بسم الله الرحمن الرحیم جلسه ۱۴۰۱/۸/۳

Chapter 1: Phase Transition in Simple system

(Cardy)

* Type of Phase Transition

→ Co-existent phases
first-order phase
Dis-continuous phase Transition
finit Correlation scale

↓ Continuouse phase transition
Infinite Correlation length scale at $T \to T_c$

* ξ : Correlation Length Scale
Time
Angular Scale.

Correlation function — تابع همبستگی

ارجاع می‌دهم درس فرآیندهای تصادفی

Weigthed Two point Correlation function

وزن دار — دو نقطه‌ای — تابع

Auto-Correlation

خود همبستگی

$$C_{\delta S}(\vec{r}_i, \vec{r}_j) = \langle \delta S(\vec{r}_i)\, \delta S(\vec{r}_j) \rangle$$

δS

$$= G(\vec{r}_i, \vec{r}_j) \rightarrow \text{Connected moment}$$

M = Magnetization

$$\delta S(\vec{r}_i) = S(\vec{r}_i) - \underbrace{\langle S(\vec{r}) \rangle_r}_{M}$$

$$G(\bar{r}_i, \bar{r}_i) = \sigma_0^2$$

Homoganiety System $\quad G(\bar{r}_i, \bar{r}_j) = G(r)$

$$r = |\vec{r}_i - \vec{r}_j|$$

$G(\bar{r}_i, \bar{r}_j)$: Response function   تابع پاسخ

Exercise 1: for $\mathcal{H} = \mathcal{H}(\{S\}) - \sum_{i=1}^{N} H_i S_i$

H_i: magnetic field at S_i

$$\boxed{\text{Ideal Gas} \quad \mathcal{H} = \sum \frac{p_i^2}{2m} + \sum_{ij} u_{ij}}$$

$$G(r_i, r_j) = \langle \delta S(i)\, \delta S(j) \rangle$$

$$= K_B T \frac{\partial}{\partial H_i} \langle S_j \rangle$$

تغییرات در محل S_j به دلیل تغییرات در محل r_i

Susceptibility χ : Global Response function

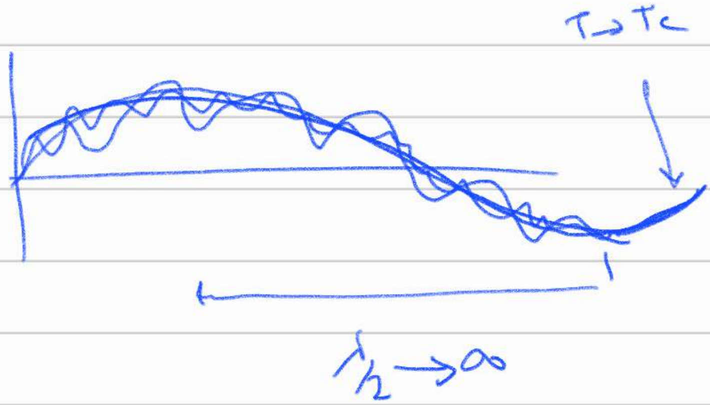
$$\sum_{ij} G(\bar{r}_i, \bar{r}_j) \propto \chi$$

$T \to T_c$ $\quad \chi$: **Static** Susceptibility

تابع پاسخ سراسری است.

Dominant mode for variation of spins $\left\{ \begin{array}{l} \chi(\omega \to 0) \\ (K \to 0) \\ \lambda \to \infty \end{array} \right.$

Langevin Model building



$\mathcal{H} \longrightarrow$ Effective Dynamical model.

$$\left\{ \frac{\partial \langle \vec{S}_i \rangle}{\partial t} = ? \leftarrow \text{Langevin} \right\}$$

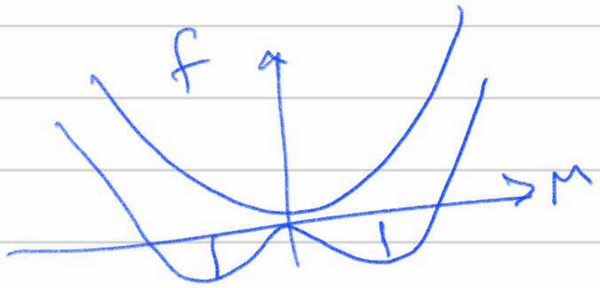
$\boxed{T \to T_c}$

تقارن های سیستم اینجا نقش می کنند

Free Energy $\to$ $f = a + btM^2 + cM^4 + dHM + \dots + O(M^6)$

$\frac{\partial f}{\partial M} = 0$

$T > T_c$ — M $\quad \boxed{F = -k_B T \ln Z}$



$G(r)$: $\boxed{r = |\bar{r}_i - \bar{r}_j|}$

$\rightarrow \chi \propto \int d^d r \, G(r) \longrightarrow \boxed{\chi \propto \xi^2}$

If $d=3$ $\quad G(r) = \underbrace{\frac{1}{r} f(r/\xi)}_{\eta=0}$

$\chi \propto \xi^{2-\eta}$ آدرس

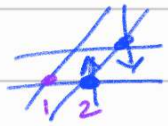
Unit cell character

فرض
$$\begin{cases} G(r) = \frac{1}{r} f(r/\xi, a/\xi) \\ f(r/\xi, a/\xi) \sim (a/\xi)^{\eta} \end{cases} \longrightarrow \boxed{\chi \propto a^{\eta} \xi^{2-\eta}}$$

Mean field theory $\eta = 0$

General Properties of Ising Model.

$$\mathcal{H} = -\vec{H} \cdot \sum_{i=1}^{N} \vec{S}_i - \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



$S_i = \pm 1$

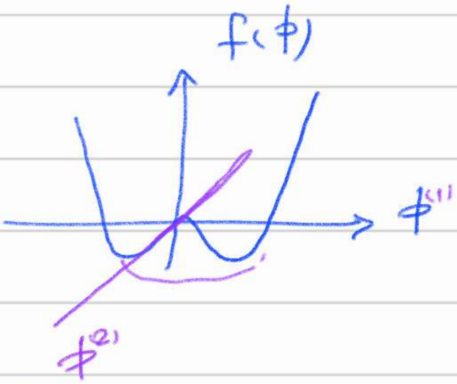
$$M = \frac{1}{N} \sum_{i=1}^{N} \langle \vec{S}_i \rangle$$

i : $\phi_i : \{\phi_i^1, \phi_i^2, \dots \phi_i^n\}$

$M = -\frac{1}{N} \partial f / \partial H$ — $O(n)$

$O(2) = \{\phi^{(1)}, \phi^{(2)}\}$

$O(1) : \{\phi\}$



$O(1) \Rightarrow \quad f = a + bt\phi^2 + c\phi^4 + \cdots$

$O(n) \Rightarrow \quad f = a + bt \sum_{a=1}^{n} (\phi^{(a)})^2 + c\sum_{a=1}^{n} (\phi^{(a)})^4 + \cdots$

free Energy.

Ⓐ $f < 0$

Ⓑ $f(H, T, J, \ldots)$ is continuous

Ⓒ $\frac{\partial f}{\partial T}, \frac{\partial f}{\partial H} \ldots$ exist every where

Ⓓ S (Entropy) $= -\frac{\partial f}{\partial T} \geq 0$

Ⓔ $\frac{\partial f}{\partial T} =$ is monotonically non-increasing.

$$\frac{\partial^2 f}{\partial T^2} \leq 0$$

$$C \propto -\frac{\partial^2 f}{\partial T^2} \geq 0$$

$$\frac{\partial f}{\partial H} \ldots \longrightarrow \frac{\partial^2 f}{\partial H^2} \leq 0$$

$$\chi \propto -\frac{\partial^2 f}{\partial H^2} \geq 0$$

$$f \overset{?}{<} 0 \longleftrightarrow$$

$$f = \frac{F_\Omega}{V(\Omega)} = -K_B T \ln Z_\Omega$$

$$Z_\Omega = \sum_{\{\}} e^{-\beta \mathcal{H}} = \overbrace{0.01 + 0.001 + 1 +}^{>1}$$

حداقل یک پیکربندی وجود دارد که همه اسپین‌ها هم‌جهت باشند ← $\mathcal{H} < 0$

$$e^{-\beta \mathcal{H}} > 1 \longrightarrow Z > 1 \longrightarrow F_\Omega < 0$$

Recall scaling Exponents for Ising model.

$$\left\{ \begin{array}{ll} M \propto |-t|^{\beta} & \text{for } H \to 0 \\ \chi \propto t^{-\gamma} & \\ M \propto H^{1/\delta} & t \to 0 \\ C \propto |t|^{-\alpha} & \\ \xi \propto |t|^{-\nu} & \end{array} \right\}$$

$$G(r) \propto \frac{1}{r^{d-2+\eta}}$$

$$\boxed{\begin{array}{c} \beta \\ \gamma \\ \delta \\ \alpha \\ \nu \\ \eta \end{array}}$$

fluid : A classical fluid

Including N-Particles

at $\vec{r}_1, \vec{r}_2 - - \vec{r}_N$

$$\mathcal{H} = \mathcal{H}_{Kinetic} + \mathcal{H}_{Intraction}$$

$$= \quad '' \quad + \sum_{ij} \widetilde{U(\bar{r}_i - \bar{r}_j)} = \quad '' \quad \overbrace{- \sum_{r_i r_j} 2 J_{ij} \cdot n(r_i) n(r_j)}^{\text{انتخاب}}$$

$$\mathcal{Z}(T, V, z) \propto \underbrace{\sum_{N=0}^{\infty} \frac{z^N}{N!} \int d^3 r_1 \, d^3 r_2 \ldots d^3 r_N \, e^{-\beta \mathcal{H}_{Int.}}}$$

fugacity $z \equiv e^{\beta\mu}$

Mapping $\longrightarrow$ Lattice Gas

Occupation No. فرض کنید فضا را گسسته می کنیم. عدد اشغال $n(r)$

$$\boxed{n(r_i) = \begin{cases} 1 \\ 0 \end{cases}}$$

Constraint $\sum_r n(r) = N$

$$\mathcal{Z}(T, V, z) = \sum_{N=0}^{\infty} \sum_{\{n(r)\}}' \frac{1}{N!} \ldots \ldots$$

$$= \sum_{\{n(r) = 0, 1\}} \boxed{\underbrace{e^{\beta\mu \sum_r n(r)}}_{z^N} \; e^{-\beta(-2) \sum_{rr'} J_{rr'} n(r) n(r')}}$$

$$S(r) \equiv 2n(r) - 1 \longrightarrow S(r) \begin{cases} -1 \\ +1 \end{cases}$$

$$z^{\sum n(r)}\, e^{2\beta \sum J_{rr'} n(r) n(r')}$$

$$= z^{N/2}\, z^{\sum \frac{s(r)}{2}}\, e^{2\beta \sum J_{rr'} \left(\frac{s(r)+1}{2}\right)\left(\frac{s(r')+1}{2}\right)}$$

$$Z = \sum_{\{s(r)=-1,+1\}} e^{\frac{\beta}{2} \sum J_{rr'} s(r') s(r) + \beta H \sum s(r)}$$

$$\boxed{H \equiv \sum_{r'} J_{rr'} + \frac{K_B T}{2} \ln z}$$

$n(r)=0$ نهایتاً با فرض اینکه فضا گسسته است و از یک حجمی کمتر

$n(r)=+1$ از یک حجمی بیشتر

معادل یک مدل آیزینگ با یک میدان خارجی در نقطه r

$$H_{ext}(r) = \frac{1}{2} K_B T \underset{\substack{\uparrow \\ e^{\beta\mu}}}{\ln z} + \sum_{r'} J_{rr'} s(r')$$

Mean field Theory

Chapter 2 : Cardy