بسم الله الرحمن الرحيم

Section 5.5: Perturbation RG (first order)

$$\mathcal{O}(u^{p+1}) \longrightarrow \mathcal{O}(u^2)$$
$$p=1$$

☆ It means that we include u'

$$\langle \mathcal{O}\rangle = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \langle \mathcal{O} u'^n \rangle_0^{\text{Connected}}$$

$$= \underbrace{\langle \mathcal{O}\rangle_0 - \underbrace{\left[\langle \mathcal{O} u'\rangle_0 - \langle \mathcal{O}\rangle_0 \langle u'\rangle_0\right]}_{\langle \mathcal{O} u'\rangle_0^{\text{Connected}}}}_{\text{first order PT}} + \mathcal{O}(u'^2)$$

(up to second order)

① $$Z = \int D\tilde{m}(q)\, e^{-\int_0^{\Lambda/\ell} \frac{d^d q}{(2\pi)^d} \frac{(t+Kq^2)}{2} |\tilde{m}(q)|^2} \times \underbrace{\int D\sigma(q)\, e^{-\int_{\Lambda/\ell}^{\Lambda} \frac{d^d q}{(2\pi)^d} \frac{(t+Kq^2)}{2} |\sigma(q)|^2}}_{\text{Gaussian}}$$

$$\times\, e^{-\tilde{u}([\tilde{m}(q), \bar{\sigma}(q)])}$$

Gaussian

② $$Z = \underbrace{\underbrace{\int D\tilde{m}(q)\, e^{-\int_0^{\Lambda/\ell} \frac{d^d q}{(2\pi)^d} \frac{(t+Kq^2)}{2} |\tilde{m}(q)|^2}}_{\text{Singular Part}}}_{} \times \underbrace{\left(e^{-\frac{nV}{2}\int_{\Lambda/\ell}^{\Lambda} \frac{d^d q}{(2\pi)^d} \ln(t+Kq^2)}\right)}_{\text{Regular part}}$$

$$\underbrace{\times \left\langle e^{-\tilde{u}([\tilde{m}(q), \sigma(q)])} \right\rangle_{\sigma(q)}}_{} \quad \int d\sigma(q)$$

(5.30)

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3.5 – Kardar

$$u = m_i m_i m_j m_j$$
$$= [\tilde{m}_i + \sigma_i][m_i + \sigma_i][\tilde{m}_j + \sigma_j][m_j + \sigma_j]$$

$$\left.\begin{array}{l}\tilde m_i\,\sigma_i\,\tilde m_j\,\tilde m_j \\ \tilde m_i\,\sigma_i\,\sigma_j\,m_j \\ \uparrow \quad \uparrow\ \uparrow\end{array}\right\}\ \text{16-terms}$$

$$③ \quad \langle O\rangle_\sigma = \int \frac{D\sigma(q)}{Z_\sigma}\, e^{-\int_{\Lambda/\ell}^{\Lambda}\frac{d^dq}{(2\pi)^d}\frac{(t+Kq^2)}{2}|\tilde\sigma(q)|^2}$$

$$Z_\sigma = \int D\sigma(q)\, e^{-\beta H_\sigma}$$

Recall that

$$④ \quad Z = \int D\overset{\tilde m(q)+\sigma(q)}{\tilde m(q)}\, e^{-\beta H[m(q)]}$$

⑤ from ② & ④ $\Rightarrow$

$$\boxed{\beta H[m(q)] = \overbrace{\underbrace{V\,\delta f_\ell^0}_{f_\ell^0}}^{\text{Regular part}} + \int_0^{\Lambda/\ell}\frac{d^dq}{(2\pi)^d}\frac{(t+Kq^2)}{2}|\tilde m(q)|^2 - \underbrace{\ln\left\langle e^{-\tilde U_1[\tilde m,\sigma]}\right\rangle_\sigma}_{2.}}$$

$$F = -k_BT\ln Z \Rightarrow H$$

$$⑥ \quad \ln\left\langle e^{-\tilde U([\tilde m(q),\sigma(q)])}\right\rangle_\sigma = ?$$

$$= \overbrace{-\langle \mathcal{U} \rangle_\sigma^{1}}^{\checkmark} + \overbrace{\frac{1}{2}\left[\langle \tilde{\mathcal{U}}^2 \rangle_\sigma - \langle \tilde{\mathcal{U}} \rangle_\sigma^{2} \right]}^{\text{Second order}} + \cdots$$

$$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}_{\text{Cumulant expansion}}$$

Cumulant expansion

(14030918)

{ To the first order, we need to compute $\langle \mathcal{U} \rangle_\sigma = ?$ }

$$\boxed{\beta \mathcal{H}(m(q)) = \underbrace{V\,\delta f_\ell^{\circ}}_{\text{Regular}} + \underbrace{\int_0^{\Lambda/\ell} \frac{d^d q}{(2\pi)^d} \left(\frac{t + k q^2}{2} \right) |\tilde{m}(q)|^2}_{\text{Singular Part}} - \underbrace{\langle \tilde{\mathcal{U}}' \rangle_\sigma}_{?} + \underbrace{\mathcal{O}(\tilde{\mathcal{U}}^2)}_{\text{ignore this part}}}$$

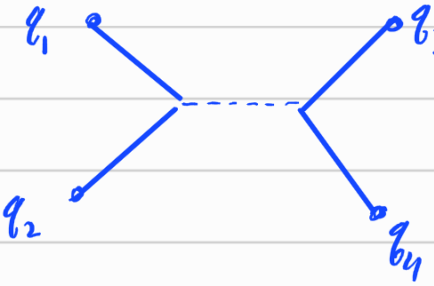
(7) $$? = \langle \tilde{\mathcal{U}} \rangle_\sigma = u \int \frac{d^d q_1\, d^d q_2\, d^d q_3\, d^d q_4}{(2\pi)^{4d}} (2\pi)^d \delta_D(q_1+q_2+q_3+q_4)$$

$$\times \Big\langle \underbrace{[\tilde{m}(q_1) + \sigma(q_1)]^2 \cdot [\tilde{m}(q_2) + \sigma(q_2)]^2}_{m \cdot m}\ \overbrace{[\tilde{m}(q_3) + \sigma(q_3)]^2 \cdot [\tilde{m}(q_4) + \sigma(q_4)]^2}^{m \cdot m} \Big\rangle_\sigma^4$$

$$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}_{(m \cdot m)^2 \text{ including } 2^4 \text{ terms} = 16}$$

گیر جمع

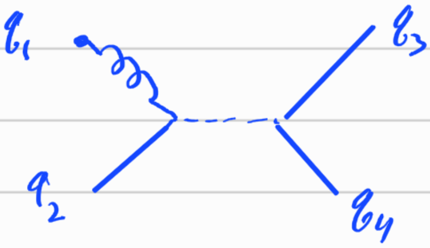
(8) $$1\, \langle \tilde{m}(q_1) \cdot \tilde{m}(q_2)\, \tilde{m}(q_3) \cdot \tilde{m}(q_4) \rangle_\sigma \neq 0$$



Result $= \underline{\mathcal{U}[\tilde m]}$

ℓ!

(9) $$4\left\langle \underbrace{\sigma(q_1)}\cdot\tilde m(q_2)\ \tilde m(q_3)\cdot\tilde m(q_4)\right\rangle_\sigma = 0$$

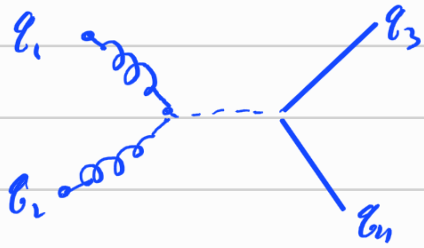


Result $= 0$

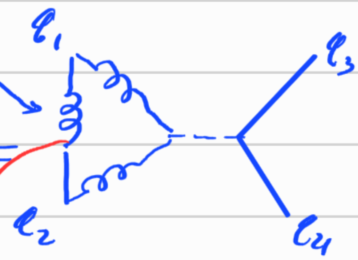
Gaussian theory with odd node

ℓ!

(10) $$2\left\langle \sigma(q_1)\cdot\sigma(q_2)\ \tilde m(q_3)\cdot\tilde m(q_4)\right\rangle_\sigma \neq 0$$



Result $=$



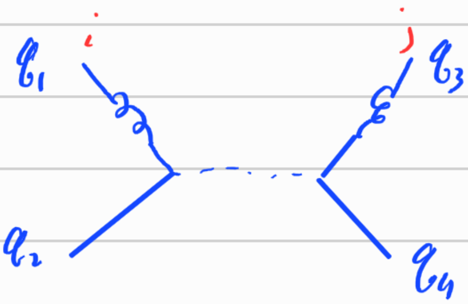
$\delta_{ii}=n$

ℓ!

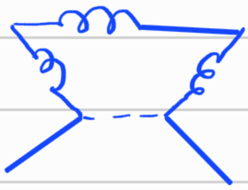
(11) $$4\left\langle \sigma(q_1)\cdot\tilde m(q_2)\ \sigma(q_3)\cdot\tilde m(q_4)\right\rangle_\sigma \neq 0$$

2 ×2

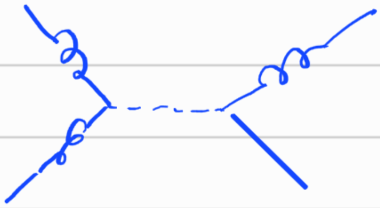
$\delta_{ij}=1$



Result $=$



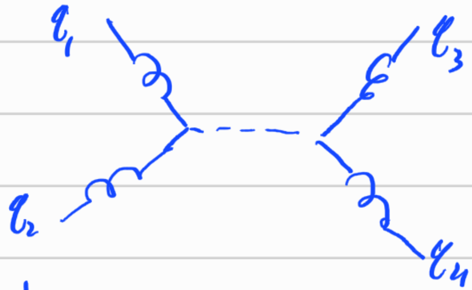
(12) $4 \left\langle \sigma(q_1)\cdot\sigma(q_2)\ \sigma(q_3)\cdot\tilde{m}(q_4) \right\rangle_\sigma = 0$



Result $= 0$

(13) $1 \left\langle \sigma(q_1)\cdot\sigma(q_2)\ \sigma(q_3)\cdot\sigma(q_4) \right\rangle_\sigma \neq 0$

[illegible] (16)



→ adding a non-Singular part
(Regular part)

Result $=$ $\delta_{ij} = n$

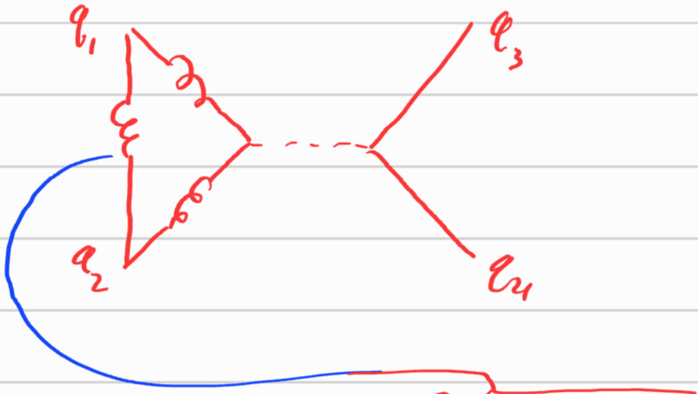
$\delta_{ij} = 1$

(8) – (13) $\Rightarrow$ 16-terms

[illegible] (12) $= 0$
[illegible] (9) $= 0$
} 8-term → Zero

[illegible] (8) $= ?$
دوم (10) $= ?$
[illegible] (11) $= ?$
[illegible] (13) $= ?$ → Regular
} 8-terms → Non-Zero.

(14) What is (10) = ?



قرار دهیم

$$2u \int \frac{d^d q_1\, d^d q_2\, d^d q_3\, d^d q_4}{(2\pi)^{4d}} (2\pi)^d \delta_D(q_1+q_2+q_3+q_4)\, \underset{\uparrow \atop n}{\delta_{ii}} \frac{(2\pi)^d \delta_D(q_1+q_2)}{t+Kq_1^2}$$

$$\times\ \tilde{m}(q_3)\cdot \tilde{m}(q_4)$$

$$\boxed{(10) = 2un \underbrace{\int_0^{\Lambda/\ell} \frac{d^d q}{(2\pi)^d} |\tilde{m}(q)|^2}_{m\cdot m} \int_{\Lambda/\ell}^{\Lambda} \frac{d^d q'}{(2\pi)^d} \frac{1}{t+Kq'^2}}$$

(15) What is (11) = ?

$$(11) = 4u \int_0^{\Lambda/\ell} \frac{d^d q\ |\tilde{m}(q)|^2}{(2\pi)^d} \int_{\Lambda/\ell}^{\Lambda} \frac{d^d q'}{(2\pi)^d} \frac{1}{t+Kq'^2}$$

(13) [illegible]

(16)

$$\beta \mathcal{H} = \bar{V}(\delta f_\ell^0 + u\,\delta f_\ell^1) + \int_0^{\Lambda/\ell} \frac{d^d q}{(2\pi)^d} \frac{\tilde{t}+Kq^2}{2} |\tilde{m}(q)|^2$$

$$
+u\int_0^{\Lambda/\ell}\underbrace{\frac{d^dq_1\,d^dq_2\,d^dq_3}{(2\pi)^{3d}}\,\tilde m(q_1)\cdot\tilde m(q_2)\,\tilde m(q_3)\cdot\tilde m(-q_1-q_2-q_3)}_{\text{جمله‌ی ⑧ ام}}
$$

جمله‌ی دوم

$$
\left\{\begin{aligned}
\tilde t &= t + 4u(n+2)\int_{\Lambda/\ell}^{\Lambda}\frac{d^dq'}{(2\pi)^d}\,\frac{1}{t+Kq'^2}\\
\tilde K &= K\\
\tilde u &= u
\end{aligned}\right\}
$$

$\boxed{\text{Coarse-graining}}$

بخش اول RG

(17) Rescaling

$$x \longrightarrow x' = x/\ell \;\longrightarrow\; x = x'\ell$$

$$q \longrightarrow q' = q\ell \;\longrightarrow\; q = q'/\ell$$

(18) Renormalization

$$m \longrightarrow m' = m/\vartheta = \ell^{x_m}\,m$$

$$\tilde m \longrightarrow \tilde m' = \frac{\tilde m}{z} = \ell^{x'_m}\,\tilde m$$

(20)

$$
\beta\tilde H \longrightarrow \beta\tilde H' = \int_0^{\Lambda}\frac{d^dq'}{(2\pi)^d}\,\ell^{-d}z^2|\tilde m'|^2\,\frac{\tilde t + Kq'^2\ell^{-2}}{2}
$$

$$
+\underline{u}\int_0^{\Lambda}\frac{d^dq'_1\,d^dq'_2\,d^dq'_3}{(2\pi)^{3d}}\,\underline{\ell^{-3d}}\,\underline{z^4}\,\tilde m'(q'_1)\cdot\tilde m'(q'_2)
$$

$$\tilde{m}'(q_3') \cdot \tilde{m}'(-q_1' - q_2' - q_3')$$

$$\left\{ \text{Dimensionless Hamiltonian after RG upto second order of P.T. } (\mathcal{O}(u^2)) \right\}$$

New Coupling Coefficients are as follows:

$$u_\ell' = \ell^{-3d} z^4 u \quad \ell^{-3d} \left(\ell^{\frac{d}{2}+1}\right)^4 = \ell^{-3d} \ell^{+2d+4} = \ell^{4-d}$$

$$t_\ell' = \ell^{-d} z^2 \tilde{t}$$

$$K_\ell' = \ell^{-d-2} z^2 K \longrightarrow K_\ell' = K \longrightarrow \boxed{z = \ell^{\frac{d+2}{2}}}$$

$$\begin{cases} t_\ell' = \ell^2 \left[t + 4u(n+2) \displaystyle\int_{\Lambda/\ell}^{\Lambda} \frac{d^d q}{(2\pi)^d} \frac{1}{t + Kq'^2} \right] \\ u_\ell' = \ell^{4-d} u \end{cases}$$

$$[K'] = R_\ell [K] \quad , \quad \ell = e^{\delta \ell} = (1 + \delta \ell)$$

$$\begin{cases} t' = t(\ell) = t(1+\delta\ell) = t + \delta\ell \dfrac{dt}{d\ell} \\ u' = u(\ell) = u(1+\delta\ell) = u + \delta\ell \dfrac{du}{d\ell} \end{cases} \Longrightarrow \beta_\ell = \frac{dK}{d\ell}$$

$$\frac{dt}{dl} = 2t + 4u(n+2)\frac{S_d}{(2\pi)^d}\frac{1}{t+K\Lambda^2}\Lambda^d$$

$$\frac{du}{dl} = (4-d)u$$

$t^* = 0$
$u^* = 0$

Gaussian fixed point

$t = t^* + \delta t$

$$\frac{d}{dl}\begin{pmatrix}\delta t\\ \delta u\end{pmatrix}\Bigg|_{\substack{t=t^*=0\\ u=u^*=0}} = \begin{pmatrix}2 & 4(n+2)\frac{S_d}{(2\pi)^d}\Lambda^{d-2}\\ 0 & 4-d\end{pmatrix}\begin{pmatrix}\delta t\\ \delta u\end{pmatrix}$$

$t' = l^{x_t} t$
$u' = l^{x_u} u$

$x_t = 2$

$x_u = 4-d \longrightarrow \boxed{\epsilon \equiv 4-d}$

(21)

$t^* = 0$
$u^* = 0$

For $d > 4$ $\quad x_u < 0 \longrightarrow u =$ Irrelevant coupling

PT works well ✓

For $d < 4$ $\quad x_u > 0 \longrightarrow u =$ Relevant couplin

$\underbrace{\text{PT fails} \times}$

قابل قبول نیست

$O(u^3)$

هنوز شرح ندادم

Chapter 12
Goldenfeld

$t^* = 0$

$u^* = \frac{\epsilon}{B}$

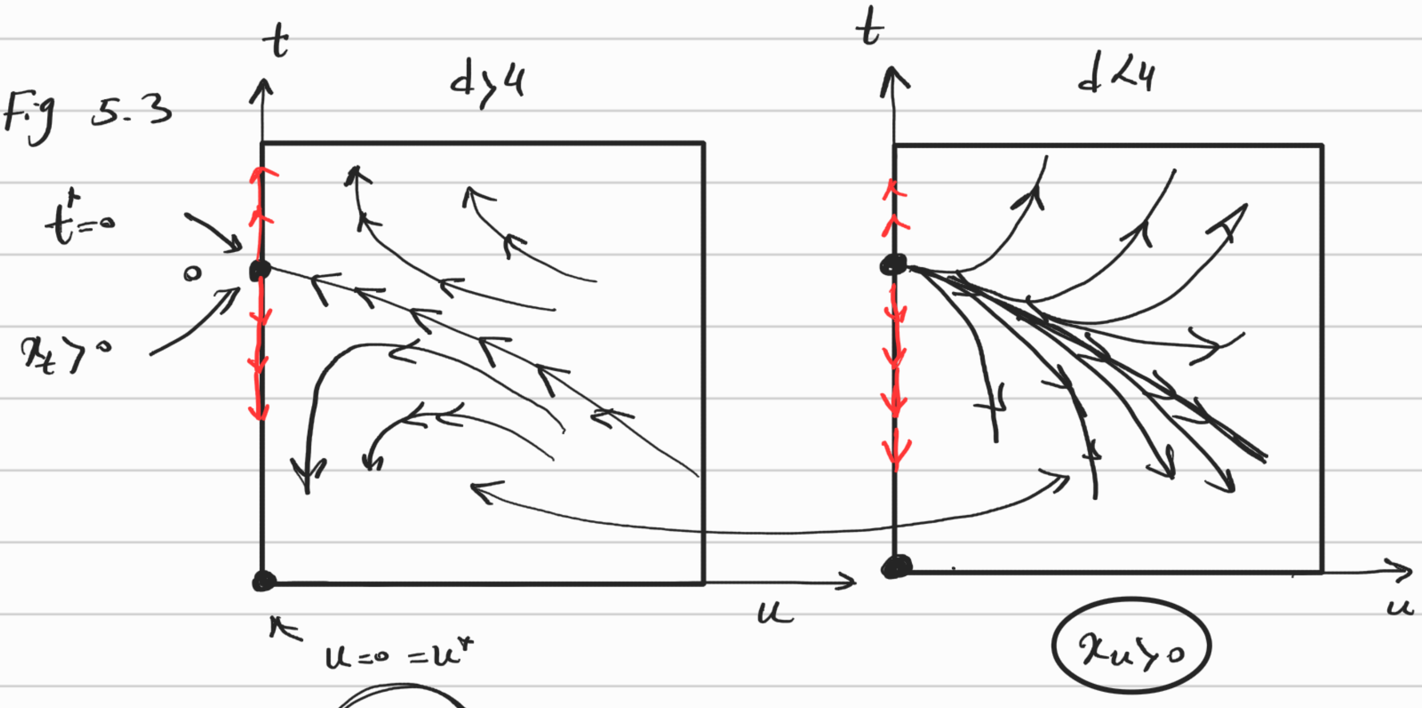
for $\epsilon > 0$ $(d<4)$

$\Big\}$ $\boxed{\text{Wilson Fisher fixed point}}$

$(t^* \neq 0, u^* \neq 0)$

$\hookrightarrow x_u < 0$ — اختلال کاری کند

Fig 5.3



Eq. (5.46) $\quad O(u^3) \Longrightarrow$

$$\frac{dt}{dl} = 2t + \frac{4u(n+2)\, S_d\, \Lambda^d}{(2\pi)^d (t+K\Lambda^2)} - Au^2$$

$$\frac{du}{dl} = \epsilon u - Bu^2$$

بدیهی

$t^*=0$
$u^*=0$

نقطه جاذب اول

↑

d>4

↓

$x_u<0$

$t^*=0$
$u^*=\frac{\epsilon}{B}$ *

نقطه جاذب دوم

↑

d<4

↓

$x_u<0$

Section 5.6 : Perturbative Second order ← Chapter 12 Goldenfeld