بسم الله الرحمن الرحیم

جبر ۱۴۰۱ /۹ /۲۲

Chapter 4: The scaling hypothesis (Kardar)

① Coarse graining (I)

② Rescale (R)

③ Renormalize (R)

$$\underbrace{R \circ R \circ I}_{RG}$$

$$e^{-\beta F} = Z = \mathrm{Tr}\, e^{-\beta \mathcal{H}} = \int Dm\, e^{-\beta \mathcal{H}}$$

فرض می‌شود عوض نشود

$$e^{-\beta F'} = Z' = \int Dm'\, e^{-\beta \mathcal{H}'}$$

{ Landau-Ginzburge Phenomenological Description }

$$f = \frac{F}{V^{(d)}} \longrightarrow \underbrace{f_s}\, : \text{ Singular part } \rightarrow a \ll x \sim L \rightarrow \infty$$

Unit cell

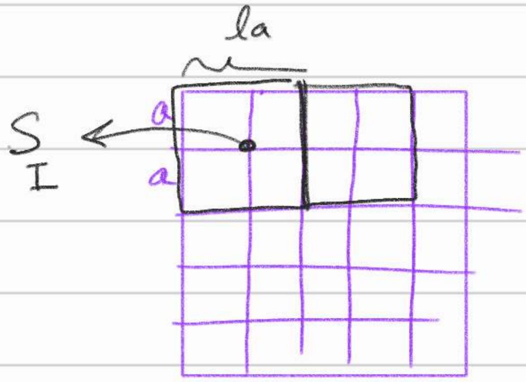
$$\underbrace{a \ll x \ll la} \quad \underbrace{la \ll x \sim L}$$

Integrate out

Recall that RG is :

$\vec{m}(x)$: Vector field $\quad m_i(x) \cdot \quad i=1 \dots n$

$$S_I = \frac{1}{l^d} \sum_{i \in I}^{l^d} s_i$$



$$\text{I} \rightarrow \quad m'_i(x) = \frac{1}{l^d} \int_{x' \in X} d^d x' \; m_i(x')$$

$$\text{II} \rightarrow \quad X_{New} = \frac{X_{old}}{l}$$

$$\text{III} \quad m_i^{New}(X_{New}) = m'_i(X_{New})\, l^{\Delta}$$

$$= \frac{m'_i(X_{New})}{v}$$

$$v = l^{-\Delta}$$

$$\Delta = \frac{x_h}{x_t}$$

{ chapter 6 : Kardar
chapter 13, 14
Pathria }

$$N_l = l^{-d} N$$

$$\xi_l = l^{-1} \xi$$

$$t_l = l^{x_t} t$$

$$h_l = l^{x_h} h$$

$$K_l = l^{x_K} K$$

$$f(t_l, h_l, K_l \dots) = l^{+d} f(t, h, K, \dots)$$

$$f(t, h, K, \dots) = l^{-d} f(t_l, h_l, K_l \dots)$$

اگه یه سیستم تحت تبدیل مقیاس طول بدر نظر گرفت

ناوردا است ← نقطه بحرانی ...

در فراکتال ها هندسی هم رفتار خود متشابهی

$N(l) \sim l^{d_f}$

$$Z = \int Dm \, e^{-\beta \mathcal{H}}$$

4.6: Gaussian model. $\quad m^2 \,, \quad \cancel{m^4}$

Without RG

$$Z = \int \mathcal{D}\vec{m}\; e^{\overbrace{-\int d^d x \left[\frac{t}{2}\vec{m}^2(x) + \frac{K}{2}(\nabla m)^2 + \frac{L}{2}(\nabla^2 m)^2 + \cdots + \frac{u}{4} m^4 - \vec{h}\cdot\vec{m} \right]}^{-\beta\mathcal{H}}}$$

چون کوچک است 0

$$\begin{pmatrix} t=0 \\ h=0 \end{pmatrix} = \text{نقطه بحرانی} \qquad t \equiv \frac{T - T_c}{T_c}$$

$\nabla \equiv -iq \rightarrow$ Wave Number

$$\vec{m}(\bar{q}) = \int \frac{d^d x}{(2\pi)^d}\, e^{-i\bar{q}\cdot\bar{x}}\, \vec{m}(\bar{q}) \,, \quad \vec{m}(\bar{x}) = \frac{1}{V^{(d)}} \int d^d q\, e^{-i\bar{q}\cdot\bar{x}}\, \vec{m}(\bar{q})$$

$$\beta\tilde{\mathcal{H}} = \sum_q \left[\frac{(t + Kq^2 + Lq^4 + \cdots)}{2V} |\tilde{m}(q)|^2 \right] - \underbrace{\tilde{h}\cdot\tilde{m}(q=0)}_{q=0}$$

\# of Vector field

$$Z = \prod_q V^{-n/2} \int \mathcal{D}\tilde{m}(q)\; e^{-\left[\left(\frac{t + Kq^2 + Lq^4 + \cdots}{2V} \right) |\tilde{m}(q)|^2 - \tilde{h}\cdot\tilde{m}(q=0) \right]}$$

$$Z = \underbrace{e^{\frac{Vh^2}{2t}}}_{q=0} \underbrace{\prod_q \left(\frac{2\pi}{t + Kq^2 + Lq^4 + \cdots} \right)^{n/2}}_{q\neq 0}$$

$1/a = \Lambda \rightarrow$ UV-Cutoff

$$f = -\left.\frac{\ln Z}{V^{(d)}}\right|_{q\neq 0} = \underbrace{\frac{n}{2} \int_0^{\Lambda} \frac{d^d q}{(2\pi)^d} \ln(t + Kq^2 + Lq^4 + \cdots)}_{L\to\infty} - \frac{h^2}{2t}$$

ضریب خمیدگی

$$f(t,h) = t^{d/2}\, \underbrace{\overbrace{g_f\left(h/t^{\Delta}\right)}} = t^{2-\alpha}\, g_f\left(h/t^{\Delta}\right)$$

$$C = \frac{\partial^2 f}{\partial t^2} \sim t^{-\alpha} \longrightarrow \quad 2-\alpha = d/2 \qquad \boxed{2 - d/2 = \alpha}$$

$$\chi \sim \frac{\partial^2 f}{\partial h^2} \sim \frac{1}{t} \longrightarrow \gamma_+ = 1$$

Gaussian Model (RG)

$$Z \sim \int D\bar{m}(\bar{q})\, e^{\left[\left\{-\int_0^{\Lambda} \frac{d^d q}{(2\pi)^d} \left(\frac{t + Kq^2 + Lq^4 + \cdots}{2}\right) |\bar{m}(q)|^2\right\} + \bar{h}\cdot\bar{m}(q=0)\right]}$$

$$I \longrightarrow R \longrightarrow R' = Z' \quad , \quad Z = Z'$$

$$\left\{\begin{array}{c} x_t \\ x_h \\ x_K \\ x_L \end{array}\right\} \leftarrow \left\{\begin{array}{c} t' \xrightarrow{?} t \\ h' \xrightarrow{?} h \\ K' \xrightarrow{?} K \\ L' \xrightarrow{?} L \end{array}\right\}$$

β-funct +

RG-flow

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{U} \longrightarrow \text{Perturbative RG}$$

Chapter 5 (Kardar)