بسم الله الرحمن الرحيم

$\langle m_\alpha(q)\, m_\beta(q')\rangle = ?$

Recall that

① $\langle O\rangle = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \langle O\, U^n\rangle_0^{\text{Connected}}$

② $Z = \int D^n m\, e^{-\beta H}$, $\beta H = \beta H_0 + U$

(5.1) $Z_0 = \int D^n m\, e^{-\beta H_0} = \int D^n m\, \underbrace{e^{-\int d^d x\, d^d x'\, \frac{1}{2}\, \underset{1\times n}{m(x)^T} \cdot \underset{n\times n}{Cov} \cdot \underset{n\times 1}{m(x')}}}_{e^{-\beta H_0}}$

$= \frac{\sqrt{(2\pi)^n \beta}}{\sqrt{\text{Det Cov}}}$

$\boxed{\text{role of } h}$

③ $Z_0 = \int D^n m\, e^{-\beta H_0}\, e^{+\int d^d x\, \lambda(x)\cdot m(x)}$

$Z_0(\lambda) = \frac{\sqrt{(2\pi)^n}}{\sqrt{\beta\, \text{Det Cov}}}\, e^{\frac{1}{2} \underset{1\times n}{\lambda^T} \cdot \underset{n\times n}{Cov^{-1}} \cdot \underset{n\times 1}{\lambda}}$

④ Using ② and ③ $\langle m_\alpha\rangle_0^{\text{Conn}} = \left.\frac{\partial \ln Z_0(\bar{\lambda})}{\partial \lambda_\alpha}\right|_{\bar{\lambda}=0}$

$\langle m_\alpha\, m_\beta\rangle_0^{\text{Con}} = \left.\frac{\partial^2 \ln Z_0(\bar{\lambda})}{\partial \lambda_\alpha\, \partial \lambda_\beta}\right|_{\bar{\lambda}=0}$

For our Gaussian Case

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$$\beta \mathcal{H}_0 = \int \frac{d^d q}{(2\pi)^d} \frac{t + Kq^2 + Lq^4 + \cdots}{2} |\tilde{m}(q)|^2$$

$$\boxed{\langle m_\alpha(q)\, m_\beta(q') \rangle_0^{\text{Conn}} = \frac{\delta_{\alpha\beta}\, \delta_D(\vec{q}+\vec{q}\,')(2\pi)^d}{t + Kq^2 + Lq^4 + \cdots}}$$ 5.6

6
$$\langle \mathcal{O} \rangle = \underbrace{\langle \mathcal{O} \rangle_0^{\text{Conn}}}_{\text{Zero-order}} - \underbrace{\langle \mathcal{O}\,\mathcal{U} \rangle_0^{\text{Conn}}}_{\text{first-order}} + \frac{1}{2!} \underbrace{\langle \mathcal{O}\,\mathcal{U}^2 \rangle_0^{\text{Conn}}}_{\text{Second order}} + \cdots$$

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$$\mathcal{U} = u \int \frac{d^d q_1}{(2\pi)^d} \int \frac{d^d q_2}{(2\pi)^d} \int \frac{d^d q_3}{(2\pi)^d}\, \tilde{m}_i(q_1)\, \tilde{m}_i(q_2)\, \tilde{m}_j(q_3)\, \tilde{m}_j(-q_1-q_2-q_3)$$

7 $\mathcal{O} \equiv m_\alpha(q)\, m_\beta(q')$ $\longrightarrow$ $\langle \mathcal{O} \rangle = ?$ up to Second order $\mathcal{O}(\mathcal{U}^2)$

8

$$q_4 = -q_1 - q_2 - q_3$$

6, 7 $\Rightarrow$ $\langle \mathcal{O} \rangle = \langle m_\alpha(q)\, m_\beta(q') \rangle$

$$= \underbrace{\langle m_\alpha(q)\, m_\beta(q') \rangle_0^{\text{Conn}}}_{A} - u \int \frac{dq_1\, dq_2\, dq_3}{(2\pi)^{3d}} \underbrace{\langle m_\alpha(q)\, m_\beta(q')\, m_i(q_1)\, m_i(q_2)\, m_j(q_3)\, m_j(q_4) \rangle_0^{\text{Conn}}}_{B}$$

$$A = \frac{\delta_{\alpha\beta}\, \delta(q+q')(2\pi)^d}{t + Kq^2 + Lq^4 + \cdots} \quad \checkmark$$

$$\textcircled{B}\ \langle \overset{1}{m_\alpha(q)}\ \overset{2}{m_\beta(q')}\ \overset{3}{m_i(q_1)}\ \overset{4}{m_i(q_2)}\ \overset{5}{m_j(q_3)}\ \overset{6}{m_j(q_4)} \rangle_0^{\text{Conn}} =$$

$$= \underbrace{\langle m_\alpha(q) m_\beta(q') m_i(q_1) m_i(q_2) m_j(q_3) m_j(q_4) \rangle_0}_{B1}$$

$$- \underbrace{\langle m_\alpha(q) m_\beta(q') \rangle_0 \langle m_i(q_1) m_i(q_2) m_j(q_3) m_j(q_4) \rangle_0}_{B2}$$

$B2 \longrightarrow$ 3 terms

Recall that Wick's theorem $\langle m_i m_j m_k m_l \rangle_0 = ?$

$$= \langle m_i m_j \rangle \langle m_k m_l \rangle + \langle m_i m_k \rangle \langle m_j m_l \rangle$$

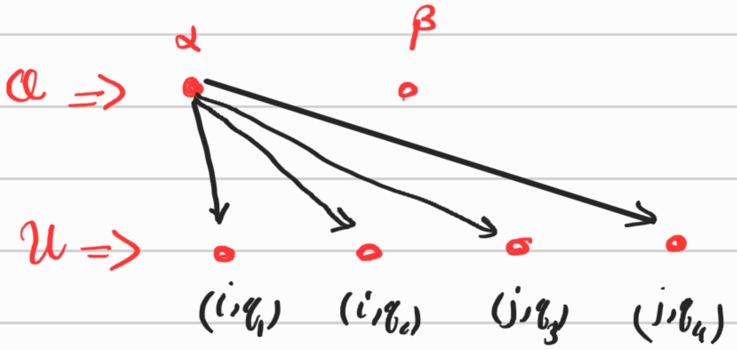
$$+ \langle m_i m_l \rangle \langle m_j m_k \rangle$$

$\textcircled{B_1} \rightarrow \binom{6}{2} = \dfrac{6 \times 5 \times 4!}{2!\,4!} = 15$ - terms

$\textcircled{B_2} \rightarrow$ 3 - terms

$\textcircled{B} =$ 15 terms - 3 term $= \underline{12 \text{ - term}}$

I   II



15 تا

$\alpha \rightarrow 4$
انتخاب دارد

I β — یک انتخاب دارد $= \underbrace{4 \times 1}_{4}$

II β — دو انتخاب دارد $= \underbrace{4 \times 2}_{8}$

12

$$\text{I} \quad \underbrace{\langle m_\alpha m_i \rangle}_{A} \underbrace{\langle m_\beta m_i \rangle}_{A} \underbrace{\langle m_j m_j \rangle}_{A} \quad \delta_{jj} \rightarrow n \longrightarrow 4n$$

$$\text{II} \quad \underbrace{\langle m_\alpha m_i \rangle}_{A} \underbrace{\langle m_\beta m_j \rangle}_{A} \underbrace{\langle m_i m_j \rangle}_{A} \quad \delta_{ij} \rightarrow 1 \longrightarrow 8$$

(9) $$\underbrace{\langle m_\alpha(q)\, m_i(q_1)\rangle_0}\,\underbrace{\langle m_\beta(q')\, m_i(q_2)\rangle_0}\,\underbrace{\langle m_j(q_2)\, m_j(q_4)\rangle_0}$$

$$\frac{\delta_{\alpha i}\,\delta_D(q+q_1)(2\pi)^d}{(t+Kq^2+Lq^4+\dots)} \times \frac{\delta_{\beta i}\,\delta_D(q'+q_2)(2\pi)^d}{(t+Kq'^2+Lq'^4+\dots)} \times \frac{\delta_{jj'}\,\delta_D(q_2+q_4)(2\pi)^2}{(t+Kq_2^2+Lq_2^4+\dots)}$$

(10) $$\langle m_\alpha(q)\, m_\beta(q')\rangle = \frac{\delta_{\alpha\beta}\,\delta_D(q+q')(2\pi)^d}{t+Kq^2+Lq^4+\dots}$$

$\delta_{\alpha\beta}$

$$-u\int \frac{dq_1\, dq_2\, dq_3}{(2\pi)^{3d}} \Bigg[4\Bigg(\frac{\delta_{\alpha i}\,\delta_D(q+q_1)(2\pi)^d}{(t+Kq^2+Lq^4+\dots)} \times \frac{\delta_{\beta i}\,\delta_D(q'+q_2)(2\pi)^2}{(t+Kq'^2+Lq'^4+\dots)}$$

$n \leftarrow$

$$\times \frac{\delta_{jj'}\,\delta_D(q_2+q_4)(2\pi)^d}{(t+Kq_2^2+Lq_2^4+\dots)}\Bigg) +$$

$$+\ 8\Bigg(\frac{\delta_{\alpha i}\,\delta_D(q+q_1)(2\pi)^d}{t+Kq^2+Lq^4+\dots} \times \frac{\delta_{\beta j}\,\delta_D(q'+q_3)(2\pi)^d}{t+Kq'^2+Lq'^4+\dots}$$

$$\times \frac{\delta_{ij}\,\delta_D(q_2+q_4)(2\pi)^d}{t+Kq_2^2+Lq_2^4+\dots}\Bigg)\Bigg]$$

$L=0$

(11) $$\underbrace{\langle m_\alpha(q) m_\beta(q') \rangle}_{\chi} = \underbrace{\frac{\delta_{\alpha\beta}\,\delta(q+q')(2\pi)^d}{(t+Kq^2)}}\left[1 - \frac{(4un+8u)}{(t+Kq^2)}\int \frac{d^d q_3}{(2\pi)^d}\frac{1}{(t+Kq_3^2)}\right]$$

$$\chi = \chi^{Gaussian}\left[1 - \underbrace{\frac{4u(n+2)}{(t+Kq^2)}\int\frac{d^d q_3}{(2\pi)^d}\frac{1}{(t+Kq_3^2)}}_{\text{Correction due to } O(u')}\right] + O(u^2)$$

(5.21)

(12) @ Critical point $\quad \chi \longrightarrow \infty$

$T_c(u\neq 0) \quad \chi^{-1} \longrightarrow 0$

$q \to 0$

@ critical point $\quad \chi^{Gau} \longrightarrow \infty$

$T_c(u=0) \quad \chi^{G^{-1}} \longrightarrow 0$

$q \to 0$

$T_c(u\neq 0) \overset{?}{=} T_c(u=0)$

$<$ ✓

$$\chi^{-1} = \langle m_\alpha(q)\, m_\beta(q')\rangle^{-1}_{q=0} = t + 4u(n+2)\int\frac{d^d q_3}{(2\pi)}\frac{1}{t+Kq_3^2} + O(u^2)$$

$$= \frac{(T-T_c^{Gau})}{T^{Gau}} + \quad //$$

$$\chi^{-1}(t=0) = \chi^{-1}(T=T_c^{Gau}) \neq 0$$

$$\chi^{-1}(t<0) = \chi^{-1}(T=T_c^{(1)}) = 0 \implies T_c^{(1)} = ?$$

$$0=\chi^{-1}(t_c^{(1)}) = t_c^{(1)} + 4u(n+2)\int \frac{d^d q_3}{(2\pi)^d}\,\frac{1}{t+Kq_3^2} + \mathcal{O}(u^2)$$

$$t_c^{(1)} = -4u(n+2)\int \frac{d^d q_3}{(2\pi)^d}\,\frac{1}{t+Kq_3^2} < 0$$

$$\frac{T_c^{(1)} - T_c^{Gau}}{T_c^{Gau}} = \quad " \quad " \quad < 0 \implies \boxed{T_c^{(1)} < T_c^{Gau}}$$

13) $\chi^{-1}(t) - \chi^{-1}(t=t_c^{(1)}) = ?$ $\quad \boxed{d<4}$ what happens?

$$\left[t + 4u(n+2)\int_0^{\Lambda} \frac{d^d q'}{(2\pi)^d}\,\frac{1}{t+Kq'^2}\right] - \left[t_c^{(1)} + 4u(n+2)\int_0^{\Lambda} \frac{d^d q'}{(2\pi)^d}\,\frac{1}{t_c^{(1)}+Kq'^2}\right]$$

$$= t - t_c^{(1)} + 4u(n+2)\int \frac{d^d q'}{(2\pi)^d}\left(\frac{1}{t+Kq'^2} - \frac{1}{t_c^{(1)}+Kq'^2}\right)$$

$$= \left(t - t_c^{(1)}\right)\left(1 - \frac{4u(n+2)}{K^2}\left(\frac{K}{t-t_c^{(1)}}\right)^{2-d/2}\right)$$

$$= \left(t - t_c^{(1)}\right)\left(1 - \underbrace{\frac{4u(n+2)}{K^2}\left(\frac{K}{t-t_c^{(1)}}\right)^{2-d/2}}\right)$$

$$\Delta\chi^{-1} \equiv -\frac{4u(n+2)}{K^2}\left(\frac{K}{t-t_c^{(1)}}\right)^{2-d/2}$$

$$\chi^{-1} = \chi^{-1}(t_c^{(1)}) + \Delta\chi^{-1}$$

$$\underbrace{2-\tfrac{d}{2} > 0}_{} \rightarrow \lim_{t \to t_c^{(1)}} \left(\frac{K}{t-t_c^{(1)}}\right)^{2-d/2} \longrightarrow 0 = \Delta\chi^{-1}$$

$$\longrightarrow \Delta\chi \longrightarrow \infty$$

$$\chi = \chi^{G} + \Delta\chi$$

Breaking Perturbation

$$2-\tfrac{d}{2} > 0 \rightarrow 2 > \tfrac{d}{2} \rightarrow d<4$$

سهم اختلال در نزدیکی خود نقطه بحرانی نسبت بزرگ می‌شود در $d<4$ اختلال می‌شکند!

$$RG + \text{Perturbation} \longrightarrow \text{log - Divergence}$$

$$+\, O(u^2) \longrightarrow$$ مشکل حل می‌گردد.

5.3 Diagramatic Representation of Perturbation theory.