

In the name of God

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ADVANCED TOPICS IN MODERN COSMOLOGY

Exercise Set 7

(Date Due: 1393/02/30)

1. To introduce metric during inflation one approach is that supposing 4-dimensional hyperboloid metric embedded in 5-dimensional Minkowski space-time as:

$$-(z_0^2) + z_1^2 + z_2^2 + z_3^2 + z_4^2 = H^{-2}$$

A :Show that for Euclidean slicing we have:

$$z_0 = \frac{\sinh Ht}{H} + \frac{H}{2} e^{Ht} \mathbf{x}^2$$
$$z_4 = \frac{\cosh Ht}{H} - \frac{H}{2} e^{Ht} \mathbf{x}^2$$
$$z_i = e^{Ht} x_i$$

in this case the 4-dimensional space-time is given by:

$$ds^2 = -dt^2 + e^{2Ht} \delta_{ij} dx^i dx^j$$

B :Show that for Spherical slicing we have:

$$z_0 = \frac{\sinh Ht}{H}$$
$$z_1 = \frac{\cosh Ht}{H} \cos \chi$$
$$z_2 = \frac{\cosh Ht}{H} \sin \chi \cos \theta$$
$$z_3 = \frac{\cosh Ht}{H} \sin \chi \sin \theta \cos \phi$$
$$z_4 = \frac{\cosh Ht}{H} \sin \chi \sin \theta \sin \phi$$

in this case the 4-dimensional space-time is given by:

$$ds^2 = -dt^2 + \frac{\cosh^2 Ht}{H^2} (d\chi^2 + \sin^2 \chi d\Omega^2)$$

C :Show that for Static slicing we have:

$$z_0 = \frac{\sinh Ht}{H} \sqrt{1 - r^2 H^2}$$
$$z_1 = \frac{\cosh Ht}{H} \sqrt{1 - r^2 H^2}$$
$$z_2 = r \cos \theta$$
$$z_3 = r \sin \theta \cos \phi$$
$$z_4 = r \sin \theta \sin \phi$$

in this case the 4-dimensional space-time is given by:

$$ds^2 = -(1 - r^2 H^2) dt^2 + \frac{dr^2}{(1 - r^2 H^2)} + r^2 d\Omega^2$$

2. As an example suppose $V(\phi) = e^{-\lambda\phi}$ and compute the Klein-Gordon and Friedmann equations. Investigate the inflationary solution. What can say about Graceful exit?
3. Solve problems 5.1, 5.2, 5.3, 5.4, 5.6, 5.7, 5.8, 5.9 of chapter 5 in "*Physical foundations of Cosmology*" written by V. Mukhanov.

Good luck, Movahed
