بسم الله الرحمن الرحیم

جلسه ۶ ,۹ ,۱۴۰۱

Sec. 5.7 : Correlation function in Landau theory

Goldenfeld

$G(r-r')$

$\downarrow$

$\chi(r-r')$

We found that:

① $e^{-\beta F} = Z = \int Dm \, \underbrace{e^{-\beta L[m]}}_{W}$ → Landau free Energy determined from phenomenological approach

② $L[m] = \int d^dx \, \mathcal{L}(m, \partial_x m, \dots)$

③ $L[m] = L[m]_0 + \Delta m^T \cdot \frac{\partial L}{\partial m}\Big|_{[m]=[m]_0} + \frac{\Delta m^T}{2} \cdot \frac{\partial^2 L}{\partial m^2}\Big|_{[m]=[m]_0} \cdot \Delta m + \dots$

کمینه می کند L را

$[m] = [m]_0 + [\psi]$ → q-vector field in d-Dimension

$\boxed{[m] = [\phi] = [\eta]}$

در بُعد $\vec{w}$ → $[\vec{m}] = \underbrace{m_l \hat{e}_l}_{i=1} + \underbrace{\sum_{i=2}^{q} m_i \hat{e}_i}_{\text{Transverse modes}}$

longitudinal mode

④ $$L[\phi] = \int d^d x \left[a_0 + a_2 \phi^2 + a_4 \phi^4 + (\nabla \phi)^2 \right]$$

$$[\phi] = [\phi]_0 + [\psi] \qquad \text{1\_vector field.}$$

for $t > 0$ $(T > T_c)$ $\quad [\phi]_0 = 0 \rightarrow [\phi] = [\psi]$

for $t < 0$ $(T < T_c)$ $\quad 0 \neq [\phi]_0 = \pm \left(-\dfrac{a_2}{2a_4} \right)^{1/2} \rightarrow [\phi] = [\phi]_0 + [\psi]$

$\nabla \phi \sim 0$

$$\frac{\partial \mathcal{L}}{\partial \phi} - \nabla \frac{\partial \mathcal{L}}{\partial \phi'} = 0 \longrightarrow [\phi]_0 = \checkmark$$

$\phi' \to \partial_x \phi$

$$\nabla^2 \phi - 2a_2 \phi - 4a_4 \phi^3 = 0$$

$\nabla^2 \phi \sim 0$

$$\boxed{[\phi]_0 = \pm \sqrt{\frac{-a_2}{2a_4}}}$$

⑤ For $t > 0$ $\quad L[\phi] = \int d^d x \left[a_0 + a_2 \psi^2 + a_4 \psi^4 + (\nabla \psi)^2 \right]$

$\phi_0 = 0$

$a_4 \sim 0$

$$L[\phi] = \int d^d x \left[a_0 + \psi (a_2 - \nabla^2) \psi \right]$$

$$e^{-\beta F} = \int D\psi \, e^{-\beta V^{(d)} a_0} \, \underbrace{e^{-\beta \int d^d x \, 2 \frac{[\psi (a_2 - \nabla^2) \psi]}{2}}}_{\text{شبیه تابع گوسی}}$$

Gaussian function form

$$e^{-\beta F} = e^{-\beta V^{(d)} a_0} \int D\psi \, e^{-\frac{\beta}{2} \int d^d x \, \psi \underbrace{2(a_2 - \nabla^2)}_{K} \psi}$$

$$= e^{-\beta V^{(d)} a_0} \frac{(2\pi)^{d/2}}{\sqrt{\mathrm{Det}(K)}}$$

$$F = V^{(d)} a_0 \underbrace{- \frac{d}{2\beta} \ln (2\pi)}_{} + \frac{1}{2} \ln \mathrm{Det}(K)$$

$T > T_c$
$t > 0$

$$= V^{(d)} a_0 + cts + \frac{1}{2\beta} \mathrm{Tr} \ln (2(a_2 - \nabla^2))$$

$$\phi = e^{i \vec{k} \cdot \vec{x}}$$

$$= V^{(d)} a_0 + cts + \frac{1}{2\beta} \underbrace{\int_{|k| \leq a^{-1}} d^d k \, \ln 2(a_2 + k^2)}_{}$$

ولی بقیه را استدلال

⑥ $t < 0$ $\quad 0 \neq [\phi]_0 = \pm \sqrt{-\frac{a_2}{2a_4}}$

$$L[\phi] = \int d^d x \left[a_0 + a_2 (\phi_0 + \psi)^2 + a_4 (\phi_0 + \psi)^4 - \nabla^2 \psi \right]$$

ع۱

$$= L[\phi]_0 + \int d^d x \left\{ -2a_2 \psi^2 + (\nabla \psi)^2 \right\}$$

تا ۱/۰ نقطه نزدیکی خود

$$= L[\phi]_0 + \int d^d x \, \psi \overbrace{\frac{2(-2a_2 - \nabla^2)}{2}}^{K} \psi$$

$$\boxed{K = 2(-2a_2 - \nabla^2)\delta_{ij}}$$

همبستگی بین عناصر غیرقطری وجود ندارد.

همبستگی کوتاه‌برد است اما نظم بلندبرد.

کلیه محاسبات بالا در فضای فوریه می‌توان انجام داد.

7 Correlation function 5.7 (Goldenfeld)

میدان مغناطیسی خارجی $H(r)$ در حالت کلی h

$[m] = [\eta]$

$$(5.55) \quad L[\eta] = \int d^d r \left[\frac{\gamma}{2} (\nabla\eta)^2 + \underbrace{at}_{a_2}\eta^2 + \underbrace{\tfrac{1}{2}b}_{a_4}\eta^4 - H\eta \right]$$

فرض همگنی

$\boxed{R = |r - r'|}$

$$\begin{cases} G(\underbrace{r-r'}) = \langle (\eta(r) - \langle\eta\rangle)(\eta(r') - \langle\eta\rangle) \rangle = ? \\ \quad = \langle \eta(r)\eta(r') \rangle - \langle\eta\rangle^2 \\ \chi(r-r') = \beta\, G(r-r') = ? \end{cases}$$

اگر معادلات حالم را نسبت را حل کنیم

$$\begin{cases} \langle \eta(r) \rangle = -\dfrac{\delta F}{\delta H(r)} \\ \chi(r-r') = \dfrac{\delta \langle \eta(r) \rangle}{\delta H(r')} = -\dfrac{\delta^2 F}{\delta H(r)\, \delta H(r')} = \beta\, G(r-r') \end{cases}$$

$$G(r-r') = K_B T\, \chi(r-r')$$

F.T

$$\tilde{G}(K) = K_B T\, \tilde{\chi}(K)$$

$K = 0$ — $\lambda \to \infty$

static Response funct

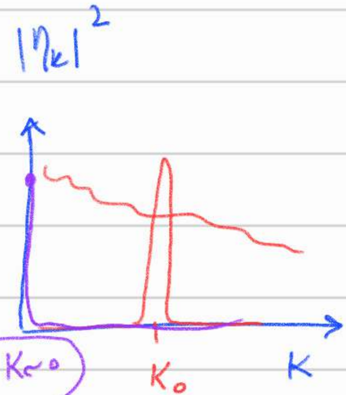
در حالت کلی می‌تواند وابسته به زمان هم باشد

$$\boxed{\tilde{G}(K,\omega) = K_B T\, \tilde{\chi}(K,\omega)}$$

$$\left\{ \begin{cases} K = 0 \ (\lambda \xrightarrow{L} \infty) \\ \omega = 0 \end{cases} \qquad \begin{matrix} K \neq 0 \\ \omega \neq 0 \end{matrix} \right\}$$

$$e^{-\beta F} = Z = \int D\eta \, e^{-\beta L[\eta]} = \int \prod_{i=1}^{N_\Omega} d\eta_i \, e^{-\beta L[\eta]}$$

$$= \underbrace{\sum_{s_1} \sum_{s_2} \sum_{s_3} \cdots \sum_{s_N}}_{\#\text{ of sites}}$$



$t=0$ در $K \to 0$

Notation

(I) $$\eta(r) = \frac{1}{V^{(d)}} \sum_k \eta_k e^{i\vec{K}\cdot\vec{r}}$$

$$\frac{V^{(d)}}{(2\pi)^d} \int d^d k$$

(II) $$\frac{1}{V^{(d)}} \sum_k e^{i\vec{K}\cdot\vec{r}} = \frac{1}{V^{(d)}} \int \frac{V^{(d)}}{(2\pi)^d} d^d K \, e^{iK\cdot r}$$

$$V^{(d)} \delta_{KK'} = (2\pi)^d \delta_D(K-K')$$

(III) $$\frac{\delta}{\delta \eta(r)} \int d^d r' \, \eta(r') = 1$$

$$\frac{\delta}{\delta \eta(r)} \eta(r') = \delta_D(r-r')$$

$$\frac{\delta}{\delta \eta(r)} \int d^d r' \, \frac{1}{2} (\nabla \eta(r'))^2 = -\nabla^2 \eta(r)$$

پس در اینجا $\langle \eta(r) \rangle$ حساب می‌کنیم.

$$L[\eta] = \int d^d r \left[\frac{\gamma}{2} (\nabla \eta)^2 + a t \eta^2 + \frac{1}{2} b \eta^4 - H(r)\eta(r) \right]$$

$$0 = \frac{\delta L}{\delta \eta} \Rightarrow \boxed{-\gamma \nabla^2 \eta(r) + 2at\, \eta(r) + 2b\eta^3(r) - H(r) = 0}$$

معادله حاکم بر $\langle \eta(r) \rangle$ $\quad -\gamma \langle \nabla^2 \eta(r) \rangle + 2bt \langle \eta(r) \rangle + 2b \langle \eta(r)^3 \rangle - H(r) = 0$

$$\to \langle \eta(r) \rangle = \checkmark$$

$$\eta^4(r) = \left(\eta^2(r)\right)^2$$

$$\frac{\delta}{\delta H(r')} \left[-\gamma \nabla^2 \eta(r) + 2at\, \eta(r) + 2b\eta^3(r) - H(r) \right] = 0$$

$$-\gamma \nabla^2 \chi(r-r') + 2at\, \chi(r-r') + 6b \langle \eta^2(r) \rangle \chi(r-r') - \delta_D(r-r') = 0$$

$$\boxed{\underbrace{\left[-\gamma \nabla^2 + 2at + 6b \langle \tilde{\eta}^2(r) \rangle \right]}_{\hat{\mathcal{L}}} \chi(r-r') = \delta_D(r-r')}$$

$$\boxed{\hat{\mathcal{L}}\, \chi(r-r') = \delta_D(r-r')}$$

$$\boxed{\beta \hat{\mathcal{L}}\, G(r-r') = \delta_D(r-r')}$$

$\langle \eta(r) \rangle = 0$

$t > 0 \to \checkmark$

$t < 0 \to \checkmark$

$\langle \eta(r) \rangle \neq 0$

For $t > 0$ $\quad \left(-\nabla^2 + \xi_>^{-2} \right) G(r-r') = \frac{K_B T}{\gamma} \delta_D(r-r')$

$$\boxed{\xi_> \equiv \left(\frac{\gamma}{2at}\right)^{1/2}} \sim t^{-1/2} \qquad \nu = 1/2$$

$t>0$

For $t<0$ $\quad \eta = \pm \left(-at/b\right)^{1/2}$

$$\left(-\nabla^2 + \xi_<^{-2}\right) G(r-r') = \frac{K_B T}{\gamma}\, \delta_D(r-r')$$

$$\boxed{\xi_< \equiv \left(\frac{-\gamma}{4at}\right)^{1/2}} \quad t<0 \quad \sim t^{-1/2} \;—\; \nu = 1/2$$

$t \sim \xi^{-2}$ طول همبستگی

پس به طور کلی

$$\boxed{\left[-\nabla^2 + \xi^{-2}\right] G(r-r') = \frac{K_B T}{\gamma}\, \delta_D(r-r')} \qquad (5.94)$$

☆ Fourier Transformation

☆ Polar coordinates

$$\text{F.T.} \longrightarrow \left(K^2 + \xi^{-2}\right)\tilde{G}(K) = \frac{K_B T}{\gamma}$$

$$\tilde{G}(K) = \frac{K_B T}{\gamma}\,\frac{1}{K^2+\xi^{-2}}$$

For $t=0$ $(T=T_c)$ $\quad \tilde{G}(K) \sim K^{-2}$

$\xi \to \infty$ $\quad G(R) \simeq \dfrac{1}{R^{d-2+\eta}}$

[illegible] $\quad \eta = 0$

$\boxed{\text{Mean field}}$

$$G(r) = \frac{1}{\beta \gamma} \frac{\Gamma\left(\frac{d-2}{2}\right)}{4\pi^{d/2}} \frac{1}{r^{d-2}} \longrightarrow \boxed{\eta = 0}$$

Chapter 8 : Goldenfeld (9.2 Kardar)

See 8.3 Dynamical Critical Exponent

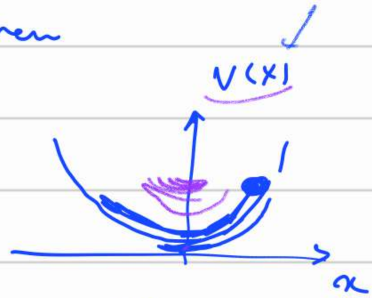
Time - Dependent Landau Ginzburg equation

Thermodynamical Equilibrium
- thermal $\quad T_1 = T_2$
- Mechanical $\to P_1 = P_2$
- chemical $- \mu_1 = \mu_2$

Stationary $\longrightarrow$ Static Results

Quasi - Static

fluctuation Dissipation theorem



$x(t) \to x = 0$

$\uparrow A e^{i\omega t}$

(زمان را نیز دمای کاهش یابنده)

$$\eta(r) \to \underline{\eta(r,t)}$$

$$L[\eta(r,t)] = \int d^d r \left[\frac{1}{2}\gamma(\nabla\eta(r,t))^2 + \overset{at}{\tilde{a}}\,\eta^2(r,t) + \frac{1}{2} b\eta^4 - H\eta\right]$$

$$\eta(r,t)\,s? \longrightarrow \chi(r,t) \longleftrightarrow \chi(k,\omega)$$

$$G(r,t) \longleftrightarrow G(k,\omega)$$

$$\frac{\partial\eta(r,t)}{\partial t} = -\Gamma\frac{\delta L}{\delta\eta(r,t)} + \underbrace{\xi(r,t)}_{\text{Noise}} \longrightarrow \begin{cases} \eta(r,t)\,s? \\ \dfrac{\partial P(\eta)}{\partial t}\,s? \end{cases}$$

Master Eq.
↳ Kramers-Moyal Expan
↳ Fokker-Planck

$$\begin{cases} \langle \xi(r,t)\rangle = 0 \\ \langle \xi(r,t)\,\xi(r',t')\rangle = D\,\delta_D(r-r')\,\delta_{tt'} \\ P_\xi(\{\xi\}) = \exp\left[-\frac{1}{2D}\int dt\, d^d r\; \xi^2(r,t)\right] \quad \text{Gaussian Distribution} \end{cases}$$

ویژگی‌های نوفه لانژون در مدل‌سازی مبتنی بر قضیه برای افت و خیز

fluctuation Dissipation theorem

این صفر

$T > T_c \qquad \langle \eta \rangle = 0 \qquad\qquad \eta(r,t) = \langle \eta \rangle + \delta\eta$

$$\frac{\partial \eta(t,r)}{\partial t} = -\Gamma \frac{\delta L}{\delta \eta} + \xi(r,t)$$

$H = 0$

$$\frac{\partial\, \delta\eta}{\partial t} = -\Gamma \left[-\gamma \nabla^2 \delta\eta + 2a\, \delta\eta \right] + \xi$$

$$\frac{\partial \delta\eta}{\partial t} = \left(-\frac{\delta\eta}{\tau_0} + \gamma \Gamma \nabla^2 \delta\eta \right) + \xi$$

$$\tau_0 \equiv (2a\Gamma)^{-1}$$

F.T.

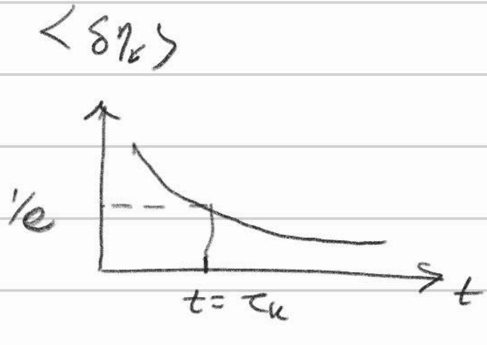
$$\boxed{\begin{gathered} \frac{\partial\, \delta\eta_k}{\partial t} = -\frac{\delta\eta_k}{\tau_k} + \xi_k \\ \tau_k^{-1} = \tau_0^{-1} + \gamma\Gamma K^2 \end{gathered}}$$

مقدار را نداریم
فقط خواص
آماری آن را
بلدهستیم

منظور $\delta\eta_k$ چیست؟ . چگونه حل کنیم؟

$$\frac{\partial \langle \delta\eta_k \rangle}{\partial t} = -\frac{\langle \delta\eta_k \rangle}{\tau_k} + \langle \xi_k \rangle$$

$\langle \xi_k \rangle = 0$



$$\langle \delta\eta_k \rangle \sim e^{-t/\tau_k}$$

$\tau_K \equiv$ Relaxation time scale

مدهای مختلف

اکنون مدهای مختلف احتمال را جزئیات بررسی کنیم.

(I) $K=0 \ (\lambda \to \infty)$ Global mode

$$\langle \delta\eta_{K=0} \rangle \sim e^{-t/\tau_0} \sim e^{-(2a\Gamma t)} \sim e^{-t/\xi}$$

مد غالب در نزدیکی نقطه بحرانی

$$T \to T_c \ (\xi \to \infty) \longrightarrow \langle \delta\eta_{K=0} \rangle \sim e^{-t/\infty}$$
$$\tau_0 \to \infty \qquad \sim cts$$

تحول زمانی ندارد

در $K \neq 0$ $T \to T_c$ برای سایر مدها $\tau_K =$ finite. یعنی سایر مدها سریع میرا می‌شوند

$$\delta\eta_{K\neq 0} \ (T \to T_c) \longrightarrow 0$$

$$\delta\eta_{K=0} \ (T \to T_c) = cts \longrightarrow 0$$

Slowing Down Criticality

Susceptibility.

$$\tilde{\chi}(K,\omega) = \frac{\delta \langle \tilde{\eta}(K,\omega) \rangle}{\delta \tilde{H}(K,\omega)} \qquad H \neq 0$$

$$\frac{\delta \chi(r,t)}{\delta t} = -\frac{\chi(r,t)}{\tau_0} + \gamma\Gamma\nabla^2\chi(r,t) + \Gamma$$

$\searrow$ F.T

$$(r,t) \longrightarrow (K,\omega)$$

$$\frac{\partial}{\partial t} \longrightarrow -i\omega$$

$$\frac{\partial}{\partial r} = iK$$

$$\tilde{\chi}(K,\omega) = \frac{\Gamma}{-i\omega + \tau_K^{-1}}$$

$T \to T_c$

$$\tilde{\chi}(K=0,\omega) = \frac{\Gamma}{-i\omega + 0}$$

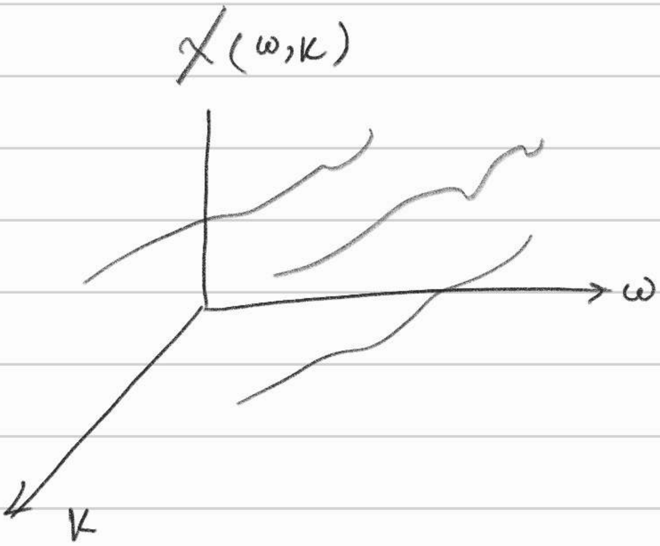
$K=0,\ \omega=0$

$$\boxed{\tilde{\chi}(0,0) \longrightarrow \infty}$$

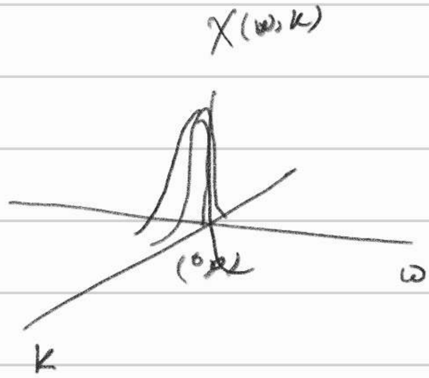
$\omega \to 0$

Static Response funct

$$\tilde{\chi}(K\neq 0,\ \omega\neq 0) = \text{finite}$$



$T = T_c$ ⟶



معادله حاکم بر تحول تابع توزیع احتمال در مقدار پارامتر نظم

$$\frac{\partial}{\partial t} P_\eta(\{\eta(r)\}, t) = \int d^d r' \frac{\delta}{\delta \eta(r')} \Big[\underbrace{\Gamma \frac{\delta L}{\delta \eta(r')}}_{D^{(1)}} P_\eta + \underbrace{\frac{D}{2} \frac{\delta P_\eta}{\delta \eta(r')}}_{D^{(2)}} \Big]$$

$\hookrightarrow$ Exercise