ListLogLogPlot[{Centrald1p5pm5},
  Joined → True, FrameLabel → {{ "", "" }, { "", "ϕ (GeV)" }},
  FrameStyle → Directive[Black, 15], PlotStyle → {{Black, Opacity[0]}},
  PlotRange → {{-2.435515 * 1018, 2.435515 * 1018}, {10-12, 100}}, ImageSize → 490,
  ImagePadding → 65, Frame → {{False, False}, {False, True}}, Axes → False, FrameTicks → All]
```

(Debug) Out[ ]:=



(Debug) In[ ]:=

```

Pixe = Import [
  "C:\\Users\\adeel\\OneDrive\\Desktop\\Notes_CS_PBH\\CSs_Bounds_Program_code\\PBHbounds
  -master\\PBHbounds-master\\bounds\\PowerSpectrum\\PIXIE_1.DAT"];
Lyalpha = Import [
  "C:\\Users\\adeel\\OneDrive\\Desktop\\Notes_CS_PBH\\CSs_Bounds_Program_code\\PBHbounds
  -master\\PBHbounds-master\\bounds\\PowerSpectrum\\Lyman-alpha_1.DAT"];
Lyalpha2 = Import [
  "C:\\Users\\adeel\\OneDrive\\Desktop\\Notes_CS_PBH\\CSs_Bounds_Program_code\\PBHbounds
  -master\\PBHbounds-master\\bounds\\PowerSpectrum\\Lyman-alpha2_1.DAT"];
PBHDM = Import [
  "C:\\Users\\adeel\\OneDrive\\Desktop\\Notes_CS_PBH\\CSs_Bounds_Program_code\\PBHbounds
  -master\\PBHbounds-master\\bounds\\PowerSpectrum\\PBHDM_1.DAT"];
Planck = Import [
  "C:\\Users\\adeel\\OneDrive\\Desktop\\Notes_CS_PBH\\CSs_Bounds_Program_code\\PBHbounds
  -master\\PBHbounds-master\\bounds\\PowerSpectrum\\planck_1.DAT"];
Planck2 = Import [
  "C:\\Users\\adeel\\OneDrive\\Desktop\\Notes_CS_PBH\\CSs_Bounds_Program_code\\PBHbounds
  -master\\PBHbounds-master\\bounds\\PowerSpectrum\\planck2_1.DAT"];
Lisa = Import [
  "C:\\Users\\adeel\\OneDrive\\Desktop\\Notes_CS_PBH\\CSs_Bounds_Program_code\\PBHbounds
  -master\\PBHbounds-master\\bounds\\PowerSpectrum\\LISA_1.DAT"];
Lyalpha2mod = Table [ {Lyalpha2[[jj, 1]], Lyalalpha2[[jj, 2]]}, {jj, 1, Length[Lyalpha2]}];
Lyalphamod = Table [ {Lyalpha[[jj, 1]], Lyalalpha[[jj, 2]]}, {jj, 1, Length[Lyalpha]}];
COBEmu = Import [
  "C:\\Users\\adeel\\OneDrive\\Desktop\\Notes_CS_PBH\\CSs_Bounds_Program_code\\PBHbounds
  -master\\PBHbounds-master\\bounds\\PowerSpectrum\\COBE_mu.csv"];

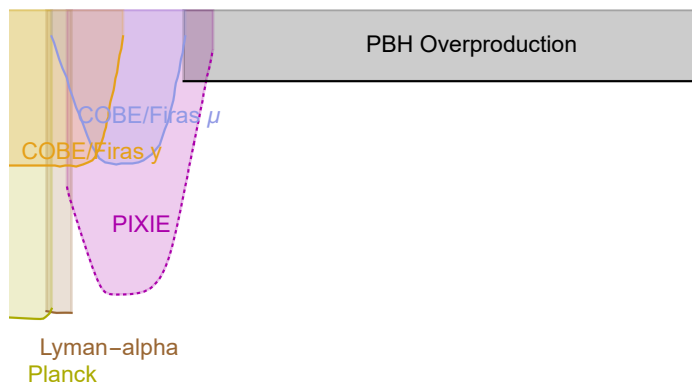
```

```

COBEy = Import[
  "C:\\Users\\adeel\\OneDrive\\Desktop\\Notes_CS_PBH\\CSs_Bounds_Program_code\\PBHbounds
  -master\\PBHbounds-master\\bounds\\PowerSpectrum\\COBE_y.csv"];
BNSBBH = Import[
  "C:\\Users\\adeel\\OneDrive\\Desktop\\PBH_An_Gh\\Neutrino_Physics\\Bounds\\2306-15555-
  fig6-SMBHB.csv"];
PSBounds = ListLogLogPlot[{Pixe, Lyalphamod, PBHDM, Planck, COBEmu, COBEy}, Joined → True,
  Frame → {False, False, False, False}, FrameLabel → {"k [Mpc]-1", "PR(k)", "Nk", None},
  FrameStyle → Directive[Black, 15], PlotStyle →
  {{Darker[Magenta], Thickness[0.003], Dashing[0.005]}, {Brown, Thickness[0.003]},
  {Black, Thickness[0.003]}, {Darker[Yellow], Thickness[0.003]},
  {Hue[0.65, 0.37, 0.9], Thickness[0.003]}, {Hue[0.11, 0.89, 0.9], Thickness[0.003]}}},
  Epilog → {Style[Text["Lyman-alpha", {Log[1 * 102], Log[3 * 10-10]}], Brown, 12],
  Style[Text["Planck", {Log[1.3], Log[6 * 10-11]}], Darker[Yellow], 12],
  Style[Text["PIXIE", {Log[2 * 103], Log[1 * 10-6]}], Darker[Magenta], 12],
  Style[Text["COBE/Firas μ", {Log[4 * 103], Log[1 * 10-3]}], Hue[0.65, 0.37, 0.9], 12],
  Style[Text["COBE/Firas γ", {Log[2 * 101], Log[1 * 10-4]}], Hue[0.11, 0.89, 0.9], 12],
  Style[Text["PBH Overproduction", {Log[4 * 1016], Log[1 * 10-1]}], Black, 12]},
  ImagePadding → 65, ImageSize → 490, Axes → False, Filling → Top,
  PlotRange → {{10-2, 6.9 * 1025}, {10-12, 100}}]

```

(Debug) Out[ ]=





(Debug) In[ ]:=

```

PSBounds1 = ListLogLogPlot[{(*Pixe,*) Lyalpha2mod, Planck2},
  Joined → True, Frame → {False, False, False, False},
  FrameLabel → {"k [Mpc]-1", "φs", "Nk", None}, FrameStyle → Directive[Black, 15],
  PlotStyle → {{Brown, Thickness[0.003]}, {Darker[Yellow], Thickness[0.003]},
    {Blue, Thickness[0.003]}, {Blue, Thickness[0.003]}}},
  (*Epilog→{Style[Text["M=0.055 mp", {Log[2*1015], Log[1*10-8]}], Darker[Gray], 15]}}, *)
  PlotRange → {{10-2, 6.9 * 1025}, {10-12, 100}}, ImagePadding → 65,
  ImageSize → 490, Axes → False, Filling → Bottom]

```

(Debug) Out[ ]:=



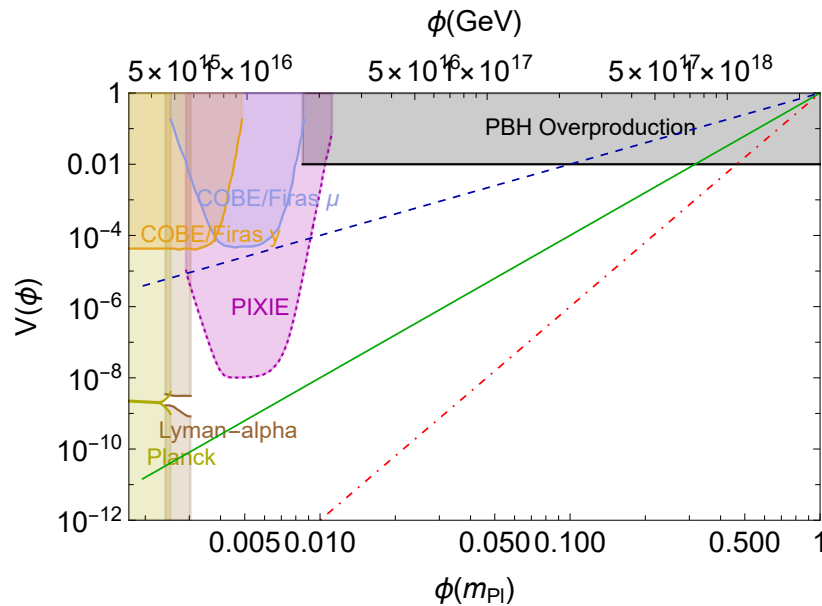
(Debug) In[ ]:=

```

ee = Overlay[{a22, PSBounds, PSBounds1, PP}]

```

(Debug) Out[ ]:=



## Plotting Residuals

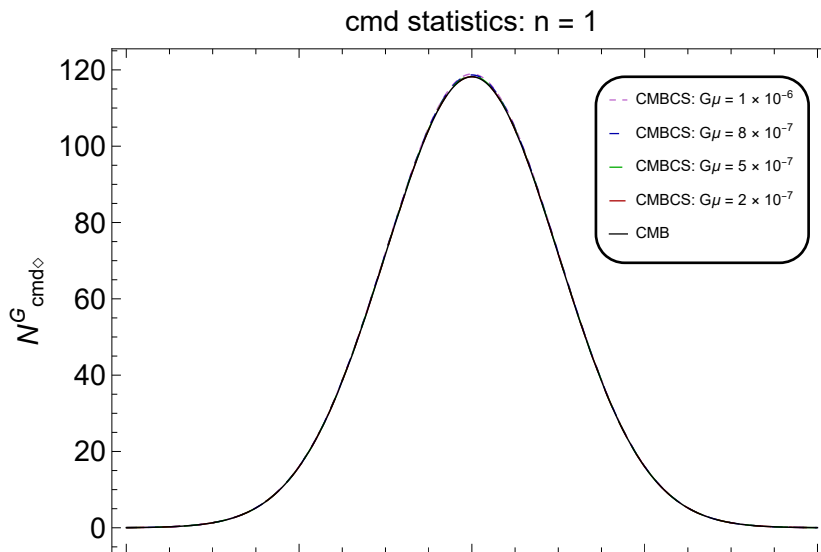
(Debug) In[ ]:=

```

n1 = 1;
PNcmdn1 = Plot[{NcmdGaussTot[Gμ, n1] /. {Gμ → 1 * 10-6},
  NcmdGaussTot[Gμ, n1] /. {Gμ → 8 * 10-7}, NcmdGaussTot[Gμ, n1] /. {Gμ → 5 * 10-7},
  NcmdGaussTot[Gμ, n1] /. {Gμ → 2 * 10-7}, NcmdGaussCMB[n1]}, {θ, -4, 4},
PlotRange → All, FrameLabel → {"", "NGcmdφ", "cmd statistics: n = 1"},
FrameStyle → Directive[Black, 15], ImageSize → 450, Axes → False,
PlotLegends → Placed[LineLegend[{"CMBCS: Gμ = 1 * 10-6",
  "CMBCS: Gμ = 8 * 10-7", "CMBCS: Gμ = 5 * 10-7", "CMBCS: Gμ = 2 * 10-7", "CMB"},
  LabelStyle → {8, Background → White}, LegendMarkerSize → {{10, 10}},
  LegendFunction → (Framed[#, RoundingRadius → 15] &), LegendMargins → 2], {0.82, 0.76}],
PlotStyle → {{Hue[0.81, 0.5, 0.8], Thickness[0.002], Dashing[0.01]},
  {Darker[Blue], Thickness[0.002], Dashing[0.015]}, {Darker[Green], Thickness[0.002],
  Dashing[0.02]}, {Darker[Red], Thickness[0.002], Dashing[0.025]},
  {Darker[Black], Thickness[0.002]}, {Darker[Yellow], Thickness[0.008], Dotted},
  {Darker[Black], Thickness[0.002], Dotted}},
Frame → True, AspectRatio → 0.7, ImagePadding → {{65, 10}, {0, 40}}]

```

```
(Debug) Out[ ]:=
```



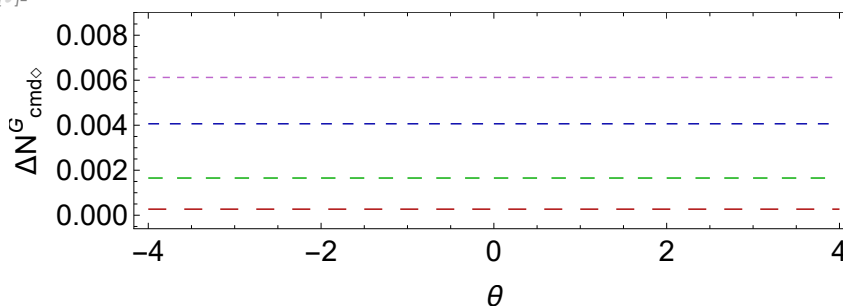
## Residual

```
(Debug) In[ ]:=
```

```
PNcmdResdn1 =
```

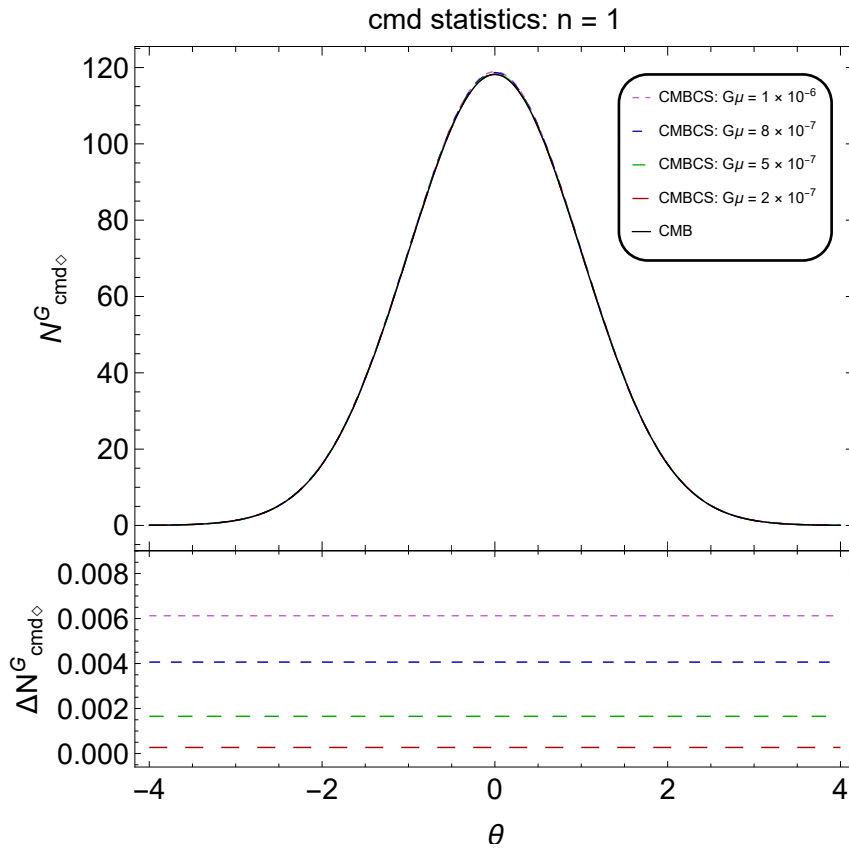
```
Plot[{NcmdGaussResd[Gμ, n1] /. {Gμ → 1 * 10-6}, NcmdGaussResd[Gμ, n1] /. {Gμ → 8 * 10-7},
      NcmdGaussResd[Gμ, n1] /. {Gμ → 5 * 10-7}, NcmdGaussResd[Gμ, n1] /. {Gμ → 2 * 10-7}},
      {θ, -4, 4}, PlotRange → {-0.0006, 0.009}, FrameLabel → {"θ", "ΔNGcmd"},
      FrameStyle → Directive[Black, 15], ImageSize → 450, Axes → False,
      PlotStyle → {{Hue[0.81, 0.5, 0.8], Thickness[0.002], Dashing[0.01]},
                  {Darker[Blue], Thickness[0.002], Dashing[0.015]}, {Darker[Green], Thickness[0.002],
                  Dashing[0.02]}, {Darker[Red], Thickness[0.002], Dashing[0.025]}}},
      Frame → True, AspectRatio → 0.3, ImagePadding → {{65, 10}, {40, 0}}]
```

```
(Debug) Out[ ]:=
```



```
(Debug) In[ ]:=
```

```
Column[{PNcmdn1, PNcmdResdn1}, Spacings → 0]
```

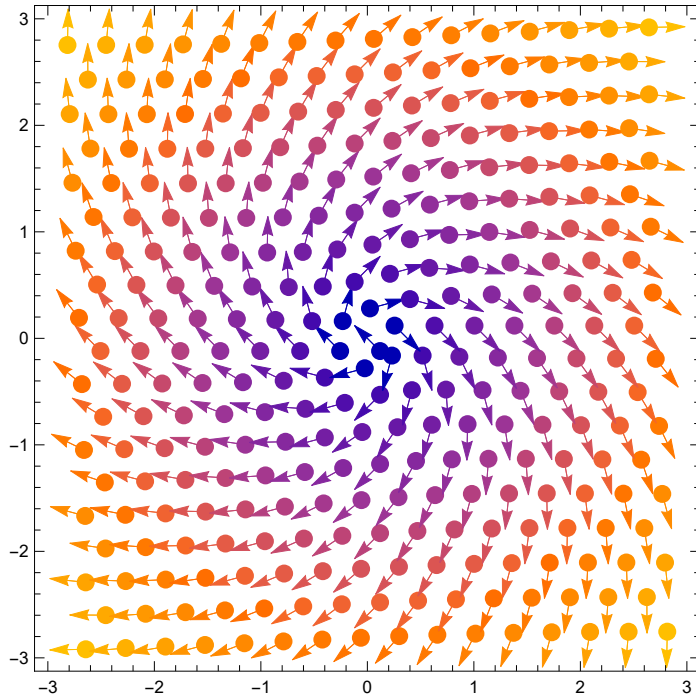
`(Debug) Out[ ]:=`

## Vector/ Stream Plots

`(Debug) In[ ]:=`

```
f1 = VectorPlot[{x1 + y1, y1 - x1}, {x1, -3, 3},
  {y1, -3, 3}, PlotLayout -> "Row", VectorMarkers -> "DotArrow"]
```

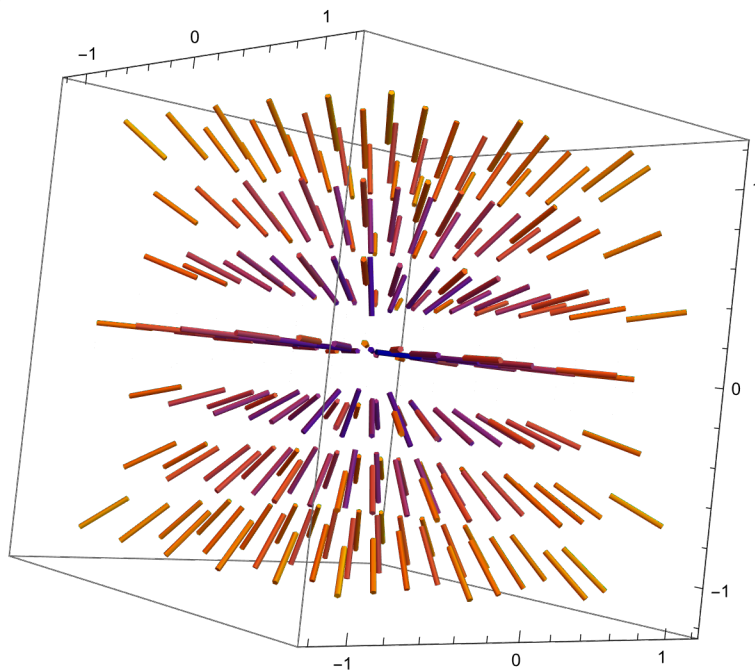
(Debug) Out[ ]:=



(Debug) In[ ]:=

```
VectorPlot3D[{x, y, z}, {x, -1, 1}, {y, -1, 1}, {z, -1, 1}, VectorMarkers -> "Tube"]
```

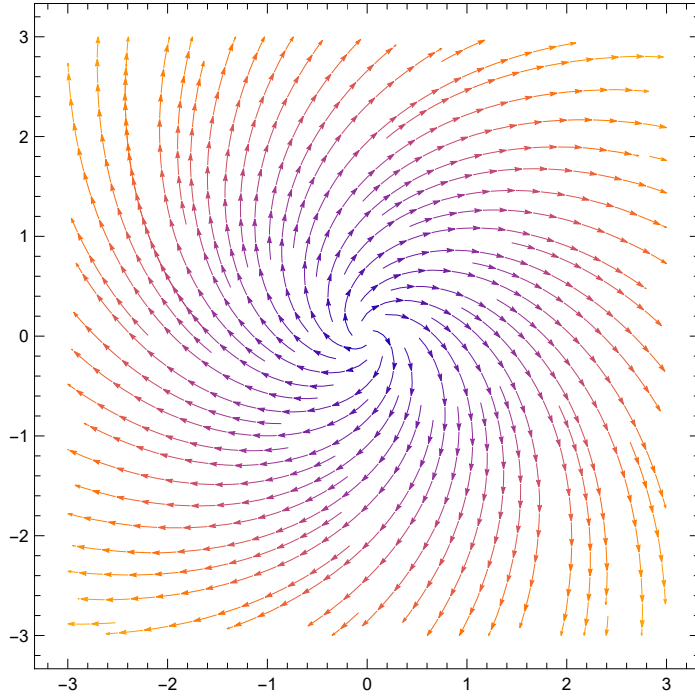
(Debug) Out[ ]:=



(Debug) In[ ]:=

```
f2 = StreamPlot[{x1 + y1, y1 - x1}, {x1, -3, 3}, {y1, -3, 3}, StreamScale -> Small]
```

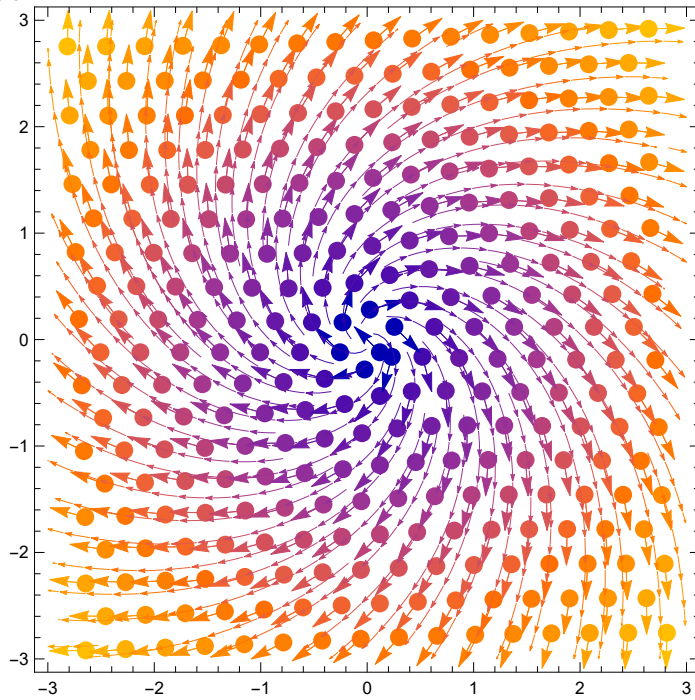
(Debug) Out[ ]:=



(Debug) In[ ]:=

**Show[f1, f2]**

(Debug) Out[ ]:=



## Plotting with L<sup>A</sup>T<sub>E</sub>X fonts

(Debug) In[ ]:=

```

SetDirectory[NotebookDirectory[]];
<< MaTeX`
ConfigureMaTeX["pdfLaTeX" → "C:\\Program Files\\MiKTeX\\miktex\\bin\\x64\\pdflatex.exe",
  "Ghostscript" → "C:\\Program Files\\gs\\gs10.01.2\\bin\\gswin64c.exe"

]
SetOptions[MaTeX, "Preamble" → {"\\usepackage{color}"}]
SetOptions[MaTeX, "Preamble" → {"\\usepackage{xcolor}"}]

```

(Debug) Out[ ]:=

```

{CacheSize → 100, WorkingDirectory → Automatic,
 pdfLaTeX → C:\Program Files\MiKTeX\miktex\bin\x64\pdflatex.exe,
 Ghostscript → C:\Program Files\gs\gs10.01.2\bin\gswin64c.exe}

```

(Debug) Out[ ]:=

```

{BasePreamble → {\usepackage{lmodern,exscale}, \usepackage{amsmath,amssymb}},
 Preamble → {\usepackage{color}}, DisplayStyle → True,
 ContentPadding → True, LineSpacing → {1.2, 0}, FontSize → 12,
 Magnification → 1, LogFileFunction → None, TeXFileFunction → None}

```

(Debug) Out[ ]:=

```

{BasePreamble → {\usepackage{lmodern,exscale}, \usepackage{amsmath,amssymb}},
 Preamble → {\usepackage{xcolor}}, DisplayStyle → True,
 ContentPadding → True, LineSpacing → {1.2, 0}, FontSize → 12,
 Magnification → 1, LogFileFunction → None, TeXFileFunction → None}

```

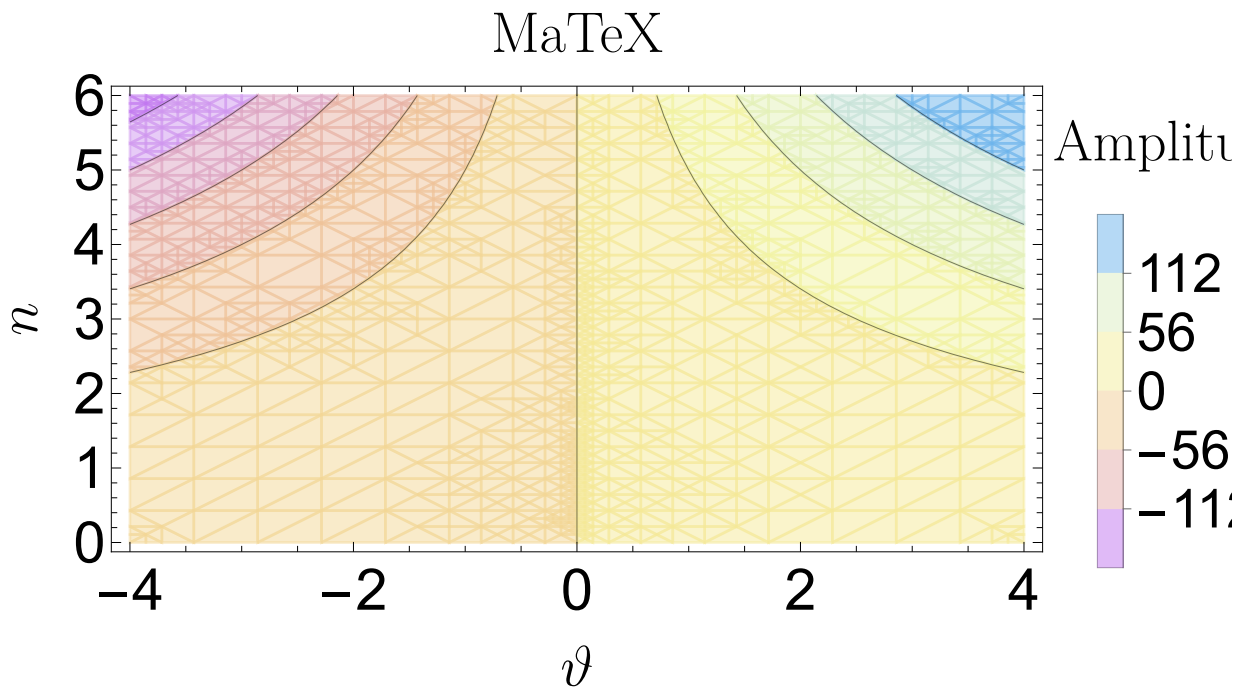
(Debug) In[ ]:=

```

ContourPlot[ $\theta * n + n^2 * \theta$ , { $\theta$ , -4, 4}, {n, 0, 6},
 ColorFunction → (Opacity[0.5, ColorData["Pastel"]][#1] &),
 Contours → 10, ColorFunctionScaling → True, PlotLegends →
   Placed[BarLegend[5, LegendLabel → MaTeX["\\text{Amplitude}"], FontSize → 30], LabelStyle →
     Directive[FontSize → 28, Black], LegendMarkerSize → 205, LegendMargins → -1], Right],
 Frame → True, FrameStyle → Directive[Black, 30], ImageSize → 550,
 AspectRatio → 0.5, FrameTicks → {True, True, False, False},
 FrameLabel → {{MaTeX["n", FontSize → 30], None},
  {MaTeX["\\vartheta", FontSize → 30], MaTeX["\\text{MaTeX}"], FontSize → 30}}]

```

(Debug) Out[ ]:=



## Plotting with Animation

### Projectile Motion

(Debug) In[ ]:=

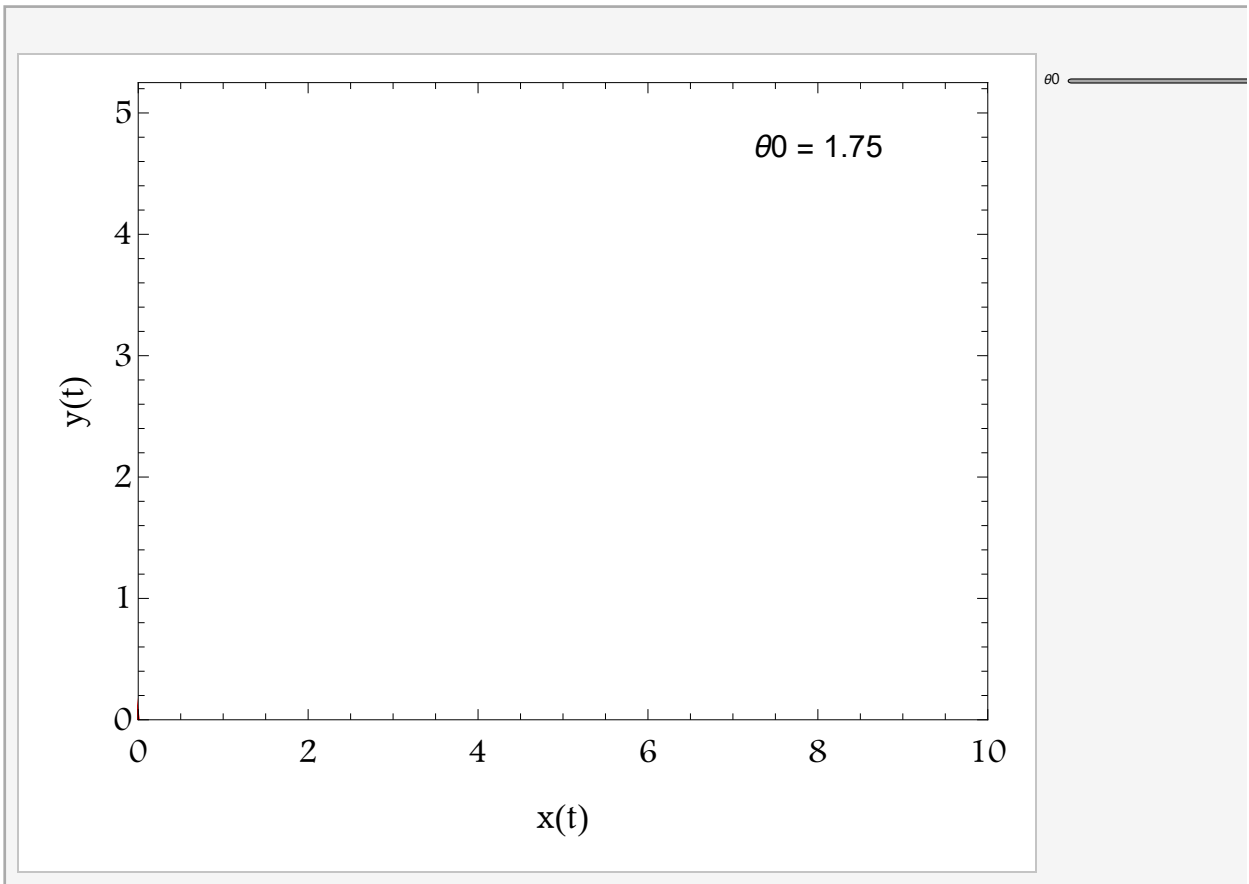
```
x[t_] = t v0 Cos[θ0];
y[t_] =  $\frac{1}{2} (-g t^2 + 2 t v0 \sin[\theta_0])$ ;
```

(Debug) In[ ]:=

```
{x[t], y[t]} /. {v0 → 10, g → 9.8};
Manipulate[ParametricPlot[%, {t, 0, 3}, AspectRatio → 0.75,
  Frame → True, FrameLabel → {"x(t)", "y(t)"}, PlotRange → {{0, 10}, {0, 5.25}},
  FrameStyle → Directive[Black, 18, FontFamily → "Andalus"],
  PlotStyle → {Red}, ImageSize → 500,
  Epilog → {Text[Style["θ0 = " <> ToString[NumberForm[θ0, {3, 2}]], 16, Black],
    Scaled[{0.8, 0.9}]]}], {θ0, -π, π}]
```



(Debug) Out[ ]:=



## Kousha Ebrahimi's Integral

$$\text{Integrate}\left[\sqrt{1 + \frac{x}{c^2 - x^2}}, x\right]$$

(Debug) In[ ]:=

$$\begin{aligned}
 dd = & \left( 2 \sqrt{\frac{c^2 + x - x^2}{c^2 - x^2}} \left( \frac{1}{8} c (1 - 2c + \sqrt{1 + 4c^2}) (1 + \sqrt{1 + 4c^2} - 2x) (c + x) (-1 + \sqrt{1 + 4c^2} + 2x) - \right. \right. \\
 & c^2 (1 + 2c + \sqrt{1 + 4c^2}) (c - x)^2 \sqrt{-\frac{c (1 + \sqrt{1 + 4c^2} - 2x)}{(1 + 2c + \sqrt{1 + 4c^2}) (-c + x)}} \\
 & \left. \left. \sqrt{\frac{(1 - 2c + \sqrt{1 + 4c^2}) (c + x)}{(1 + 2c + \sqrt{1 + 4c^2}) (-c + x)}} \sqrt{\frac{c (-1 + \sqrt{1 + 4c^2} + 2x)}{(-1 - 2c + \sqrt{1 + 4c^2}) (c - x)}} \right) \right)
 \end{aligned}$$

$$\begin{aligned}
& \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(1-2c+\sqrt{1+4c^2})(c+x)}{(1+2c+\sqrt{1+4c^2})(-c+x)}}}\right], \frac{2c+\sqrt{1+4c^2}}{2c-\sqrt{1+4c^2}}\right] - \\
& c(1+2c+\sqrt{1+4c^2})(c-x)^2 \sqrt{-\frac{c(1+\sqrt{1+4c^2}-2x)}{(1+2c+\sqrt{1+4c^2})(-c+x)}} \\
& \sqrt{\frac{(1-2c+\sqrt{1+4c^2})(c+x)}{(1+2c+\sqrt{1+4c^2})(-c+x)}} \sqrt{\frac{c(-1+\sqrt{1+4c^2}+2x)}{(-1-2c+\sqrt{1+4c^2})(c-x)}} \\
& \left( \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(1-2c+\sqrt{1+4c^2})(c+x)}{(1+2c+\sqrt{1+4c^2})(-c+x)}}}\right], \frac{2c+\sqrt{1+4c^2}}{2c-\sqrt{1+4c^2}}\right] - 2 \text{EllipticPi}\left[ \right. \\
& \quad \left. \frac{1+2c+\sqrt{1+4c^2}}{1-2c+\sqrt{1+4c^2}}, \text{ArcSin}\left[\sqrt{\frac{(1-2c+\sqrt{1+4c^2})(c+x)}{(1+2c+\sqrt{1+4c^2})(-c+x)}}}\right], \frac{2c+\sqrt{1+4c^2}}{2c-\sqrt{1+4c^2}}\right] \right) - \\
& \frac{1}{2} c(1+2c+\sqrt{1+4c^2})(c-x)^2 \sqrt{-\frac{c(1+\sqrt{1+4c^2}-2x)}{(1+2c+\sqrt{1+4c^2})(-c+x)}} \\
& \sqrt{\frac{(1-2c+\sqrt{1+4c^2})(c+x)}{(1+2c+\sqrt{1+4c^2})(-c+x)}} \sqrt{\frac{c(-1+\sqrt{1+4c^2}+2x)}{(-1-2c+\sqrt{1+4c^2})(c-x)}} \left( (-2c+\sqrt{1+4c^2}) \right. \\
& \left. \text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{(1-2c+\sqrt{1+4c^2})(c+x)}{(1+2c+\sqrt{1+4c^2})(-c+x)}}}\right], \frac{2c+\sqrt{1+4c^2}}{2c-\sqrt{1+4c^2}}\right] - (1+ \right. \\
& \quad \left. \sqrt{1+4c^2}) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(1-2c+\sqrt{1+4c^2})(c+x)}{(1+2c+\sqrt{1+4c^2})(-c+x)}}}\right], \frac{2c+\sqrt{1+4c^2}}{2c-\sqrt{1+4c^2}}\right] \right) + \\
& 2 \text{EllipticPi}\left[\frac{1+2c+\sqrt{1+4c^2}}{1-2c+\sqrt{1+4c^2}}, \text{ArcSin}\left[\sqrt{\frac{(1-2c+\sqrt{1+4c^2})(c+x)}{(1+2c+\sqrt{1+4c^2})(-c+x)}}}\right], \right.
\end{aligned}$$

$$\left. \left. \left. \left. \frac{2c + \sqrt{1+4c^2}}{2c - \sqrt{1+4c^2}} \right] \right) \right) \right) \right) / \left( c \left( 1 - 2c + \sqrt{1+4c^2} \right) (c^2 + x - x^2) \right);$$

$$\text{Solve} \left[ \left( \text{dd} /. x \rightarrow \frac{1}{2} - \text{dd} /. x \rightarrow -\frac{1}{2} \right) == \frac{\pi}{4}, c \right]$$

(Debug) Out[ ]:=

\$Aborted

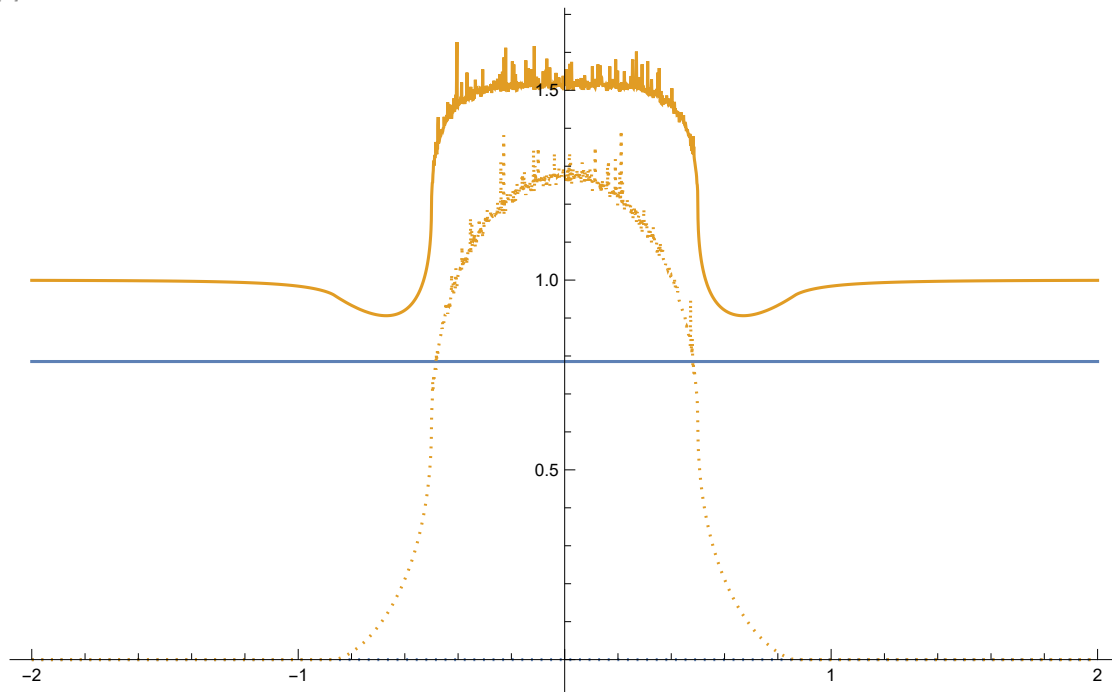
(Debug) In[ ]:=

$$\text{ff}[c_?NumericQ] := \text{NIntegrate} \left[ \sqrt{1 + \frac{x}{c^2 - x^2}}, \{x, \frac{-1}{2}, \frac{1}{2}\} \right]$$

(Debug) In[ ]:=

$$\text{ReImPlot} \left[ \left\{ \frac{\pi}{4}, \text{ff}[c] \right\}, \{c, -2, 2\} \right]$$

(Debug) Out[ ]:=



(Debug) In[ ]:=

```
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  FontFamily → "Cambria Math", FontSize → 37, FontWeight → "Bold"]
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(Debug) Out[ ]:=

**End of the Begenning → نایاپ زاغآ**

(Debug) In[ ]:=

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Speak["End of the Begenning"]
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