بسم الله الرحمن الرحیم   جلسه ۱۳ / ۹ / ۱۴۰۱

chapter 14: Phase Transitions: The Renormalization Group approach

Pathria

دیدیم که سیستم ها در نقطه بحرانی و خیلی نزدیک به نقطه بحرانی تحت تبدیل مبتنی بر طول تغییر نمی کنند

غیردقیق

رفتار مقیاسی مشهود است

تقارن داریم

$$\xi_\ell = \ell^{-1}\xi \quad \begin{cases} \xi = 0 \\ \xi = \infty \end{cases}$$

$$\left\{ \begin{aligned} \mathcal{H}_\ell &\overset{\Delta}{=} \mathcal{H} \\ Z_\ell &= Z \end{aligned} \right\} \Bigg|_{T=T_c}$$

$$N_\ell = \ell^{-d} N$$

$$\xi_\ell = \ell^{-1}\xi$$

$$t_\ell = \ell^{x_t} t$$

$$h_\ell = \ell^{x_h} h$$

$$u_\ell = \ell^{x_u} u$$

$$f(t_\ell, h_\ell) = \ell^{d} f(t, h)$$

$$f = \sum_{n=0}^{\infty} a_n[K]\, m^n + \dots$$

ضرایب جفت شدگی

$$[K] = [K_0, K_1, K_2, K_3, \dots]$$

$$= [a_0, a_1, a_2, a_3, a_4, \dots]$$

$$= [a_0, t, h, u, \dots]$$

$$\xi \sim t^{-\nu} \longrightarrow \nu = \frac{1}{x_t}$$

$$\Delta \equiv \frac{x_h}{x_t}$$

$$M \sim t^{\beta} \longrightarrow \beta = \frac{\nu x_h}{x_t}$$

$$G(r) \sim r^{-(d-2+\eta)} e^{-r/\xi}$$

$$M \sim h^{1/\delta} \longrightarrow \delta = \frac{d-\Delta}{\Delta}$$

$$2\Delta = d - 2 - \eta$$

$$\chi \sim t^{\gamma} \longrightarrow \gamma = \nu(2\Delta - d)$$

اگر بتوانیم Δ و $(x_t, x_h, \dots)$ را حساب کنیم نماهای بحرانی بدست می‌آید

$$[K]_l = R_l [K]$$ به طور کلی تحت تبدیلات بستنی بهجول

Recursive Relations

$$[K=K^*]_l = R_l [K=K^*]$$ دقیقا بنا به تعریف در نقطه بحرانی

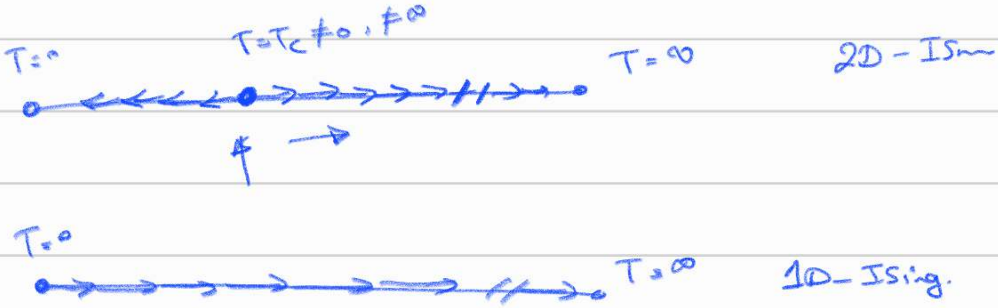
از این رابطه دقیقا نقطه بحرانی بدست می‌آید $\leftarrow$ $$[K^*] = R_l [K^*]$$ $\xrightarrow{\text{fixed point}}$ $T_c = ?$

پس یافتن روابط بازگشتی

نماهای بحرانی $\{x_t, x_h, x_u, \dots\}$ بدست می‌آید.

نوع نقطه ثابت $\leftarrow$ RG شار RG flow

$h=0$

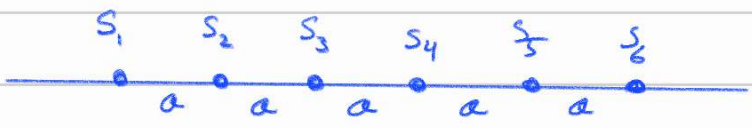


$h \neq 0$ شکل فرق می‌کند.

fig 14.6

Ex 1: Ising Model in 1D according to RG in Real space

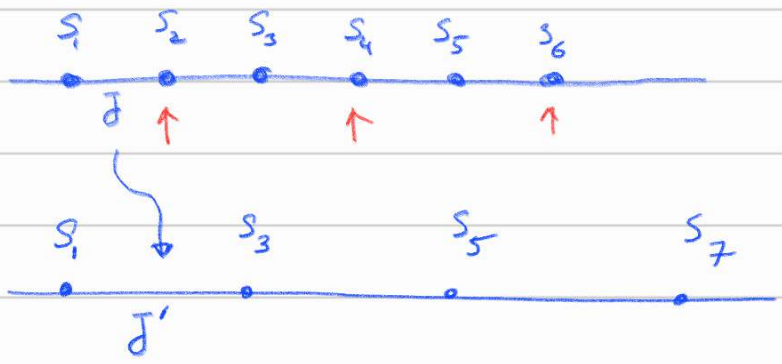
Periodic BC



$$Z_N(t,h) = \sum_{\{S\}, S=\pm 1} e^{-\beta \mathcal{H}} \quad , \quad \mathcal{H} = -J\sum_{i=1}^{N} S_i S_{i+1} - h\sum S_i$$

$$\boxed{K_0 = 0} \quad , \quad K_1 \equiv +\beta J \quad , \quad K_2 = \beta h$$

$$Z_N(t,h) = \sum_{\{S\}=\pm1} e^{\sum_{i=1}^{N}\left[K_0 + K_1 S_i S_{i+1} + \frac{K_2}{2}(S_i + S_{i+1})\right]}$$



$$\left\{\begin{aligned} & J \sum S_i S_{i+1} \\ & J' \sum_{i=1}^{N/2} S_i S_{i+2} + 0 \\ & J'' \sum_{i}^{N/4} S_i S_{i+4} + 0 \end{aligned}\right.$$

$$\boxed{\begin{aligned} K_0 &= 0 \\ K_1 &= \beta J \\ K_2 &= \beta h \end{aligned}}$$

در مدل آیزینگ ساده

$$Z_N = \sum_{\substack{\{S_i\} \\ i=1 \dots N}} \prod_{i=1}^{N} e^{\left\{K_0 + K_1 S_i S_{i+1} + \frac{K_2}{2}\left[S_i + S_{i+1}\right]\right\}}$$

$$Z_N = \sum_{\substack{\{S_i\} \\ i=1,N}} \prod_{i=1}^{N/2} e^{2K_0 + K_1\left(S_{2i-1} S_{2i} + S_{2i} S_{2i+1}\right) + \frac{1}{2} K_2 \left(S_{2i-1} + 2 S_{2i} + S_{2i+1}\right)}$$

روی $S_{2i} = \pm 1$ جمع بزنیم

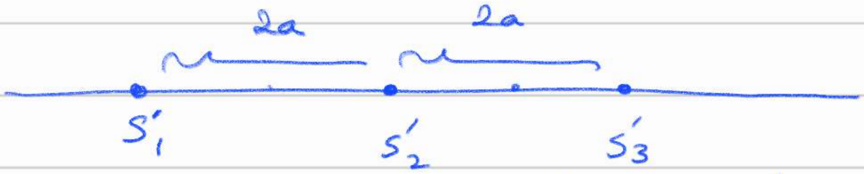
$$\prod_{j=1}^{N/2} e^{2K_0}\, 2\cosh\left(K_1\left(\underbrace{S_{2j-1}}_{i} + \underbrace{S_{2j+1}}_{i+1}\right) + K_2\right) e^{\frac{K_2}{2}\left(S_{2j-1} + S_{2j+1}\right)}$$

$$Z_{N/2} = \sum_{\{S'_i\}} \prod_{i=1}^{N/2} \Big[\underbrace{e^{2K_0}\, 2\cosh\{ K_1(S'_i + S'_{i+1}) + K_2 \}\, e^{\frac{K_2}{2}(S'_i + S'_{i+1})}} \Big]$$

$i=1, N/2$

$\boxed{l=2}$

مقایسه کنیم



$$Z_{N/2} = \sum_{\{S'_i\}} \prod_{i=1}^{N/2} \underbrace{e^{K'_0 + K'_1 S'_i S'_{i+1} + \frac{K'_2}{2}(S'_i + S'_{i+1})}}$$

$$\underbrace{e^{K'_0 + K'_1(S'_i S'_{i+1}) + \frac{1}{2}K'_2(S'_i + S'_{i+1})} = e^{2K_0}\, 2\cosh\{ K_1(S'_i + S'_{i+1}) + K_2 \}\, e^{\frac{K_2}{2}(S'_i + S'_{i+1})}}$$

$$[K]_2 = R_2[K]$$

$\uparrow$

$l=2$

$S'_i = S'_{i+1} = +1 \longrightarrow$ $\boxed{e^{(K'_0 + K'_1 + K'_2)} = e^{2K_0 + K_2}\, 2\cosh(2K_1 + K_2)}$

$S'_i = S'_{i+1} = -1 \longrightarrow$ $\boxed{e^{(K'_0 + K'_1 - K'_2)} = e^{2K_0 - K_2}\, 2\cosh(2K_1 - K_2)}$

$S'_i = -S'_{i+1} = \pm 1 \longrightarrow$ $\boxed{e^{K'_0 - K'_1} = e^{2K_0}\, 2\cosh(K_2)}$

for $h=0$ $\qquad K_2 = 0$

$$\begin{cases} K_0' = \ln\left\{2\sqrt{\cosh(2K_1)}\right\} \\ K_1' = \ln\left\{\sqrt{\cosh(2K_1)}\right\} = \frac{1}{2}\ln\cosh(2K_1) \\ K_2' = 0 \end{cases}$$

نقطه بحرانی پیدا کنیم

fixed point

$$[K'] = R_2[K]$$

$$[K' = K^*] = R_2[K = K^*]$$

A

$$[K^*] = R_2[K^*]$$

$$K_1' = \frac{1}{2}\ln\cosh 2K_1 \longrightarrow \begin{cases} K_1^* = 0 \to T_c = \infty \\ K_1^* = \infty \to T_c = 0 \end{cases}$$

$$\begin{cases} y_1 = K^* \\ y_2 = \frac{1}{2}\ln\cosh 2K^* \end{cases}$$

B

$$\boxed{\underset{\substack{\uparrow \\ l=2}}{t_2} = 2^{x_t}\, t}$$

خب $x_t = ?$

$$\boxed{K_1 = \frac{J}{K_B T} = \beta J}$$

$$t = \frac{T - T_c}{T_c}$$ تعریف خوبی نیست

$\beta > 0$ ضریب دلخواه

$$t = e^{-\beta K_1}$$ تعریف می‌کنیم

$T = T_c = 0$, $K_1 = \infty$ $\to$ $t = e^{-\beta\infty} = 0$

پس این تعریف بهتر است.

$$t = e^{-pK_1}$$

$$t_\ell = \ell^{x_t}\, t \qquad t \to 0,\quad T = T_c,\quad K_1 \to \infty$$

σ_c

حول $K_1=\infty$

$$K_1' = \frac{1}{2}\ln\cosh(2K_1)$$

$$\boxed{K_1' = K_1 - \frac{1}{2}\ln 2}$$

$$t' = e^{-pK_1'} = e^{-p\left[K_1 - \frac{1}{2}\ln 2\right]}$$

$$= e^{-pK_1}\, e^{\frac{p}{2}\ln 2}$$

$$t_2 = t\, 2^{p/2} = \ell^{x_t}\, t$$

$$t_2 = 2^{p/2}\, t \;\longrightarrow\; \boxed{x_t = p/2}$$

اگر $\boxed{p=2}$ در نتیجه $\boxed{x_t = 1}$

(C)

$$\boxed{\begin{aligned} &\alpha = 2 - 2/p \\ &\beta = 0 \\ &\gamma = \frac{2}{p} \\ &\delta = \infty \\ &\eta = 1 \end{aligned}}$$

(D) RG flow

stable attractive fixed point — Repulsive fixed point

$$\bullet \longleftarrow\longleftarrow\longleftarrow\longleftarrow\longleftarrow \bullet$$

$K_1^* = 0$, $T_c = \infty$ — $K_1^* = \infty$, $T_c = 0$

$$K_1' \simeq K_1 - \underbrace{\frac{1}{2}\ln 2}_{\text{مثبت}} \Rightarrow K_1' < K_1$$

(منفی)

$$K_1'' < K_1'$$

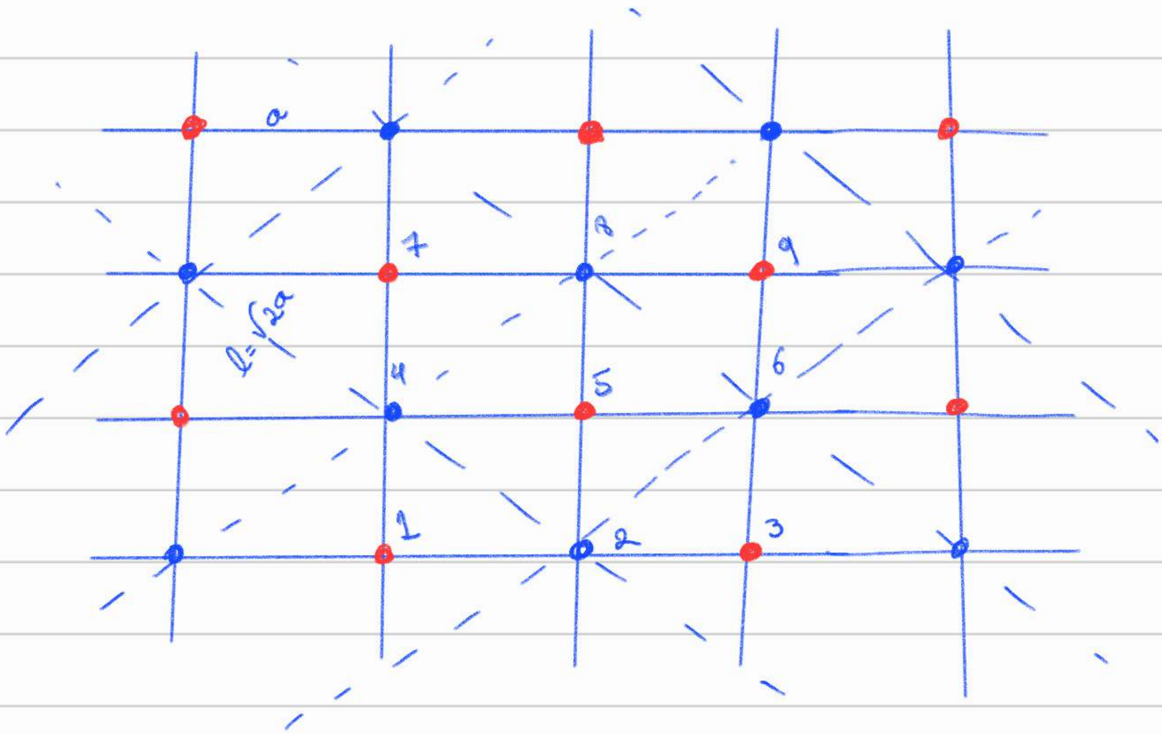
$$K_1''' < K_2''$$

Ex 2: Ising Model in 2D

{ Exercise 14.5 Migdal – Kadanoff }

$$Z(N,[K]) = \sum_{\{S_i\}} e^{K \sum_{n.n} S_i S_j}$$

n.n. = nearest Neighbor Interact



$$\sum_{S_5=\pm1} e^{K S_5 S_4 + K S_5 S_6 + K S_5 S_2 + K S_5 S_8} = 2\cosh K[S_2+S_4+S_6+S_8]$$

$S_2, S_4, S_6, S_8 \to \pm1$

$2^4 = 16$

① $S_2 = S_4 = S_6 = S_8$

② $S_2 = S_4 = S_6 = -S_8$

③ $S_2 = S_4 = -S_6 = -S_8$

④ $S_2 = -S_4 = -S_6 = S_8$

$$e^{K_0' + K_1'(S_2 S_4 + S_2 S_6 + S_6 S_8 + S_8 S_4)} \stackrel{?}{=} 2\cosh K[S_2+S_4+S_6+S_8]$$

$$2\cosh K[s_2+s_4+s_6+s_8] = e^{[K_0' + \frac{1}{2}K_1'[s_2 s_4 + s_2 s_6 + s_6 s_8 + s_8 s_4] + K_2'(s_2 s_8 + s_4 s_6) + K_3'(s_2 s_4 s_6 s_8)}$$

$$\left\{\begin{aligned} K_0' &= \ln 2 + \frac{1}{2}\ln\cosh 2K + \frac{1}{8}\ln\cosh 4K \\ K_1' &= \frac{1}{4}\ln\cosh 4K \\ K_2' &= \frac{1}{8}\ln\cosh 4K \\ K_3' &= \frac{1}{8}\ln\cosh 4K - \frac{1}{2}\ln\cosh 2K \end{aligned}\right\}$$

$$[K]_l = R_l[K]$$

یعنی با این وضعیت

$$Z(N,[K]) = \sum e^{K_1 \sum_{n.n} s_i s_j}$$

$$Z(N/2,[K']) = e^{\frac{N}{2}K_0'} \sum e^{K_1' \sum_{n.n} s_i' s_j' + K_2' \sum_{\substack{mn \\ n.n.n}} s_m' s_n' + K_3' \sum_{squar} s_j' s_k' s_l' s_m'}$$

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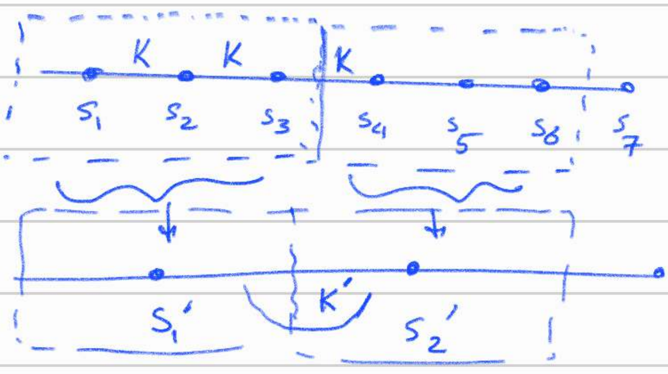
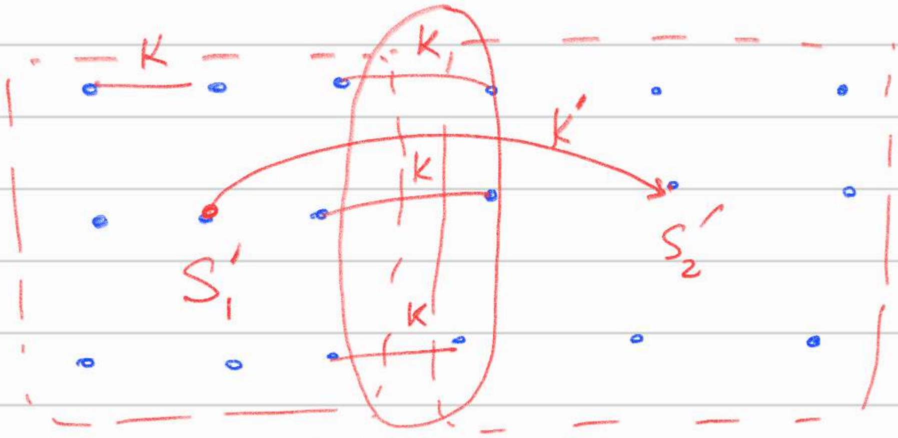
1D



$K' \simeq K \left(-\frac{1}{2}\ln 2\right)$ 1D

دقیق



تخمین غیردقیق $\boxed{K' \sim \ell^{d-1} K}$

$K' > K$

$d > 1$

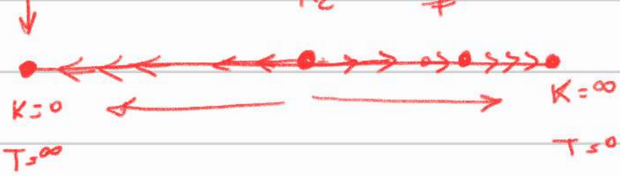
$K = 0 \longrightarrow T_c = \infty$

$K = \infty \longrightarrow T_c = 0$

$h = 0$

نقطه ثابت غیر بدیهی

$K_c \rightarrow T_c \neq 0, \neq \infty$



$K' > K$

$\boxed{[K]_\ell = R_\ell [K]}$

β-function
RG-Flow