بسم الله الرحمن الرحیم

Correlation
(۱۴۰۳)

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دانشکده فیزیک - دانشگاه شهید بهشتی

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مهمترین منابع مورد استفاده

Books:

موارد قرمز شده بسیار مرتبط است خصوصا آخرین مرجع قرمز رنگ

- Data analysis: A Bayesian Tutorial, by D.S. Sivia & J. Skilling, Oxford science Publication, 2010
- Data reduction and error analysis for the physical sciences, P. R. Bevington & D. K. Robinson, McGrawHill, 2003
- Error of Observations and their Treatment, J. Topping, 1972.
- Practical Physics, G. L. Squires, 1985.
- An Introduction to to Error Analysis: The Study of Uncertanties in Physical Measurements, Taylor, J. R. 2nd Ed.; University Science Books: Sausalito, CA, 1997.
- Data analysis in cosmology, "Lecture notes in physics 665"
- Mathematical statistics with applications, J.E. Freund's, Pearson education, 2005,
- All of Statistics, L. Wasserman, Springer, 2004
- Statistics, R.J. Barlow, Wiley, 2002.
- Introduction to statistics and Data analysis for Physicist, Gerhard Bohm, Günter Zech, 2010.
- Statistics, Data Mining, and Machine Learning in Astronomy: A Practical Python Guide for the Analysis of Survey Data (Princeton Series in Modern Observational Astronomy), Željko Ivezic´, Andrew J. Connolly,
  Jacob T. VanderPlas, and Alexander Gray, PRINCETON UNIVERSITY PRESS, 2014

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Papers and Lectures:

- Verde, Licia. "A practical guide to basic statistical techniques for data analysis in cosmology." arXiv preprint arXiv:0712.3028 (2007).
- Verde, Licia. "Statistical methods in cosmology." Lectures on Cosmology. Springer, Berlin, Heidelberg, 2010. 147-177.
- Liddle, Andrew R. "How many cosmological parameters." Monthly Notices of the Royal Astronomical Society 351.3 (2004): L49-L53.
- Heavens, Alan. "Statistical techniques in cosmology." arXiv preprint arXiv:0906.0664 (2009).
- Efron, Bradley. "Bayesians, frequentists, and physicists." PHYSTAT2003: Statistical Problems in Particle Physics, Astrophysics, and Cosmology, SLAC, Stanford CA (2003): 17-24
- BARLOW, ROGER J. "Asymmetric statistical errors." Statistical Problems In Particle Physics, Astrophysics And Cosmology. 2006. 56-59, arXiv:physics/0406120
- Betancourt, Michael. "A conceptual introduction to Hamiltonian Monte Carlo." arXiv preprint arXiv:1701.02434 (2017).
- Neal, Radford M. "MCMC using Hamiltonian dynamics." Handbook of Markov Chain Monte Carlo 2.11 (2011).

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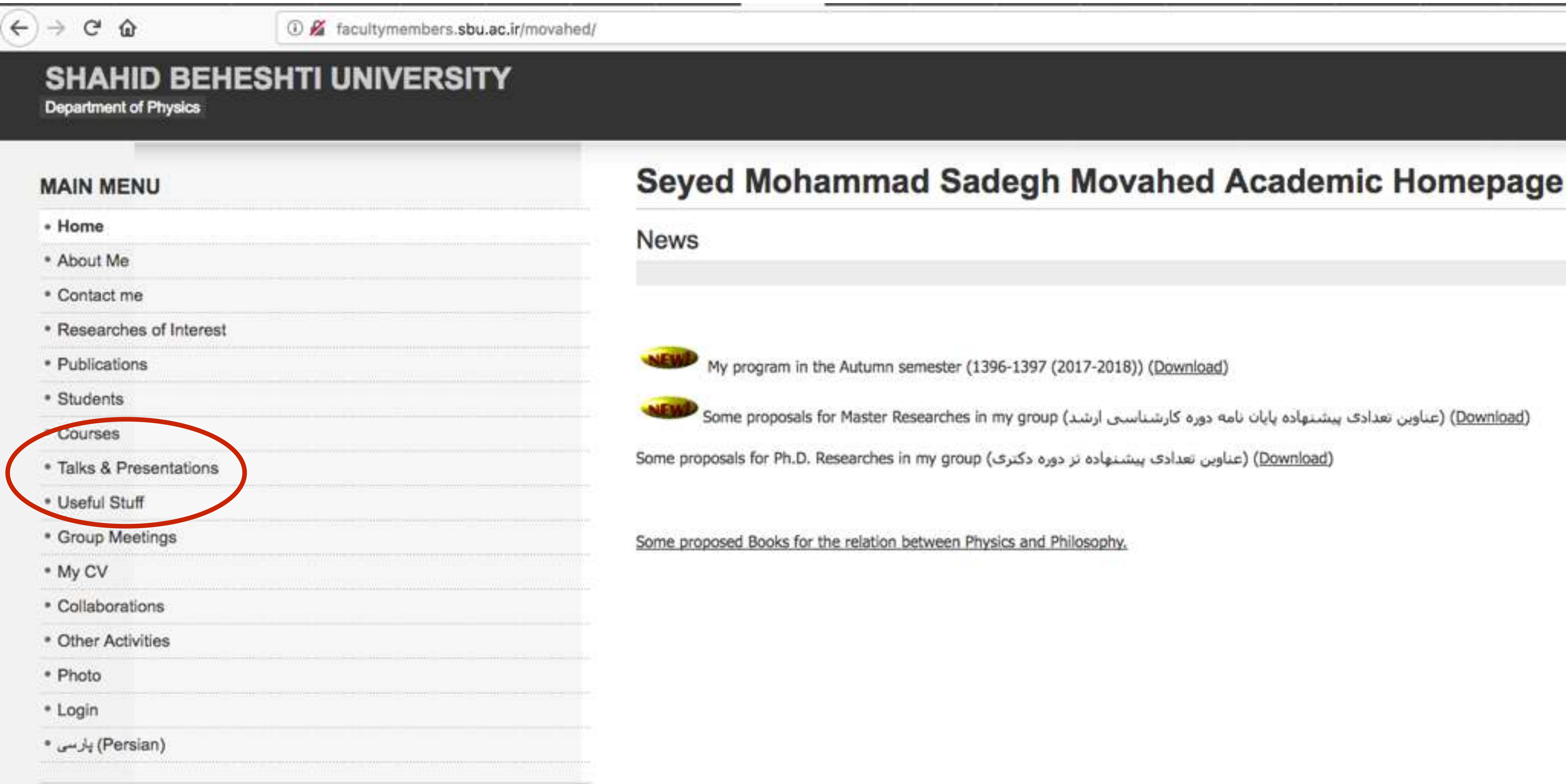

فهرست مطالب

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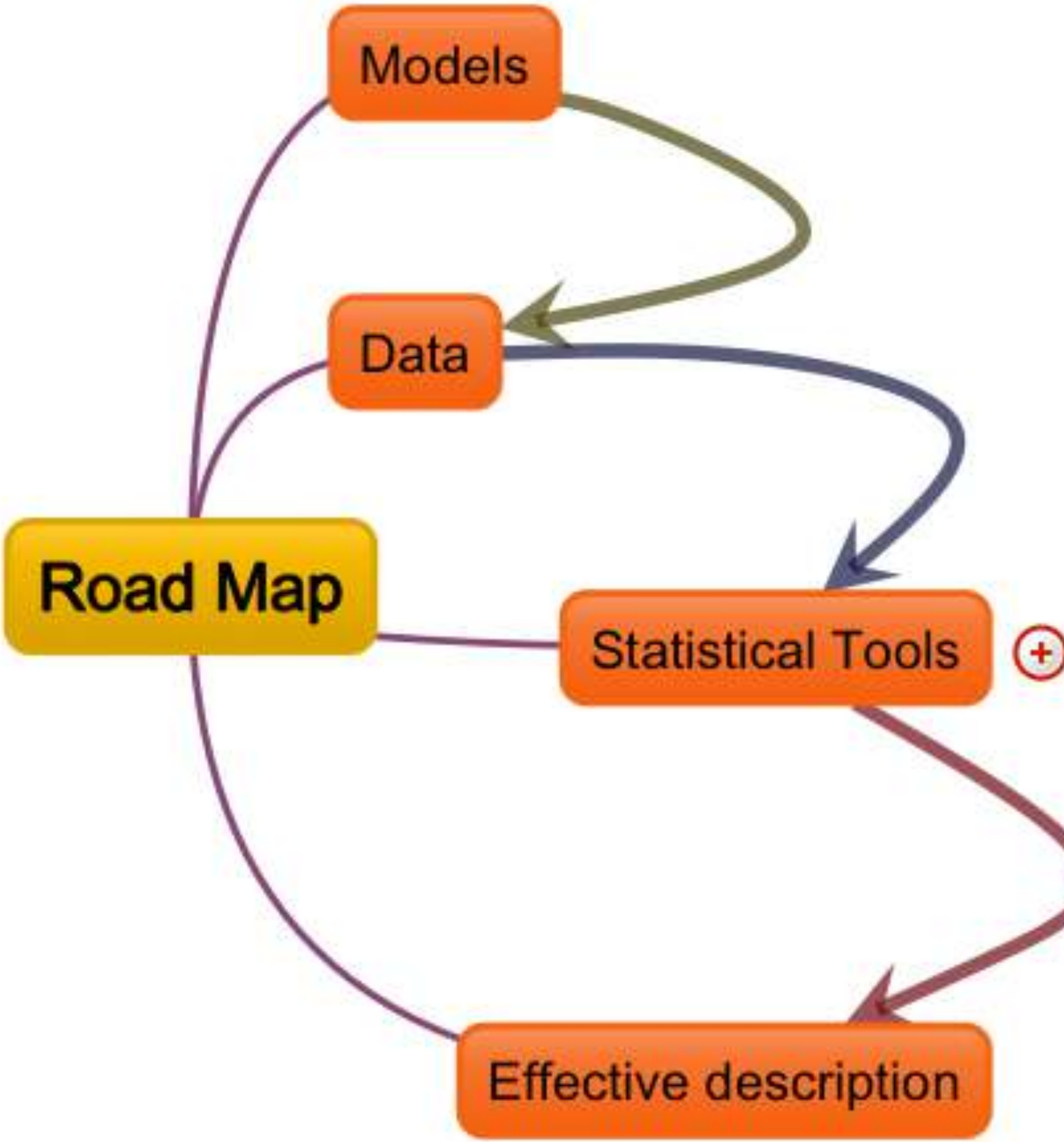
۲) همبستگی (Correlation)
جمع بندی

نقشه راه من


Models
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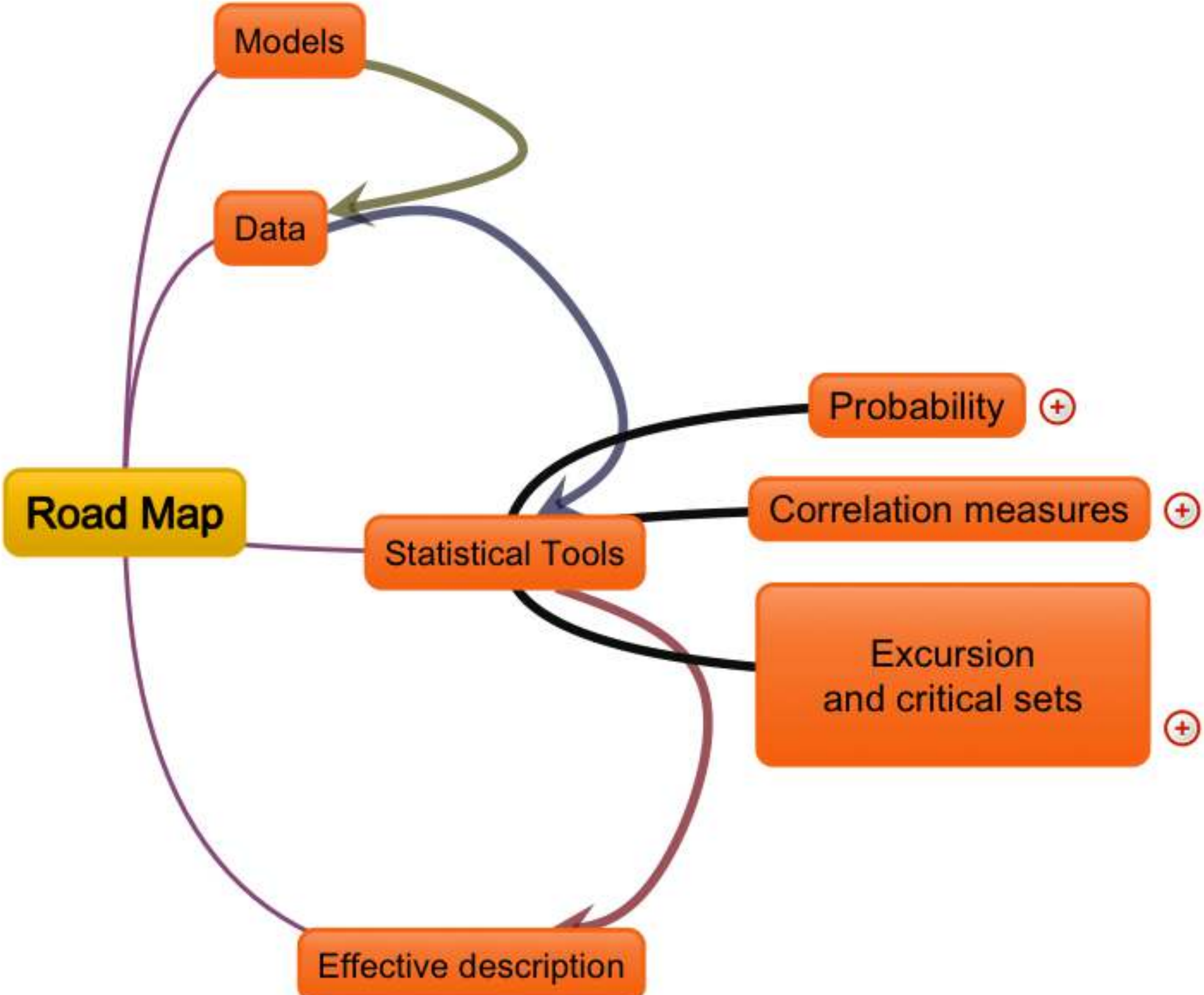
Road Map
Statistical Tools
Effective description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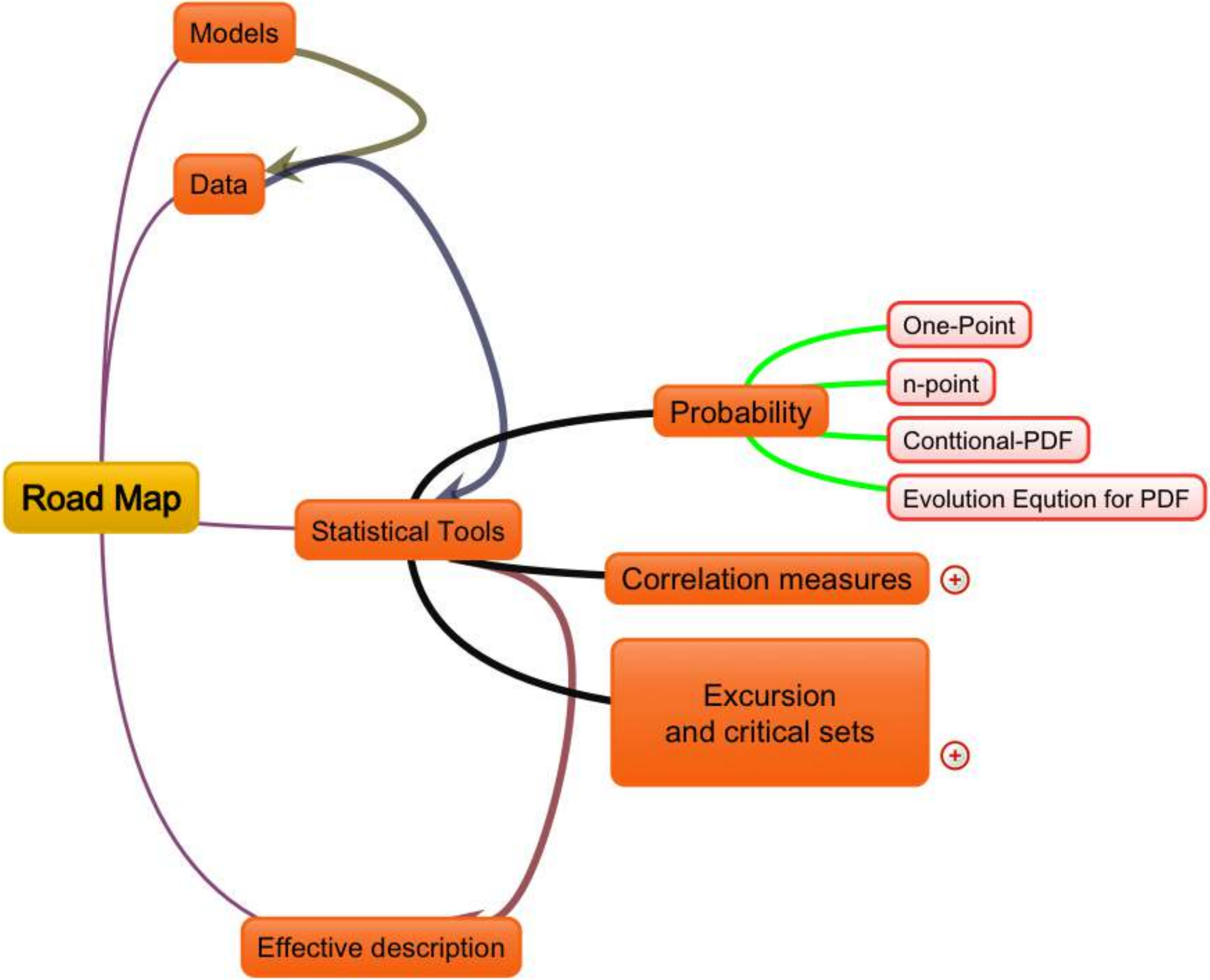
and critical sets
Effective description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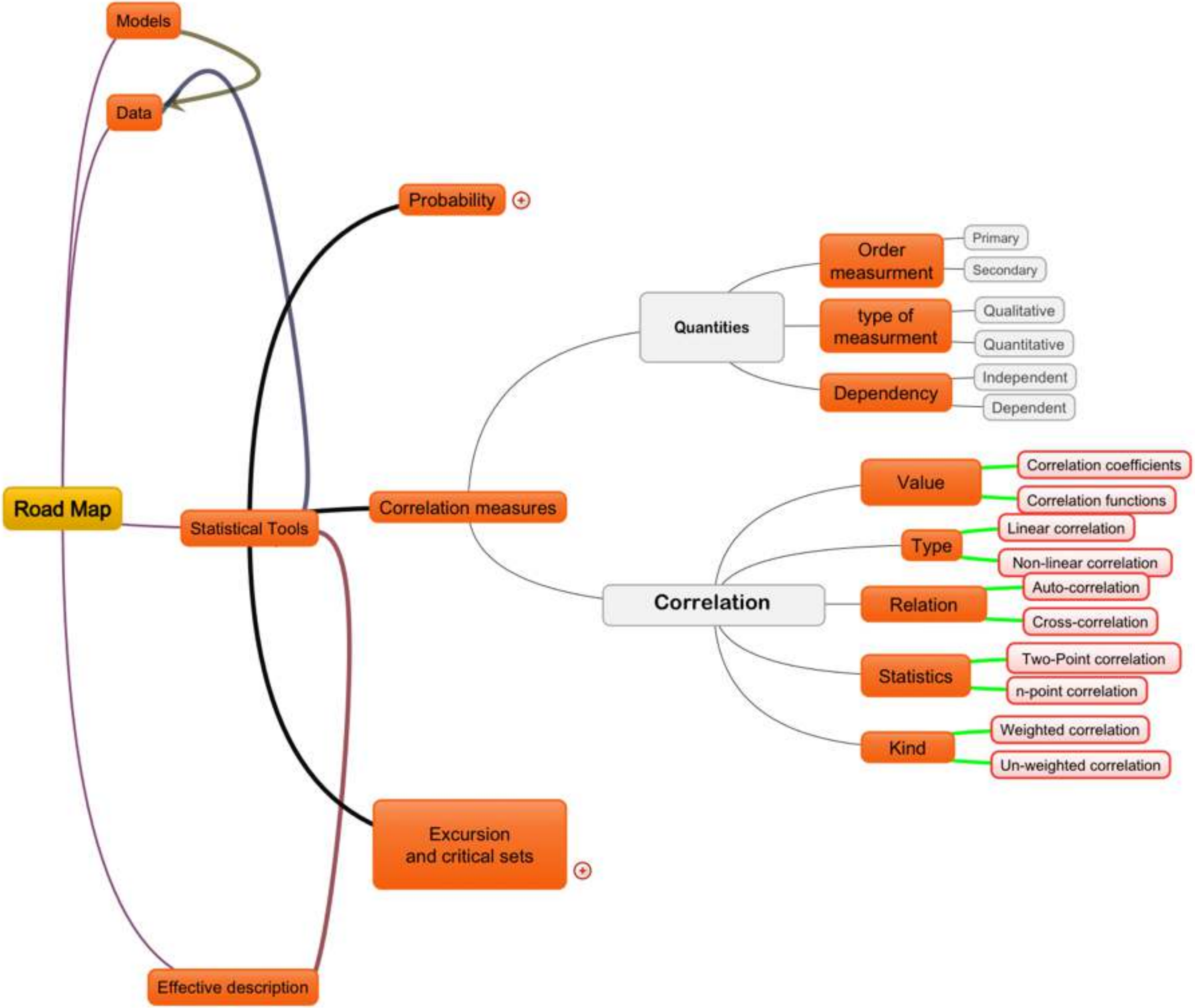
Models
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Quantities
Order measurment
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Secondary
type of measurment
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Quantitative
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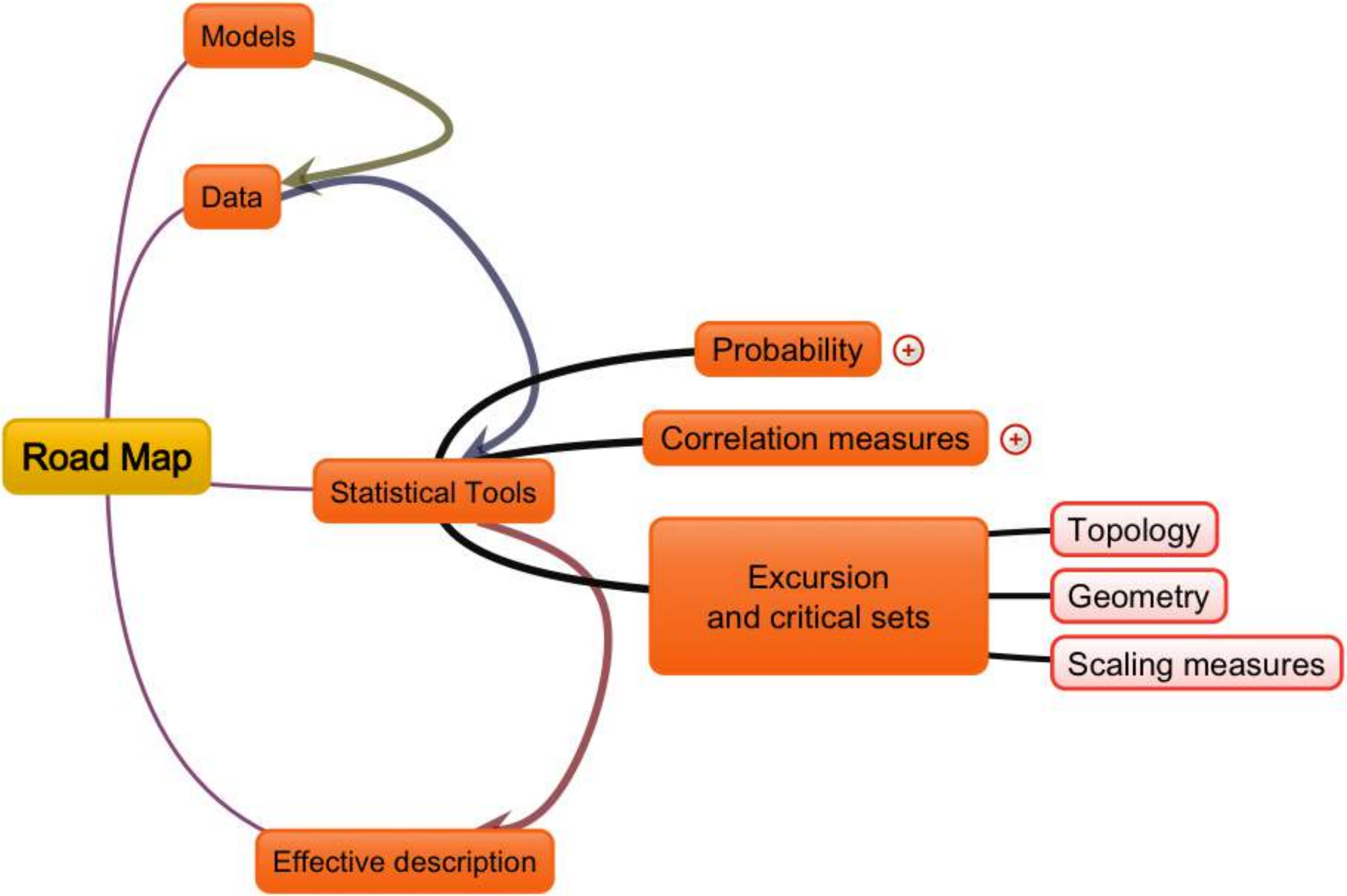
Models
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Probability
Road Map
Correlation measures
Statistical Tools
Topology
Excursion
and critical sets
Geometry
Scaling measures
Effective description

Random Processes

- Purely Random Process

$$p(x_n, t_n \mid x_{n-1}, t_{n-1}; \ldots; x_1, t_1) = p(x_n, t_n)$$

- Dependent Process

$$p(x_n, t_n; x_{n-1}, t_{n-1}; \ldots; x_1, t_1) =$$

$$p(x_n, t_n \mid x_{n-1}, t_{n-1}; \ldots; x_1, t_1) \times p(x_{n-1}, t_{n-1} \mid x_{n-2}, t_{n-2}; \ldots; x_1, t_1)$$

$$\ldots \times p(x_2, t_2 \mid x_1, t_1) \times p(x_1, t_1)$$

$$t_1 < t_2 < \ldots < t_n$$

- Markov Process

$$p(x_n, t_n; x_{n-1}, t_{n-1}; \ldots; x_1, t_1) = p(x_n, t_n \mid x_{n-1}, t_{n-1})$$

$$\times p(x_{n-1}, t_{n-1}; x_{n-2}, t_{n-2}; \ldots; x_1, t_1)$$

Probability distribution

$$\zeta(t) :: [\ \zeta_1(t_1), \zeta_2(t_2), \quad \dots \quad , \zeta_N(t_N)\ ]$$

$$\zeta_1(t_1) = x_1\ , \zeta_1(t_1) = x_1\ , \dots , \zeta_N(t_N) = x_N$$

- Probability Density Function (PDF)
- $$P(x \le \zeta < x + dx) = p(x)dx$$
- Joint Probability Density

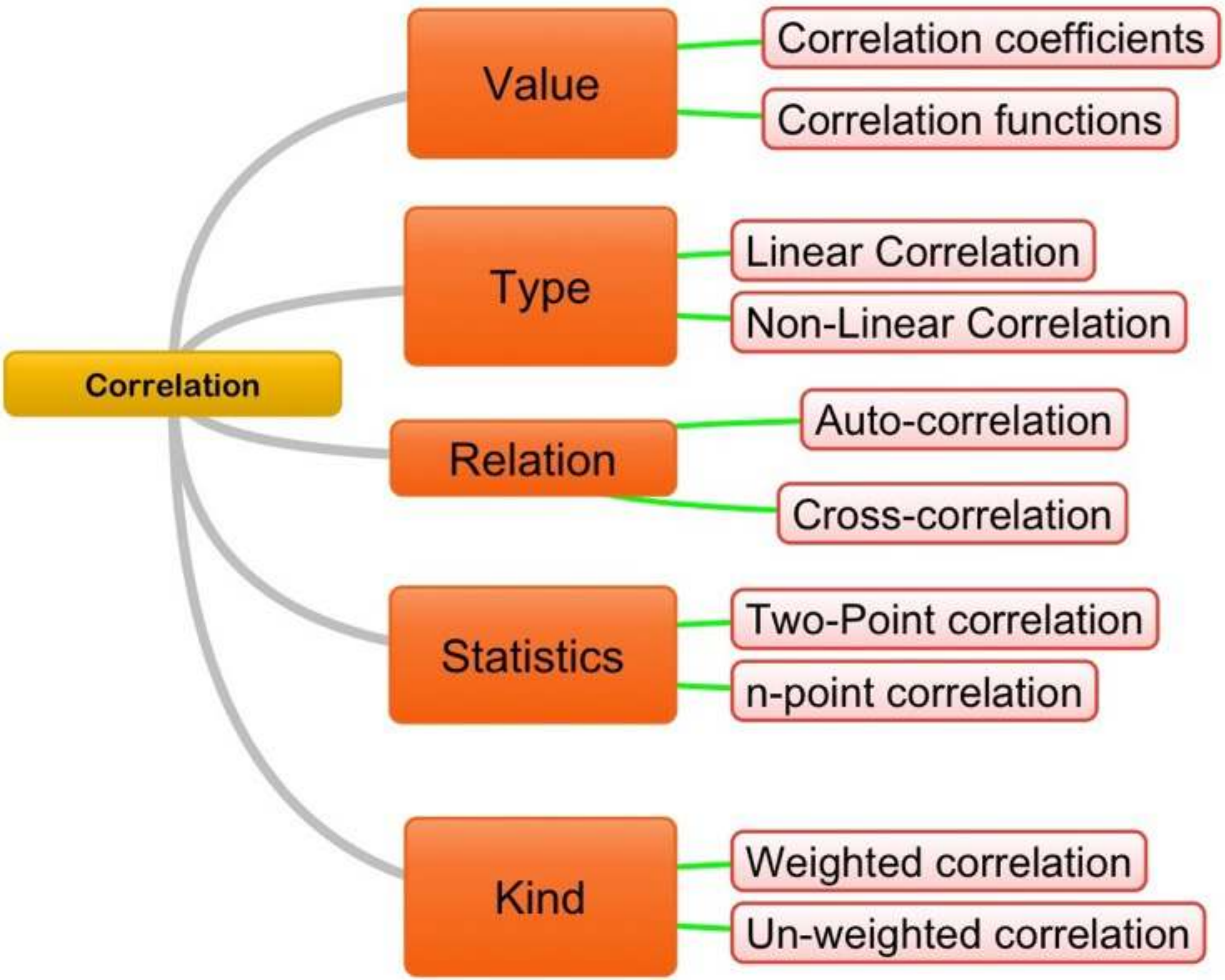
$$p(x_i, t_i\ ; x_j, t_j)$$

- Conditional Probability Density

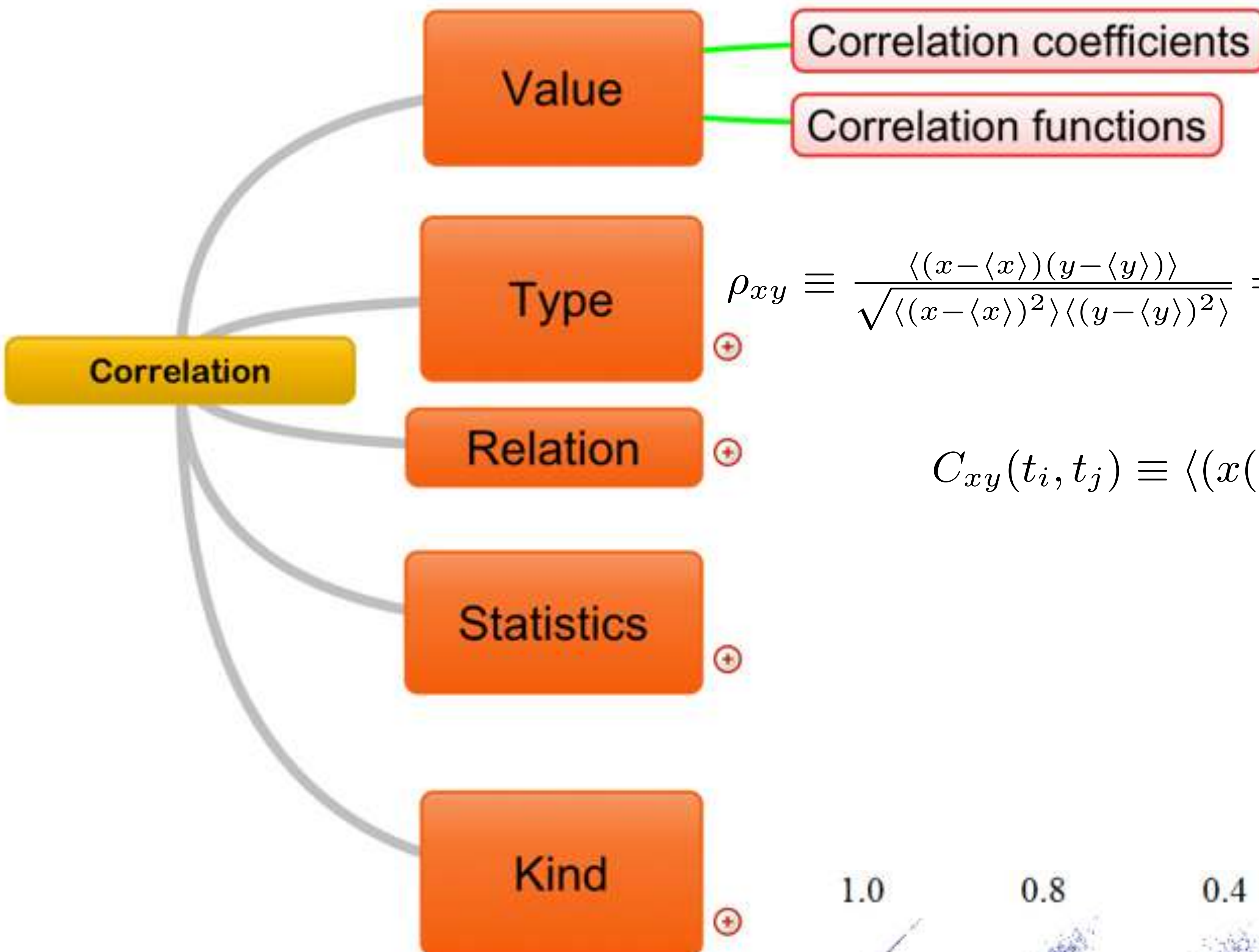
$$p(x_i, t_i \mid x_j, t_j) = \frac{p(x_i, t_i\ ; x_j, t_j)}{p(x_j, t_j)}$$

همبستگی آماری
Statistical Correlation

Classification of correlations

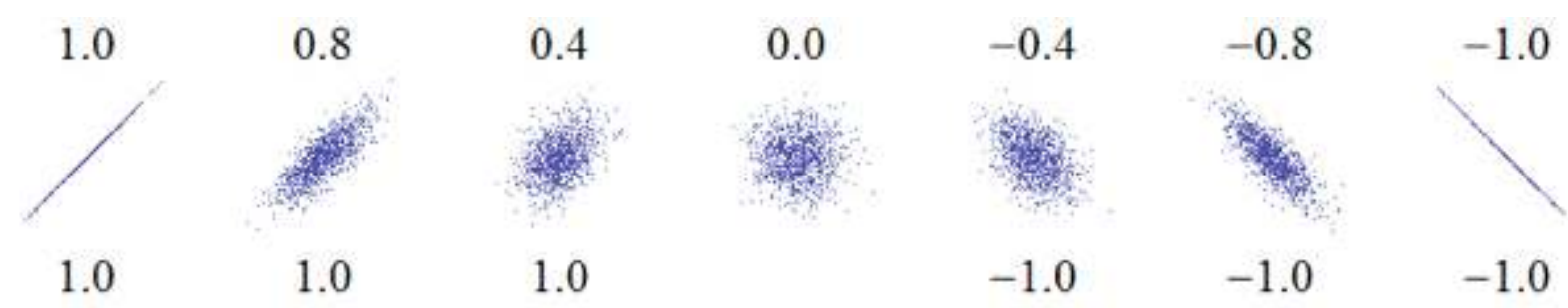


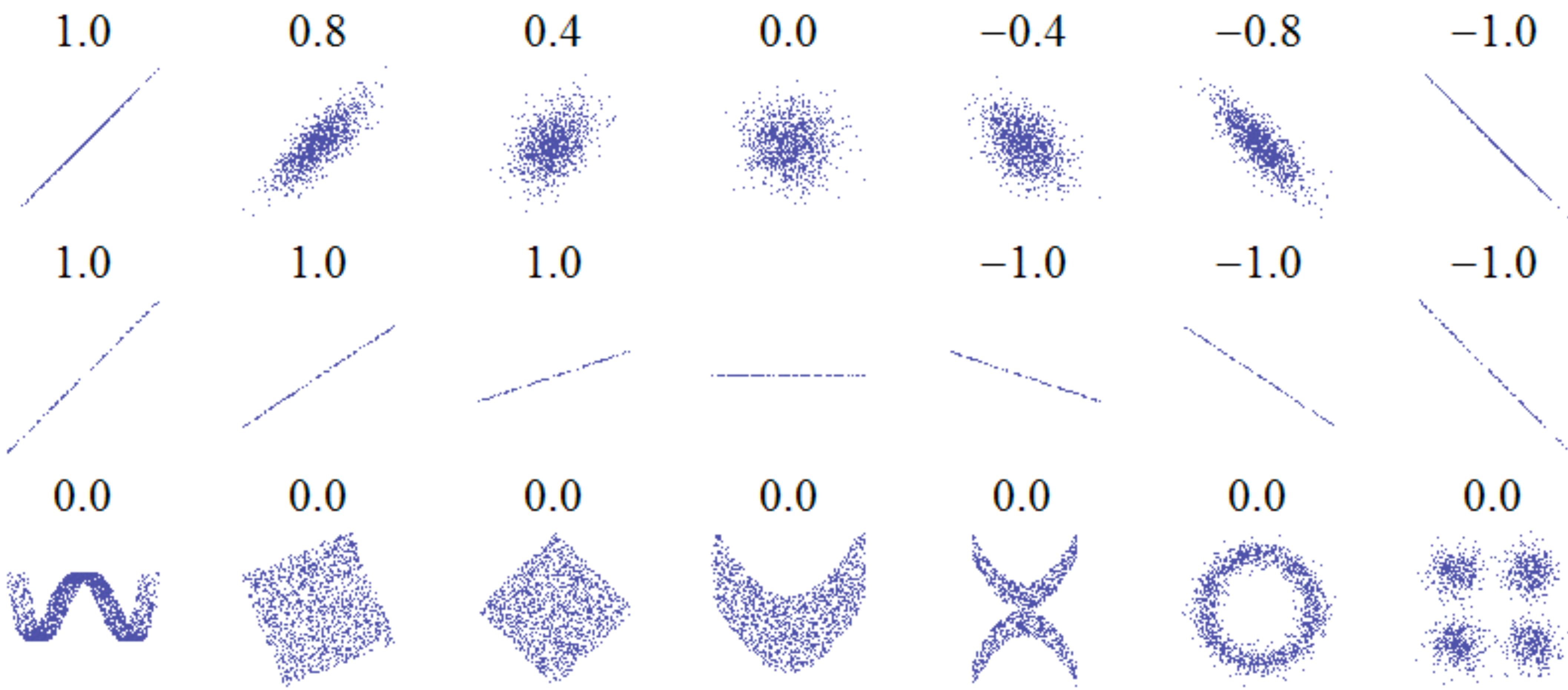
Classification of correlations



$$\rho_{xy} \equiv \frac{\langle (x-\langle x\rangle)(y-\langle y\rangle)\rangle}{\sqrt{\langle (x-\langle x\rangle)^2\rangle \langle (y-\langle y\rangle)^2\rangle}} = \begin{cases} 1 & \text{for} \quad x = y \\ 0 & \text{un}-\text{correlated data} \\ -1 & \text{anti}-\text{correlated data} \end{cases}$$

$$C_{xy}(t_i, t_j) \equiv \langle (x(t_i) - \langle x(t_i)\rangle)\,(y(t_j) - \langle y(t_j)\rangle)\rangle$$




1.0
0.8
0.4
0.0
−0.4
−0.8
−1.0
1.0
1.0
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−1.0
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0.0
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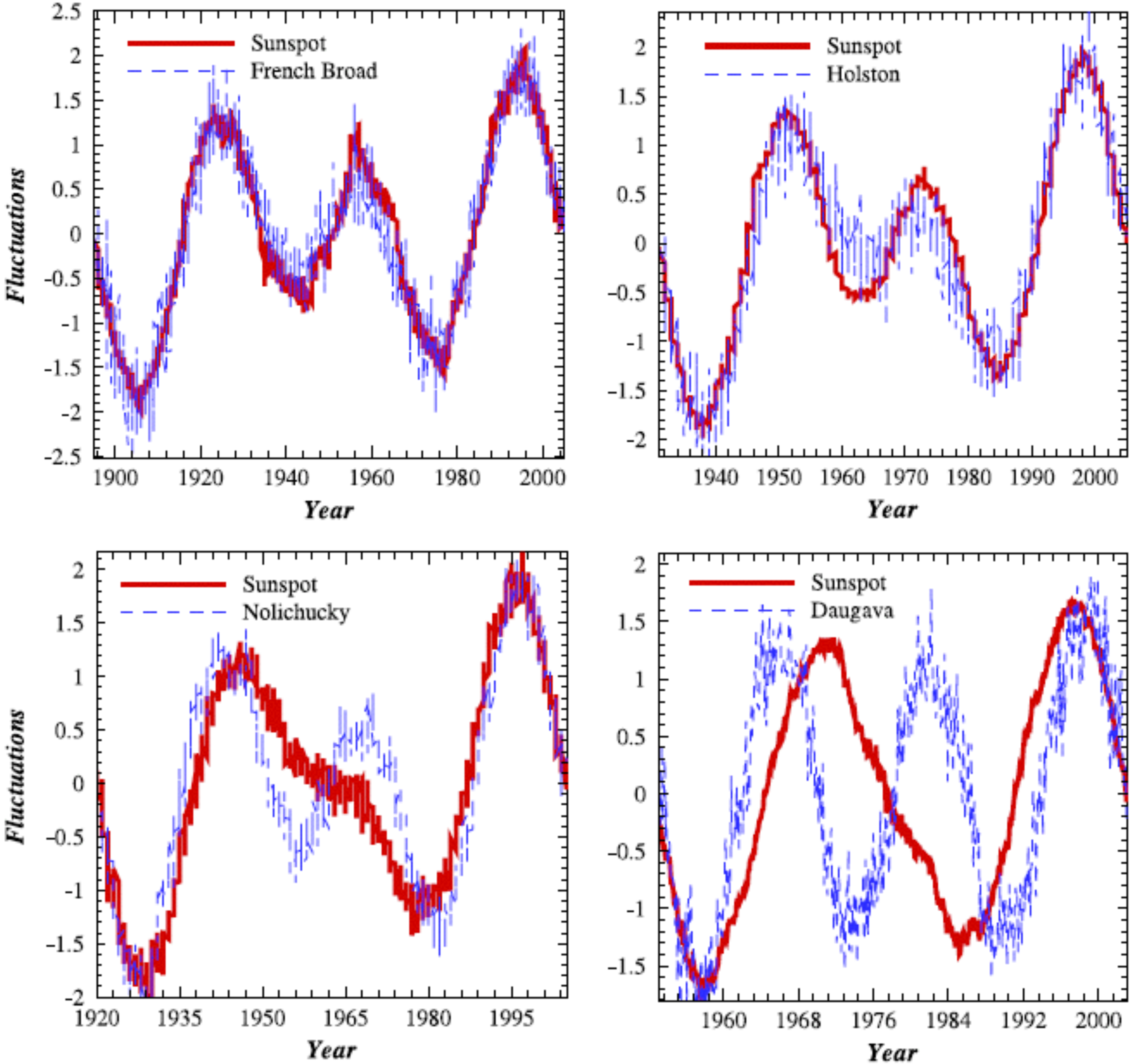
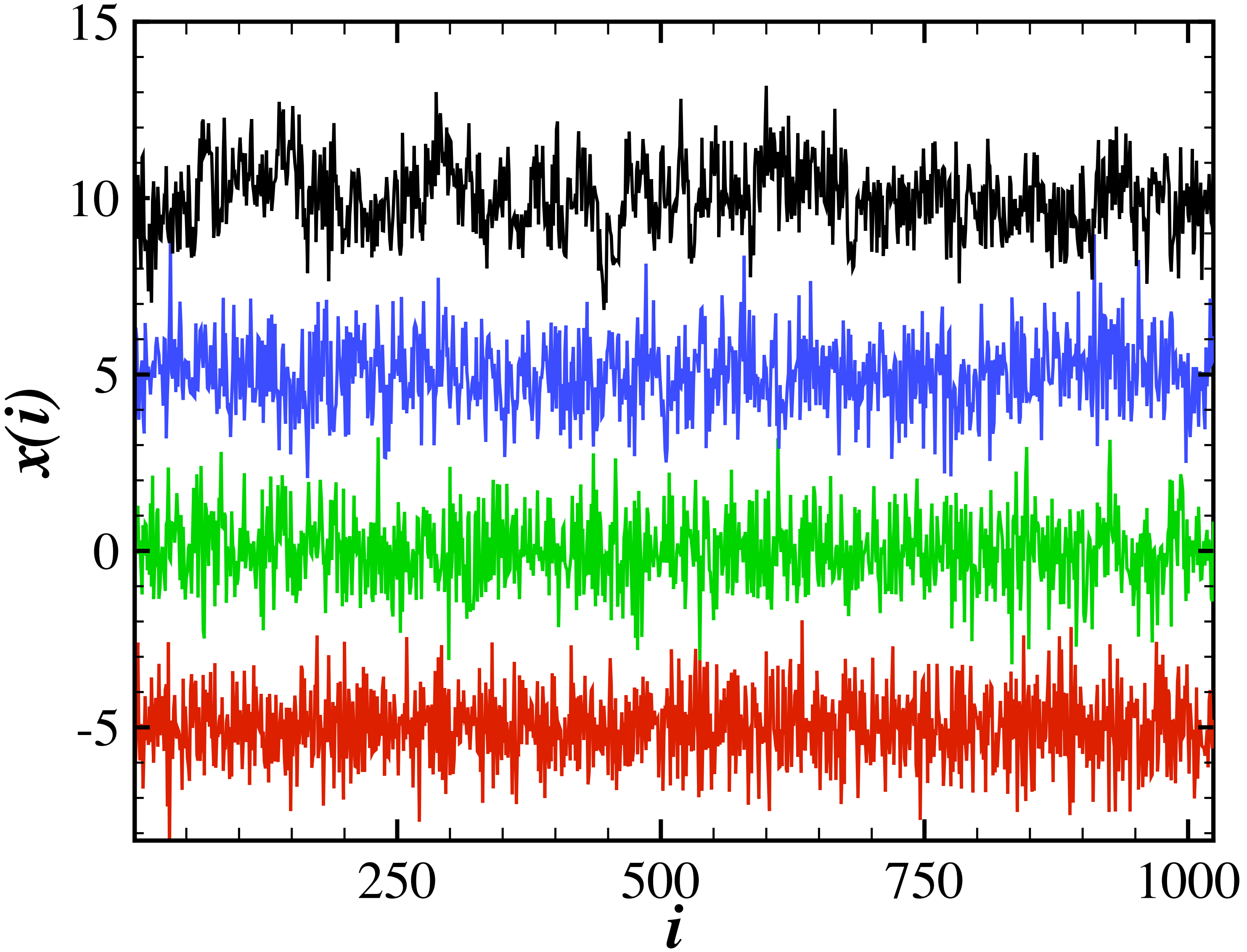
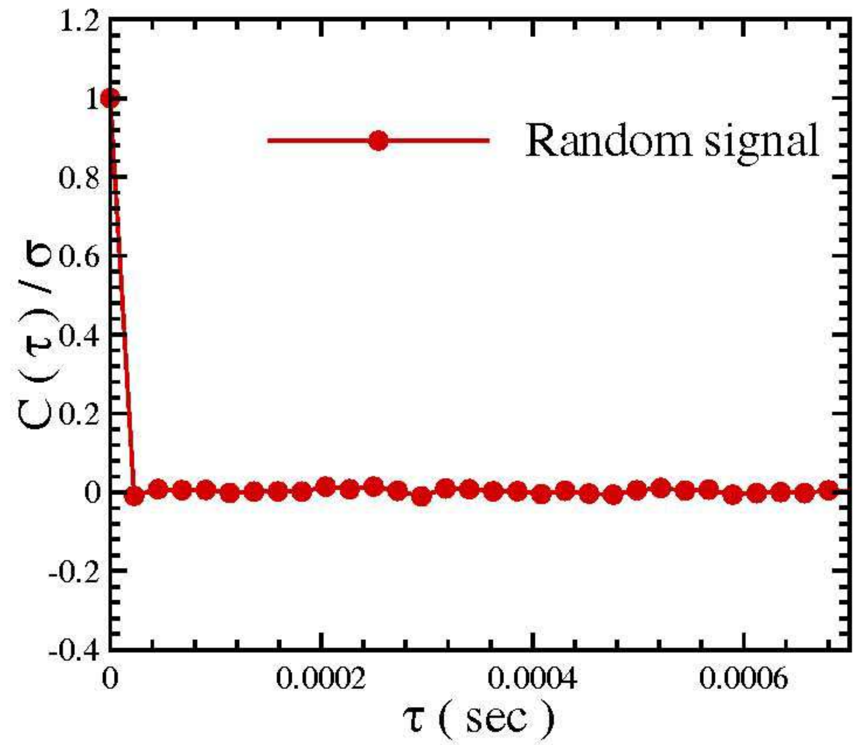
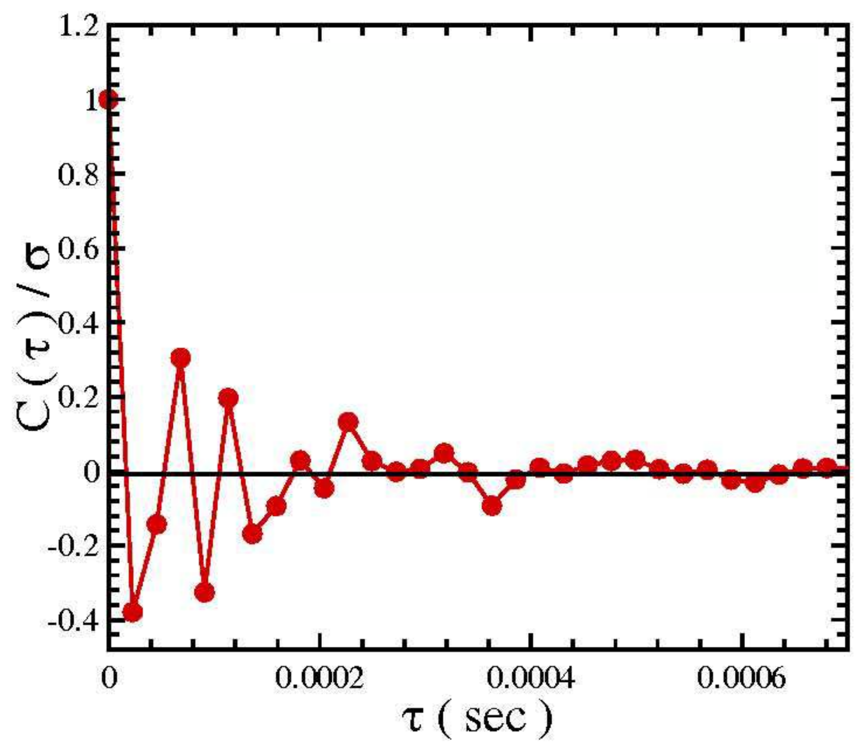
0.0
0.0


Sunspot
French Broad
Fluctuations
Year
Sunspot
Holston
Year
Sunspot
Nolichucky
Fluctuations
Year
Sunspot
Daugava
Year

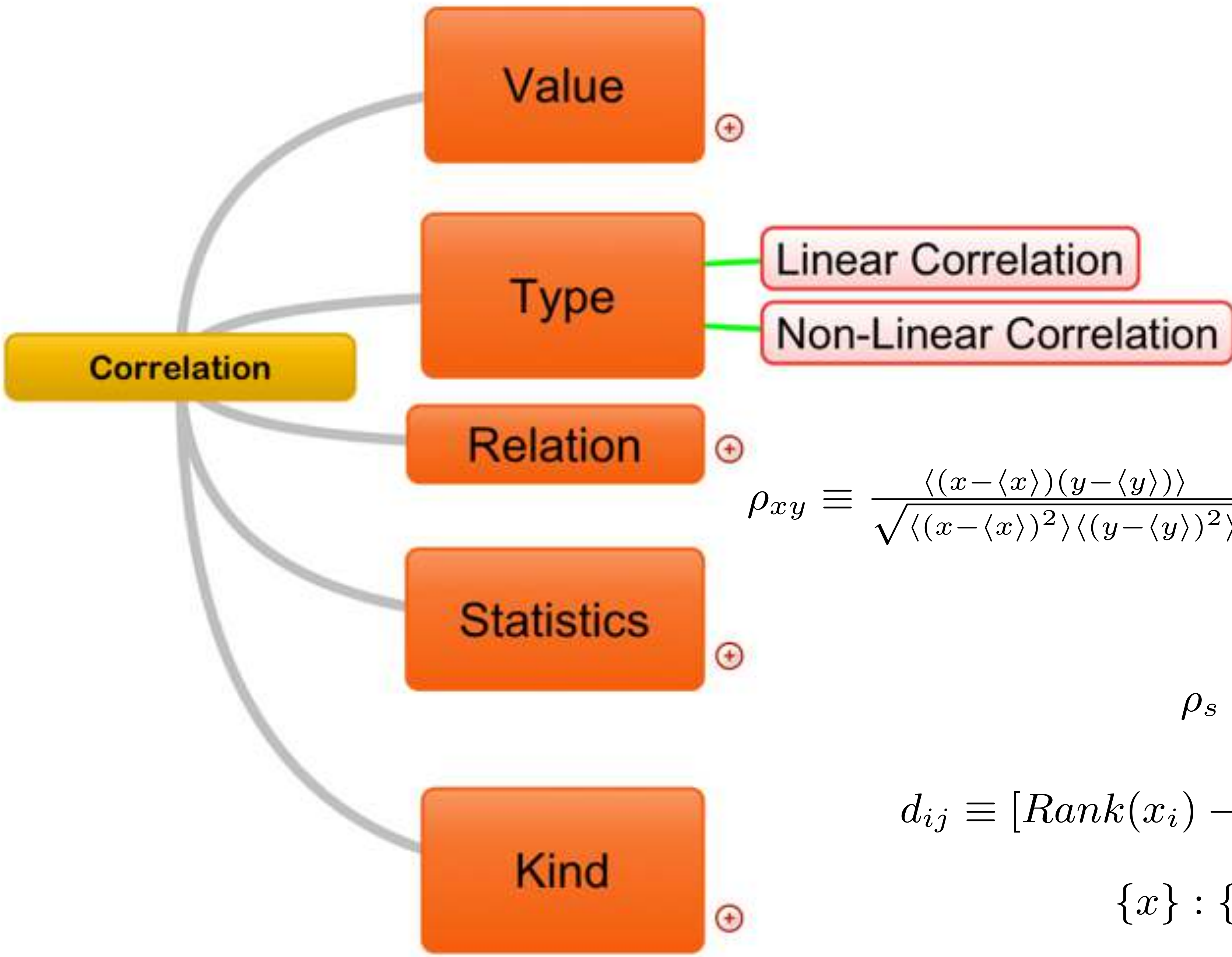
Anti-correlated, Uncorrelated and Correlated Gaussian noise







Classification of correlations



Pearson's coefficient
Spearman's coefficient

$$\rho_{xy} \equiv \frac{\langle (x-\langle x\rangle)(y-\langle y\rangle)\rangle}{\sqrt{\langle (x-\langle x\rangle)^2\rangle \langle (y-\langle y\rangle)^2\rangle}} = \begin{cases} 1 & \text{for} \quad x = y \\ 0 & \text{un} - \text{correlated data} \\ -1 & \text{anti} - \text{correlated data} \end{cases}$$

$$\rho_s \equiv 1 - 6\frac{\sum_{i,j} d_{ij}^2}{N(N^2-1)}$$

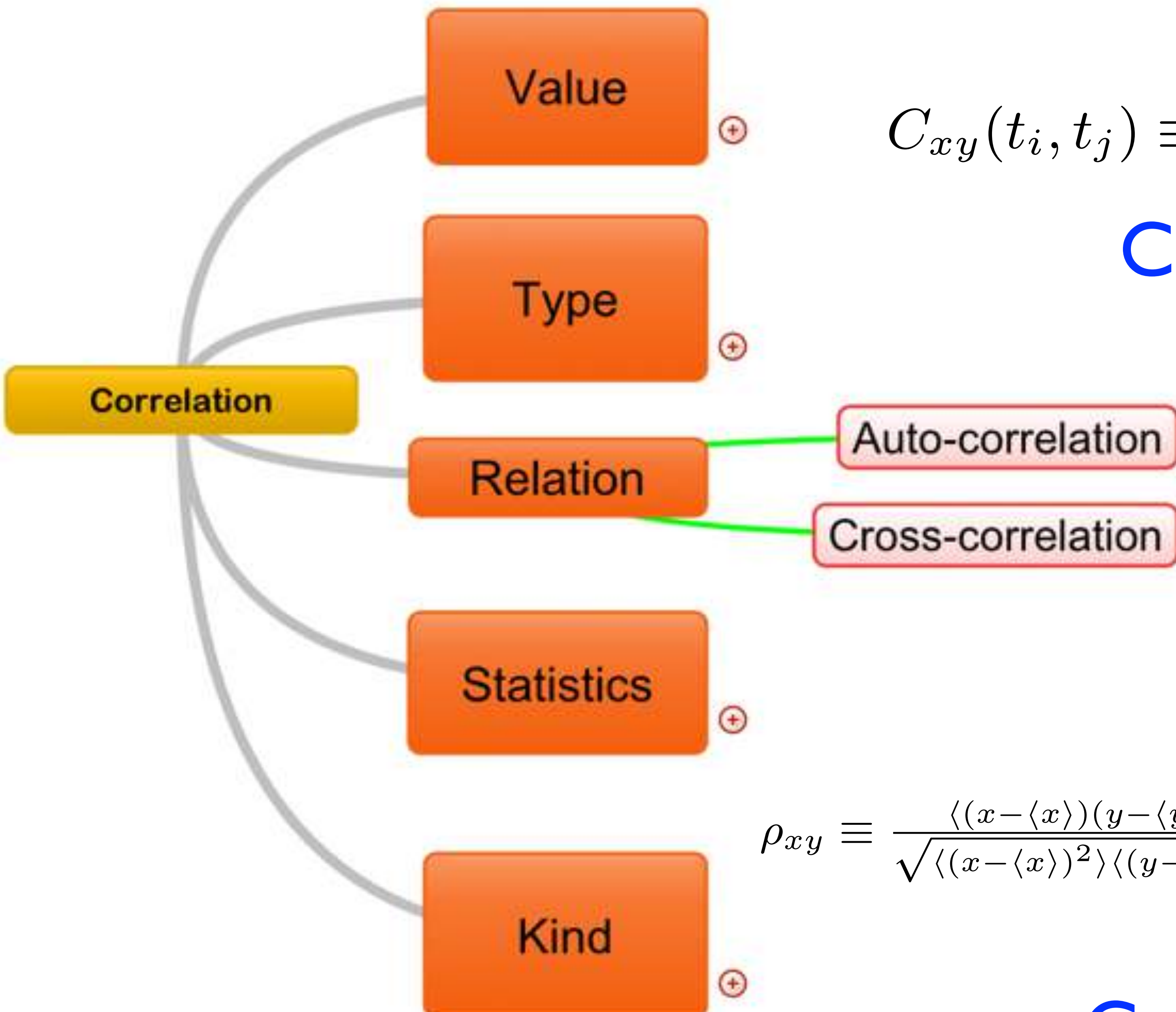
$$d_{ij} \equiv [Rank(x_i) - Rank(x_j)][Rank(y_i) - Rank(y_j)]$$

$$\{x\} : \{20, 100, 30, 50, 160, 10\}$$

$$Rank(x) : \{5, 2, 4, 3, 1, 6\}$$

Auto-correlation function

$$C_{xx}(t_i, t_j) \equiv \langle (x(t_i) - \langle x(t_i) \rangle)\,(x(t_j) - \langle x(t_j) \rangle) \rangle$$



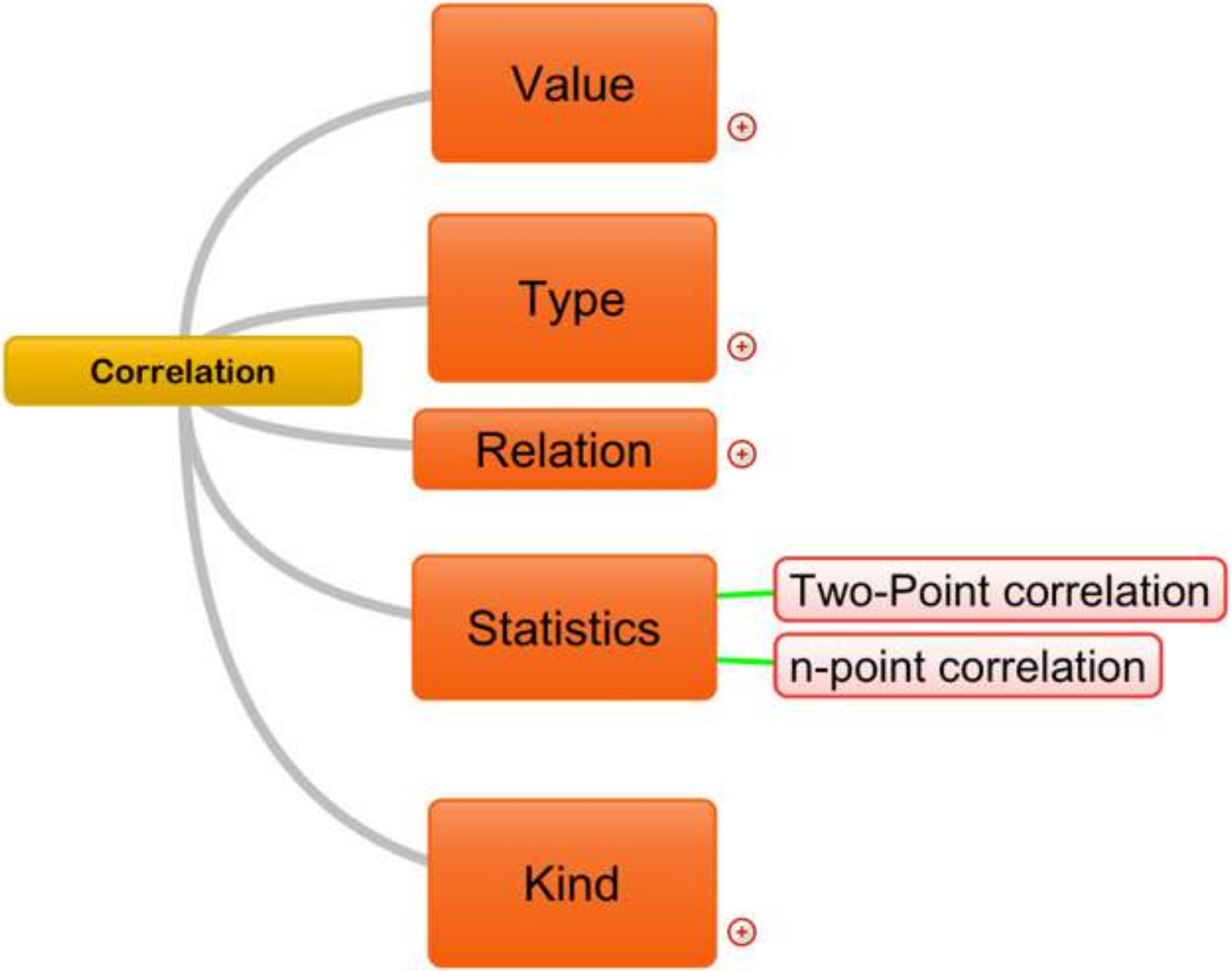
$$C_{xy}(t_i, t_j) \equiv \langle (x(t_i) - \langle x(t_i) \rangle)\,(y(t_j) - \langle y(t_j) \rangle) \rangle$$

Cross-correlation function

$$\rho_{xy} \equiv \frac{\langle (x - \langle x \rangle)(y - \langle y \rangle) \rangle}{\sqrt{\langle (x - \langle x \rangle)^2 \rangle \langle (y - \langle y \rangle)^2 \rangle}} = \begin{cases} 1 & \text{for} \quad x = y \\ 0 & \text{un} - \text{correlated data} \\ -1 & \text{anti} - \text{correlated data} \end{cases}$$

Cross-correlation coefficient

Classification of correlations



$$C_{xx}(t_i, t_j) \equiv \langle (x(t_i) - \langle x(t_i) \rangle) (x(t_j) - \langle x(t_j) \rangle) \rangle$$

$$C_{xxx}(t_i, t_j, t_k) \equiv \langle (x(t_i) - \langle x(t_i) \rangle) (x(t_j) - \langle x(t_j) \rangle) (x(t_k) - \langle x(t_k) \rangle) \rangle$$

تابع همبستگی وزن دار

Weighted Correlation function

$$C_{xy}^{\times}(t_1,t_2) = \langle x(t_1)y(t_2)\rangle_{ensemble}$$

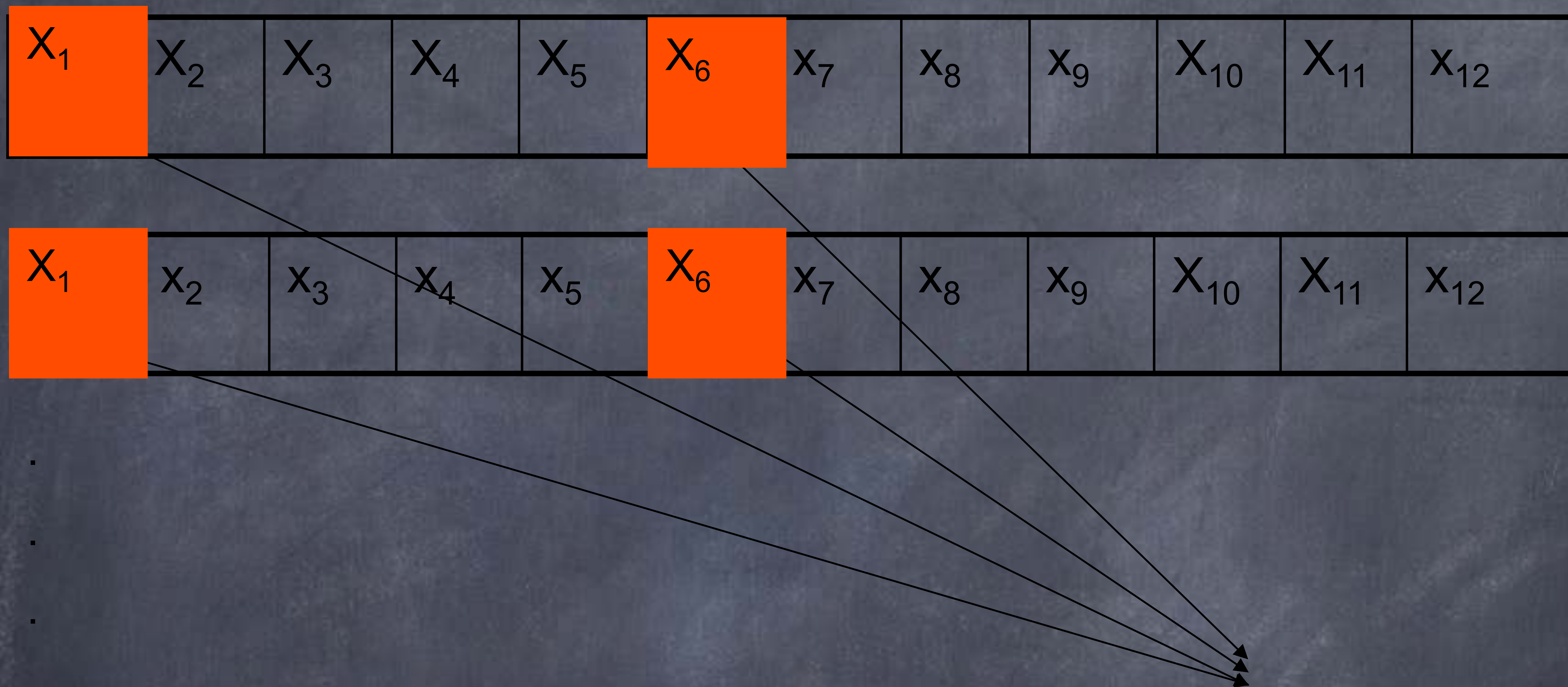
$$C_x(t_1,t_2) = \langle x(t_1)x(t_2)\rangle_{ensemble}$$

For stationary processes

$$C_x(\tau) = C_x(t_1,t_2)$$

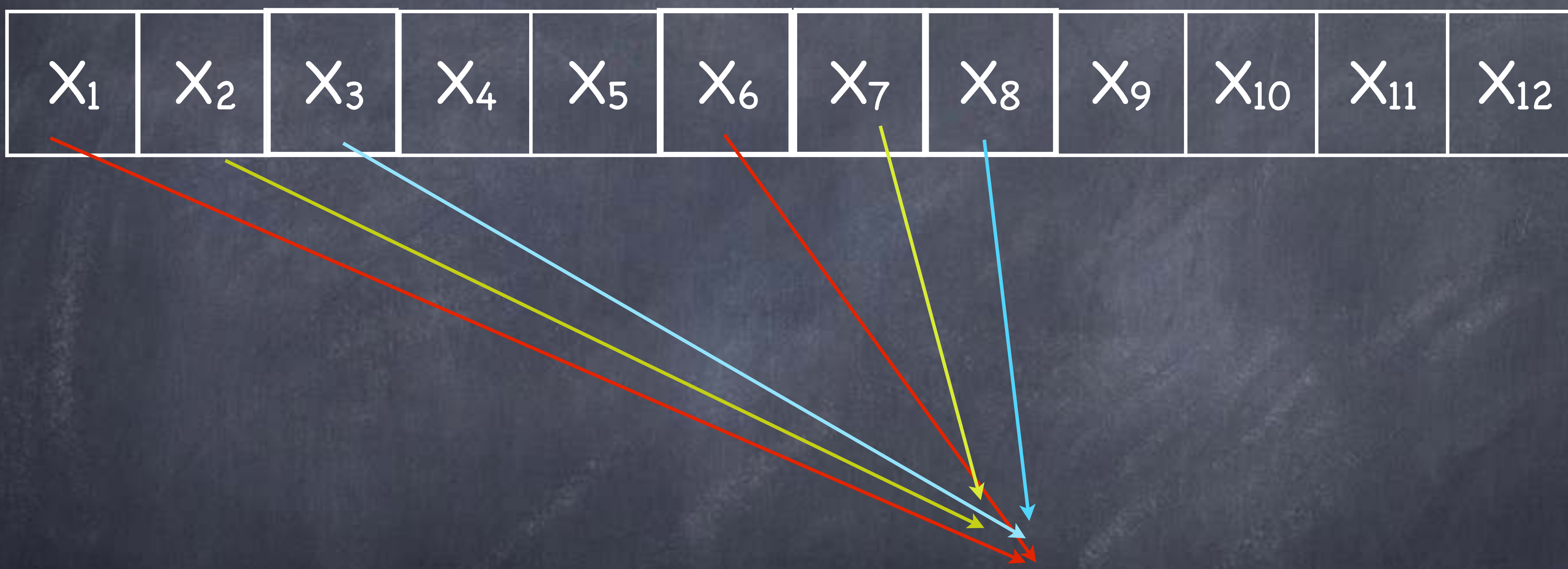
$$C_x(\tau) = \langle x(t+\tau)x(t)\rangle$$

Auto-correlation (Ensemble averaging)



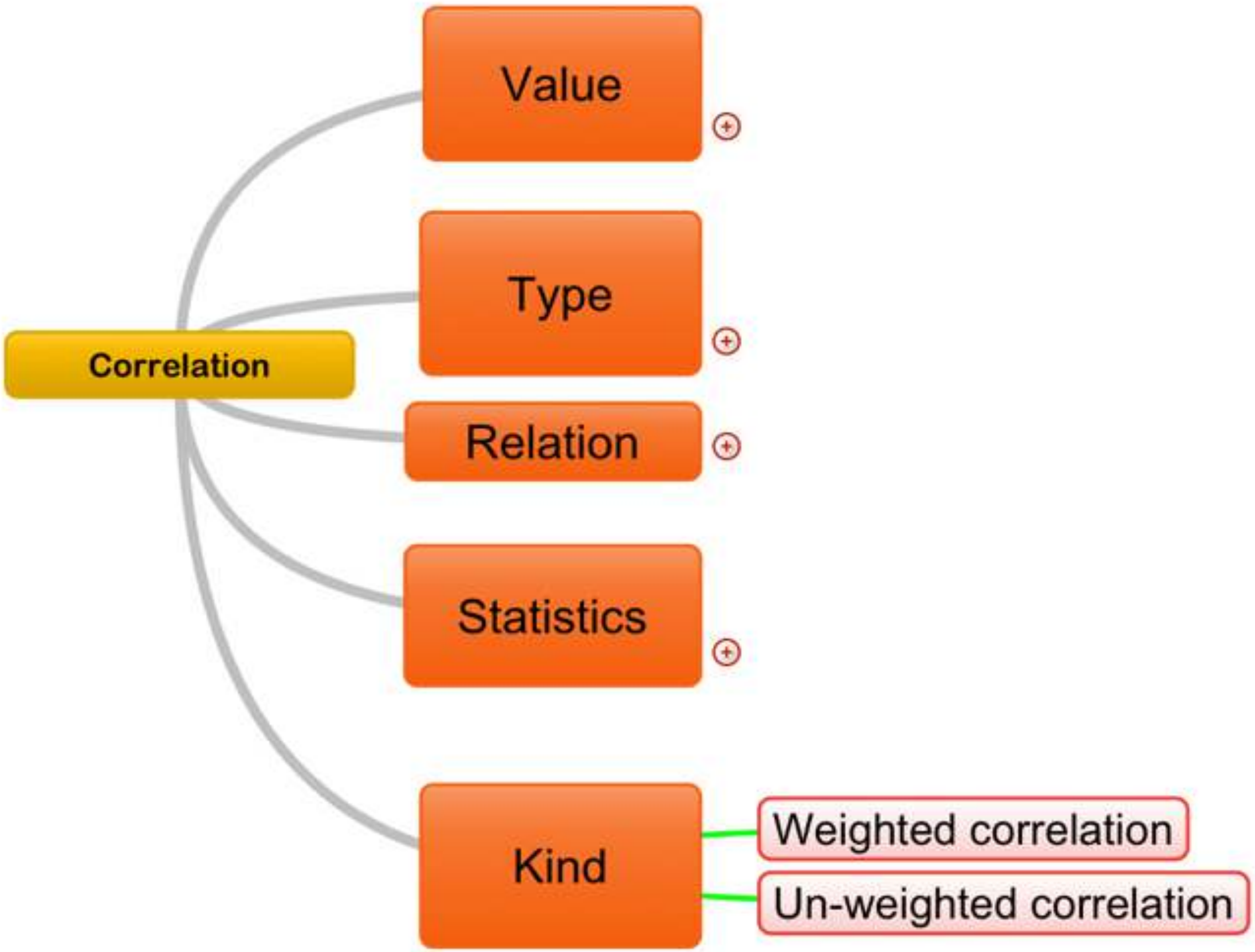
$$C(1,6) = \left\langle (X(1) - \bar{X})(X(6) - \bar{X}) \right\rangle = \frac{1}{N} \sum_{k=1}^{N} X_k(1) X_k(6)$$

Auto-correlation (Time averaging)



$$C(1,6) = \left\langle (X(1) - \bar{X})(X(6) - \bar{X}) \right\rangle = \left\langle X(k+1)X(k) \right\rangle = \frac{1}{N-5} \sum_{k=1}^{N-5} X(k+5)X(k)$$

Classification of correlations



در تابع همبستگی وزن دار مقدار داده ها در تولید این تابع نقش مستقیم دارند. اتفاقاً به همین دلیل هم تنوعی در آن نداریم. اما می خواهم اشاره کنم به اینکه در نوع غیروزن دار دقیقاً داستان برعکس است.

Weighted $\underbrace{\text{Two-Point Correlation function}}_{\text{TPCF}}$

Un-Weighted $\underbrace{\text{Two-Point Correlation function}}_{\text{TPCF}}$

- با توجه به مفهوم بدون وزن که یک دید جدیدی و البته گسترده برای ما فراهم می‌گردد که بتوانیم توابع همبستگی متنوعی را تعریف کنیم که در نتیجه می‌توان خواص آماری متنوعی را برای یک فرآیند تعریف و تعیین کنیم.

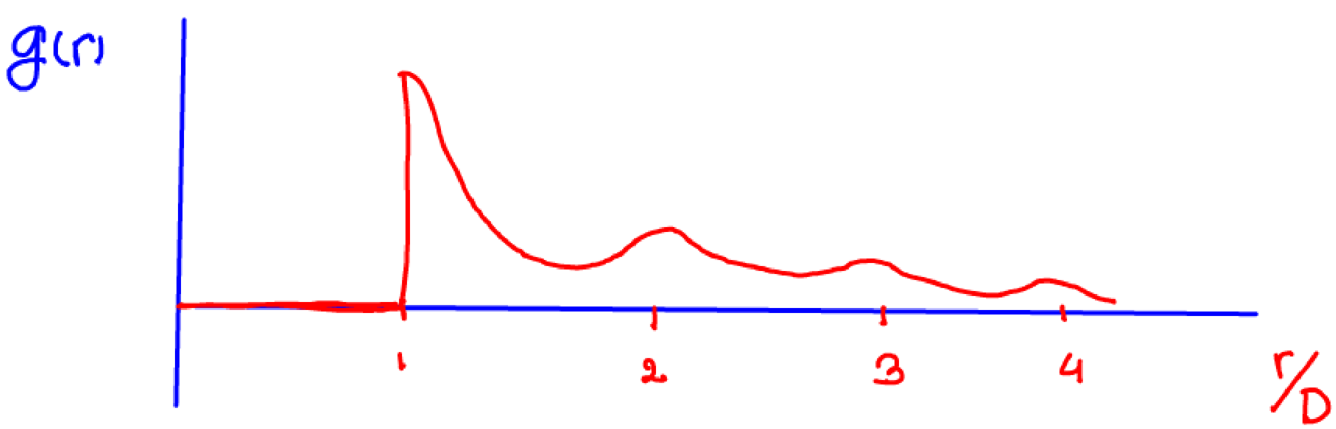
- عبارت کلیدی برای تعریف تابع همبستگی بدون وزن $\boxed{\text{احتمال اضافی Excess Probability}}$

g(R) and its relation to thermodynamics

$$H = \sum_{i=1}^{3N} \frac{p_i^2}{2m} + \frac{1}{2} \sum_{ij}{}' u_{ij}$$

$$u(r) = u(r_{ij}) = \begin{cases} 0 & \text{for} \quad r > D \\ \infty & \text{for} \quad r < D \end{cases}$$

Steric Repulsion



احتمال یافتن ذره با فاصله جدایی r

$$U(N,V,T) = \langle H \rangle = \frac{dNkT}{2} + \frac{nN}{2} \int u(r) g(r) d\mathbf{r}. \qquad P = nkT\left(1 - \frac{n}{2dkT} \int \frac{du}{dr} r g(r) d\mathbf{r}\right)$$

Statistical Mechanics, R. K. Pathria, Paul D. Beale, 2011.

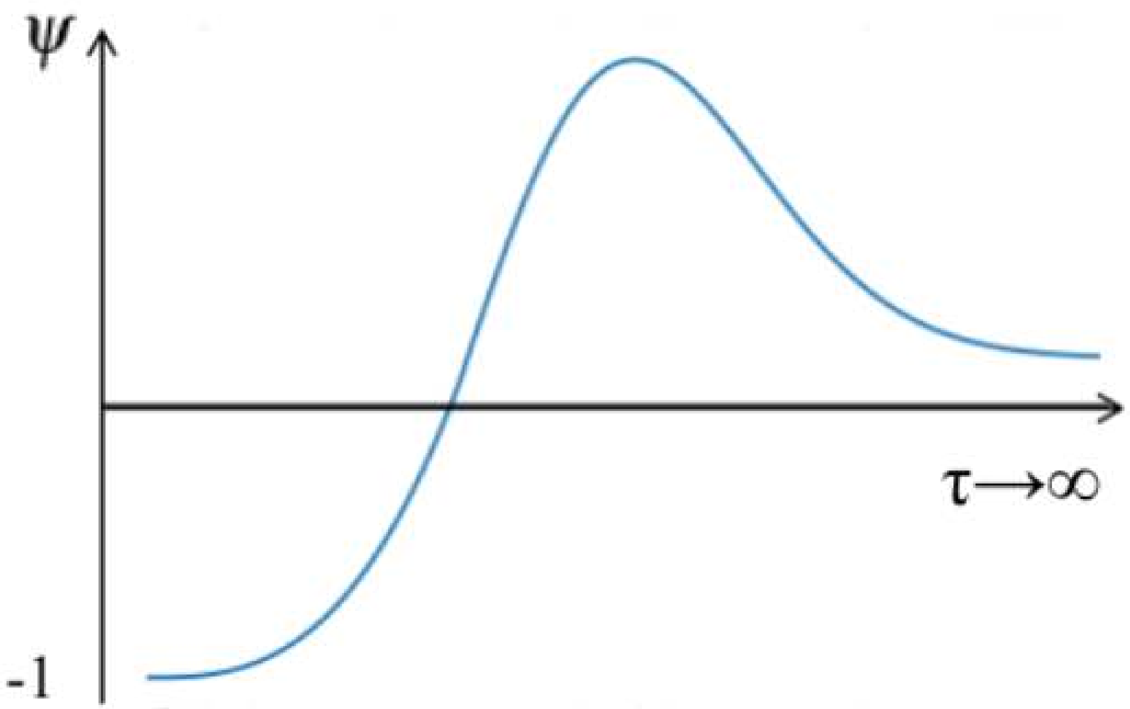
$$C_{xx}(t_i, t_j) \equiv \langle (x(t_i) - \langle x(t_i) \rangle) (x(t_j) - \langle x(t_j) \rangle) \rangle$$

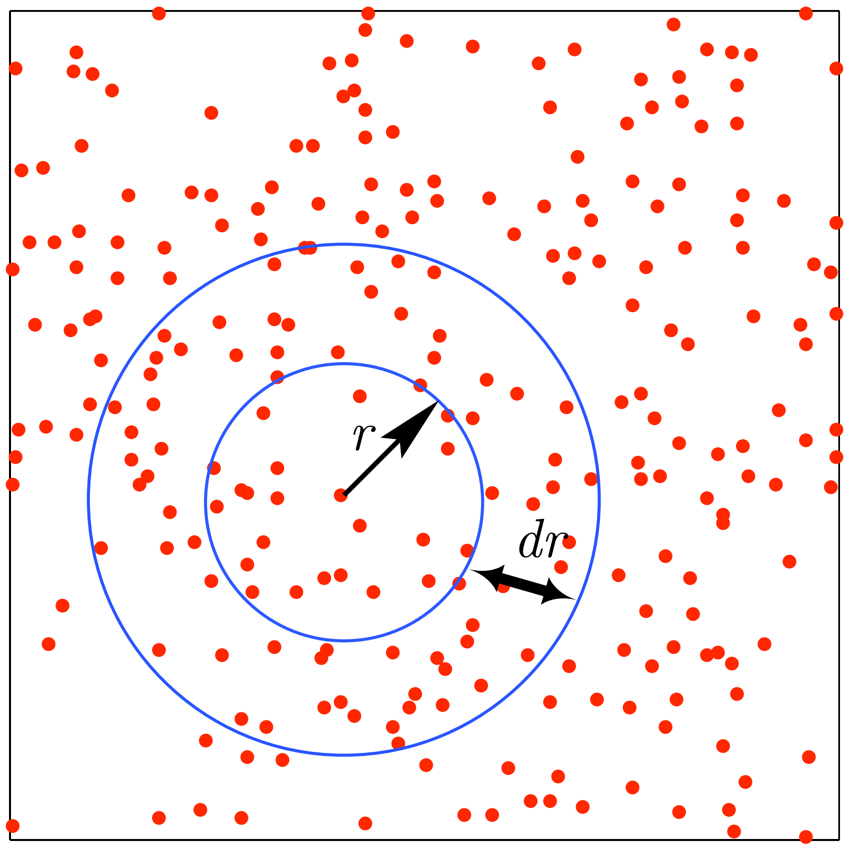
$$C_{xx}(\tau) \equiv \langle (x(t_i) - \langle x(t_i) \rangle) (x(t_i + \tau) - \langle x(t_i + \tau) \rangle) \rangle$$

$$p_{joint}(x_i, x_j; \tau) \equiv p(x_i, t_i) p(x_j, t_i + \tau) \left[1 + \Psi(x_i, x_j; \tau)\right]$$

$$\langle \mathcal{N}(R) \rangle_{pair} = N_{pair}(1 + \Psi_{pair}(R))$$

$$\Psi_{pair}(R) = \frac{\langle \mathcal{N}(R) \rangle_{pair}}{N_{pair}} - 1$$



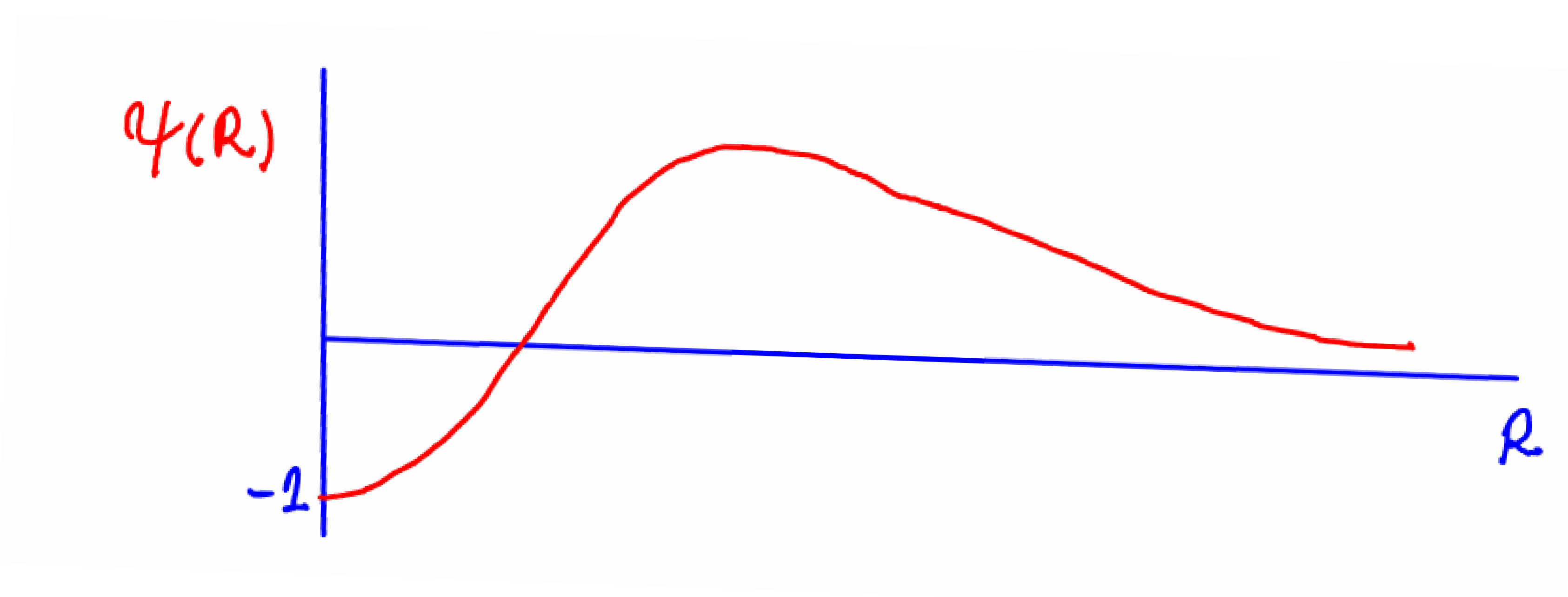


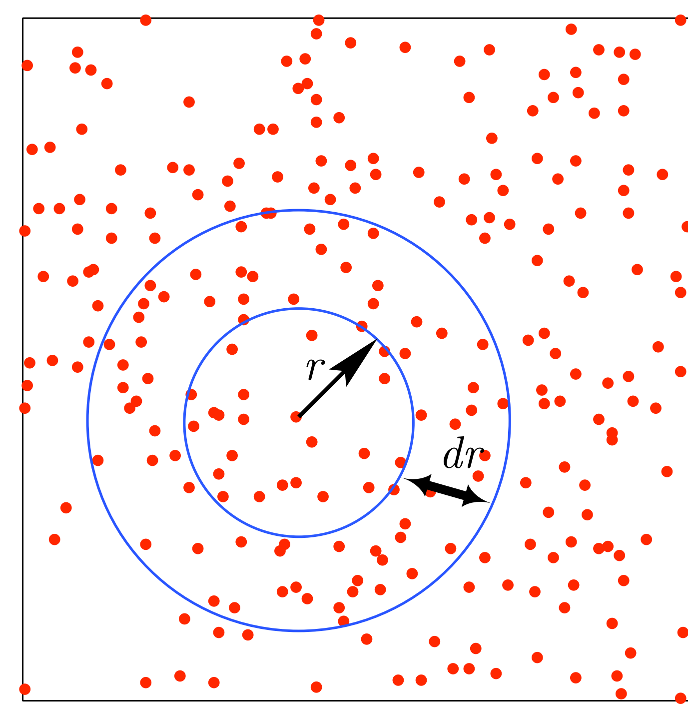
تابع همبستگی غیروزندار
Un-weighted TPCF

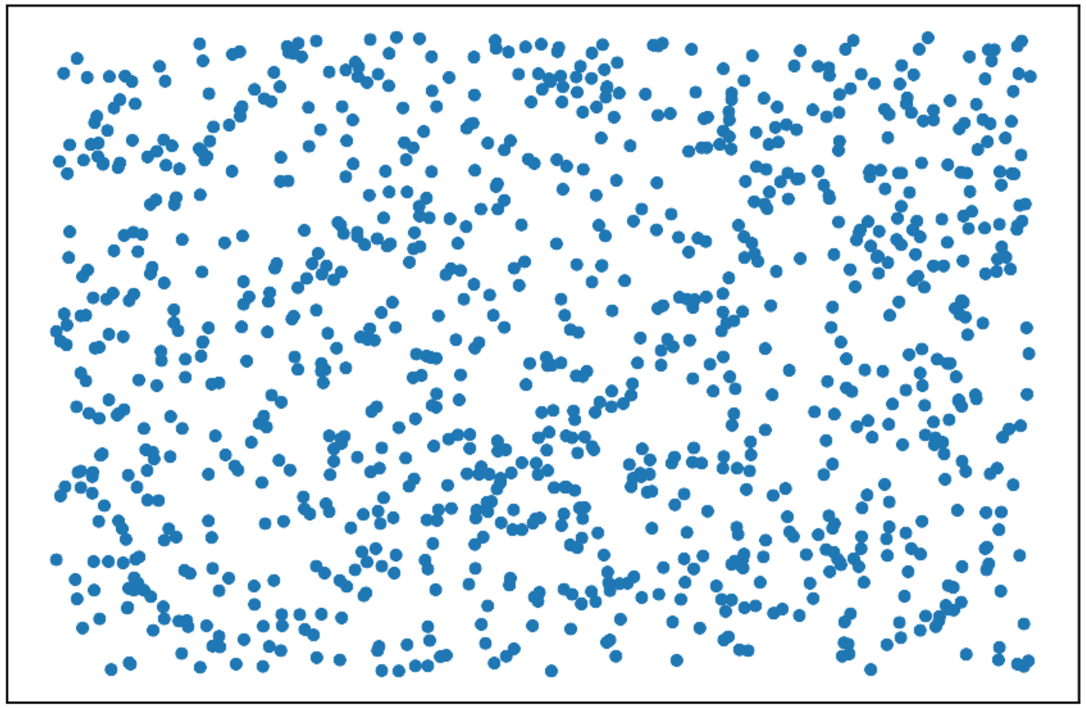
- این تابع مبتنی بر مفهوم احتمال اضافی یافتن یک ویژگی دلخواه تعریف می شود.

$$P(x(t_i); x(t_j)) = P(x(t_i))\, P(x(t_j)) \left[1 + \Psi_x(t_i, t_j)\right]$$

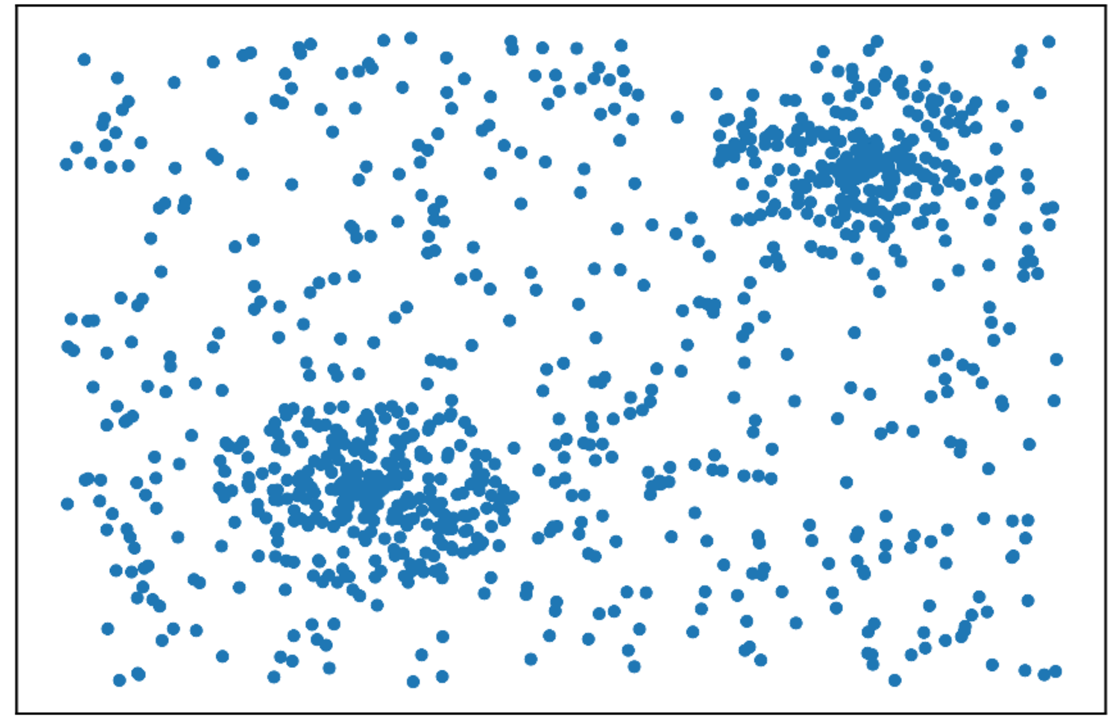
$$N_x^{Pair}(t_i, t_j) = \bar{n}_{pair}\, |t_j - t_i| \left(1 + \Psi_{pair}(t_i, t_j)\right)$$







$$\Psi_{pair}(R) = 0 \quad \text{for all } R$$



$$\Psi_{pair}(R) \geq 0 \quad \text{for } R = R_0$$

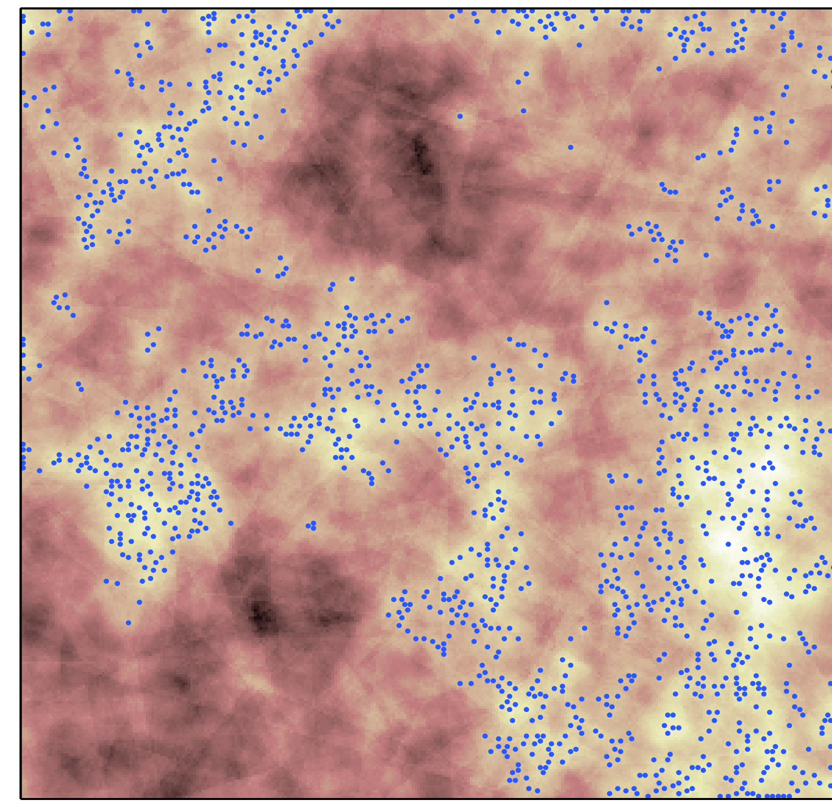
Clustering of features
Un-weighted correlation function

This is unweighted pixel correlations with particular conditions

$$\langle n(\vec{r}_1)\, n(\vec{r}_2)\rangle = \langle n(\vec{r}_1)\rangle \langle n(\vec{r}_2)\rangle \left[1 + \Psi(R)\right]$$

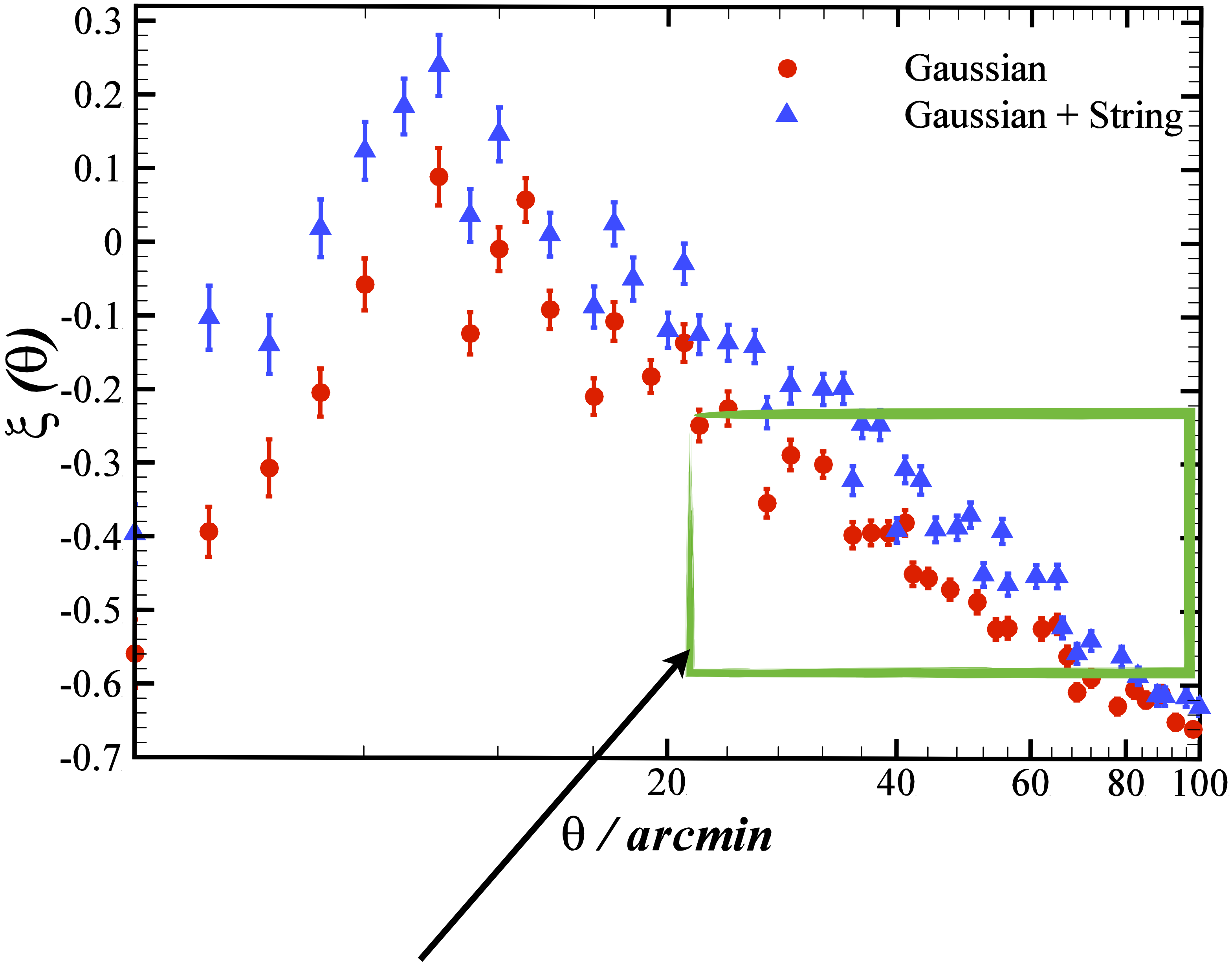
$$\Psi(R) = \frac{\langle n(\vec{r}_1)\, n(\vec{r}_2)\rangle}{\langle n(\vec{r}_1)\rangle \langle n(\vec{r}_2)\rangle} - 1$$

$$\Psi(R) + 1 = \frac{1}{\langle n(\vec{r}_1)\rangle \langle n(\vec{r}_2)\rangle} \int dA^{\mu}\, dA^{\nu}\, \mathrm{T.F.}\; P(A^{\mu}, A^{\nu})$$



جلسه بعداز ظهر را ببینید

A challenge: Finite size effect:



not reliable part

A challenge: Finite size effect: Solution

$$\Psi^{N}_{\diamond-\diamond}(r;\vartheta_1,\vartheta_2) = \left(\frac{D_\diamond(\mathbf{r}_1,\vartheta_1)D_\diamond(\mathbf{r}_2,\vartheta_2)}{R_\diamond(\mathbf{r}_1,\vartheta_1)R_\diamond(\mathbf{r}_2,\vartheta_2)}\right)\frac{N_R^\diamond(N_R^\diamond-1)}{N_D^\diamond(N_D^\diamond-1)} - 1$$

$$\Psi^{H}_{\diamond-\diamond}(r;\vartheta_1,\vartheta_2) = \frac{R_\diamond(\mathbf{r}_1,\vartheta_1)R_\diamond(\mathbf{r}_2,\vartheta_2)D_\diamond(\mathbf{r}_1,\vartheta_1)D_\diamond(\mathbf{r}_2,\vartheta_2)}{\left[D_\diamond(\mathbf{r}_1,\vartheta_1)R_\diamond(\mathbf{r}_2,\vartheta_2)\right]^2} - 1$$

$$\Psi^{LS}_{\diamond-\diamond}(r;\vartheta_1,\vartheta_2) = \left(\frac{D_\diamond(\mathbf{r}_1,\vartheta_1)D_\diamond(\mathbf{r}_2,\vartheta_2)}{R_\diamond(\mathbf{r}_1,\vartheta_1)R_\diamond(\mathbf{r}_2,\vartheta_2)}\right)\frac{N_R^\diamond(N_R^\diamond-1)}{N_D^\diamond(N_D^\diamond-1)} - \left(\frac{D_\diamond(\mathbf{r}_1,\vartheta_1)R_\diamond(\mathbf{r}_2,\vartheta_2)}{R_\diamond(\mathbf{r}_1,\vartheta_1)R_\diamond(\mathbf{r}_2,\vartheta_2)}\right)\frac{N_R^\diamond(N_R^\diamond-1)}{N_D^\diamond N_R^\diamond} + 1$$

Some examples

Clustering of local extrema in *Planck* CMB maps

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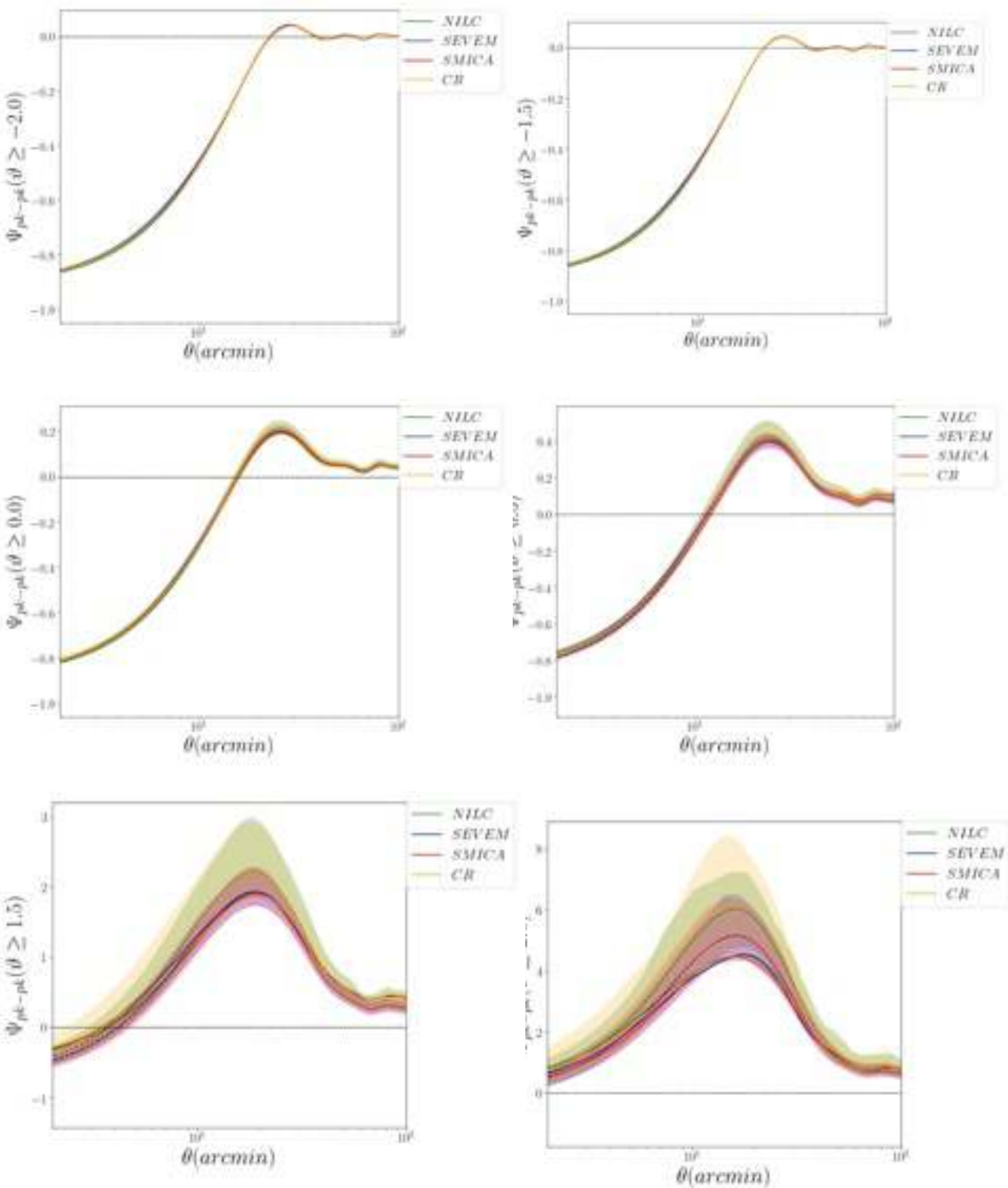

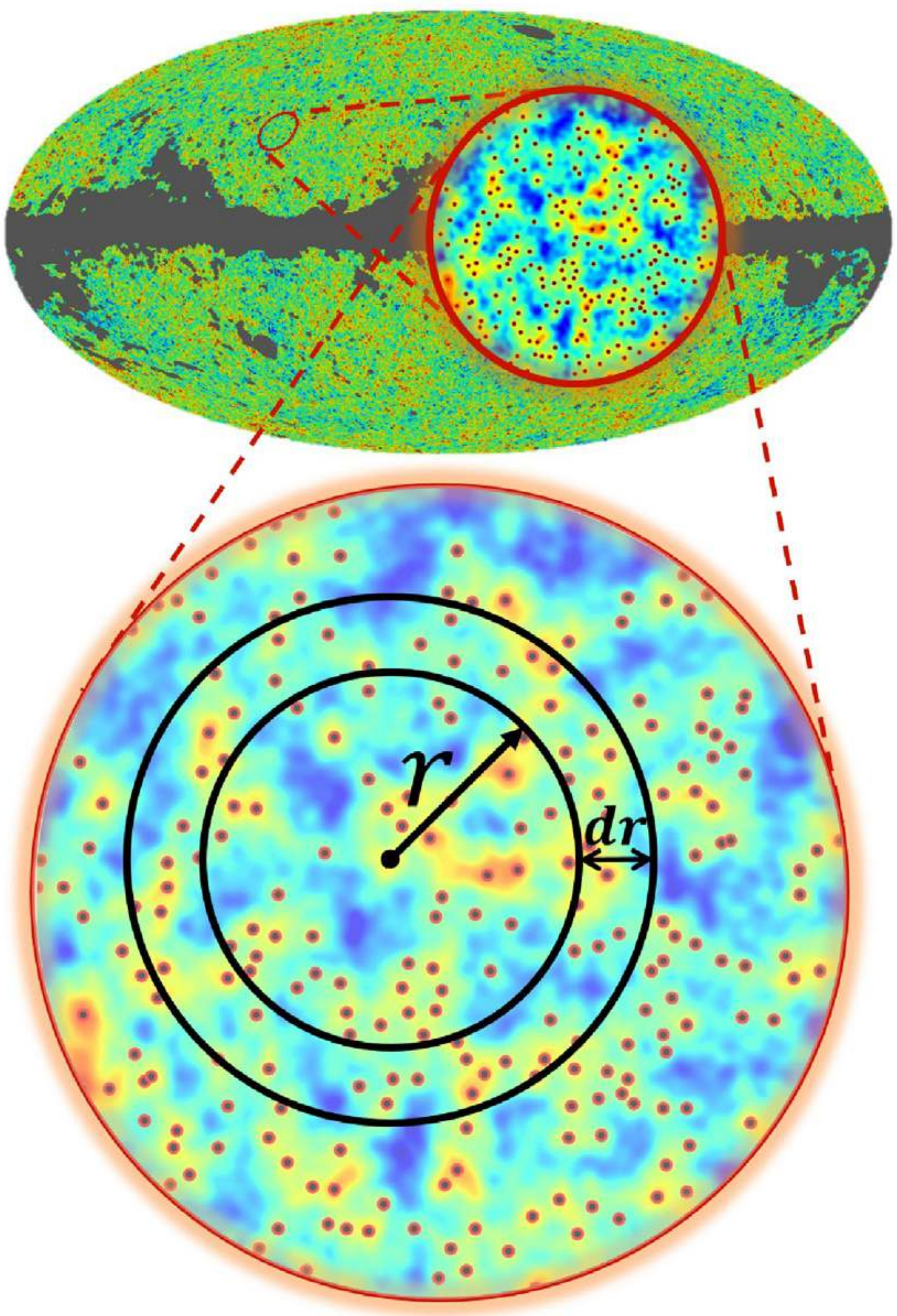

Probing cosmology via the clustering of critical points

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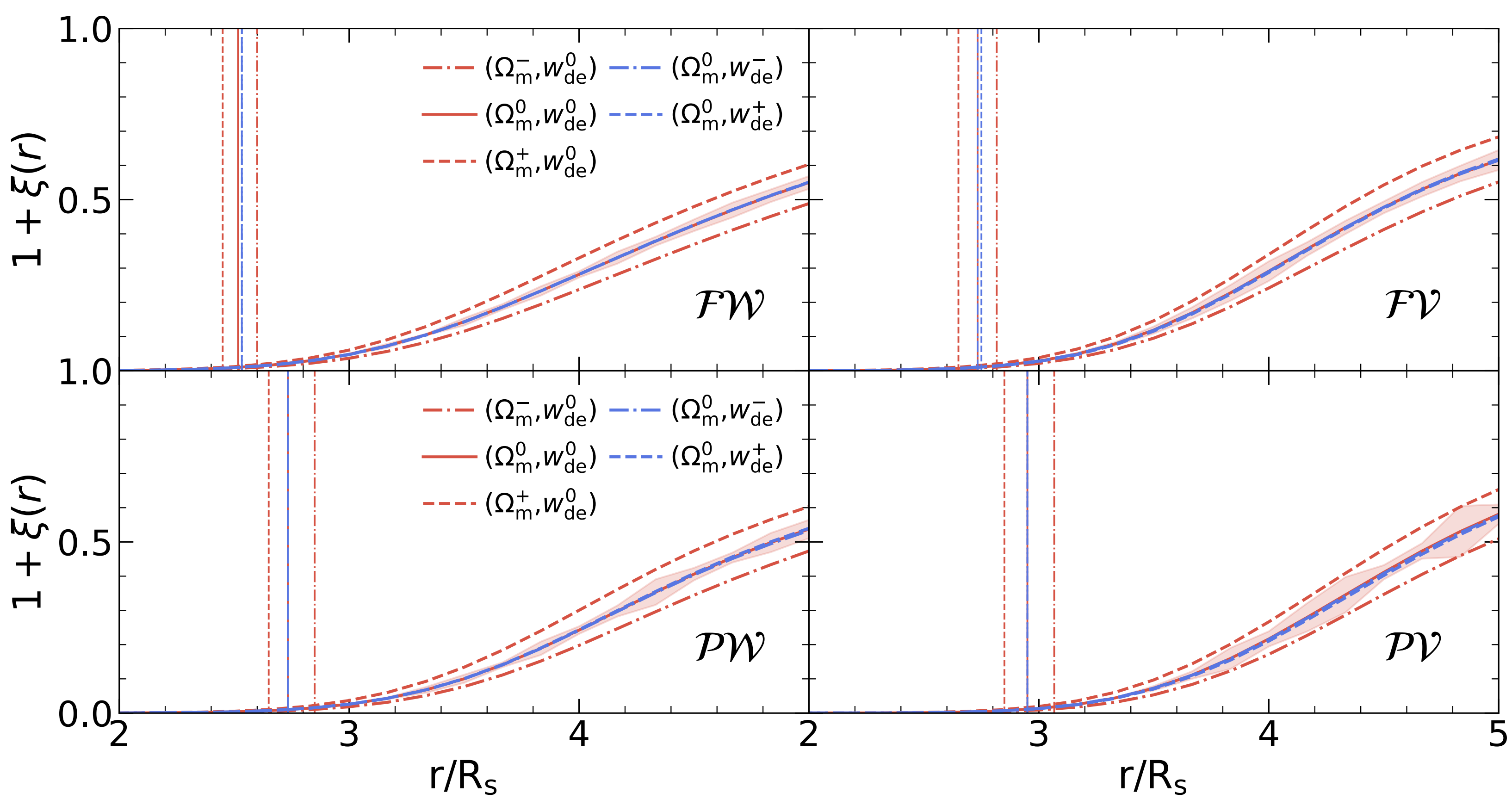


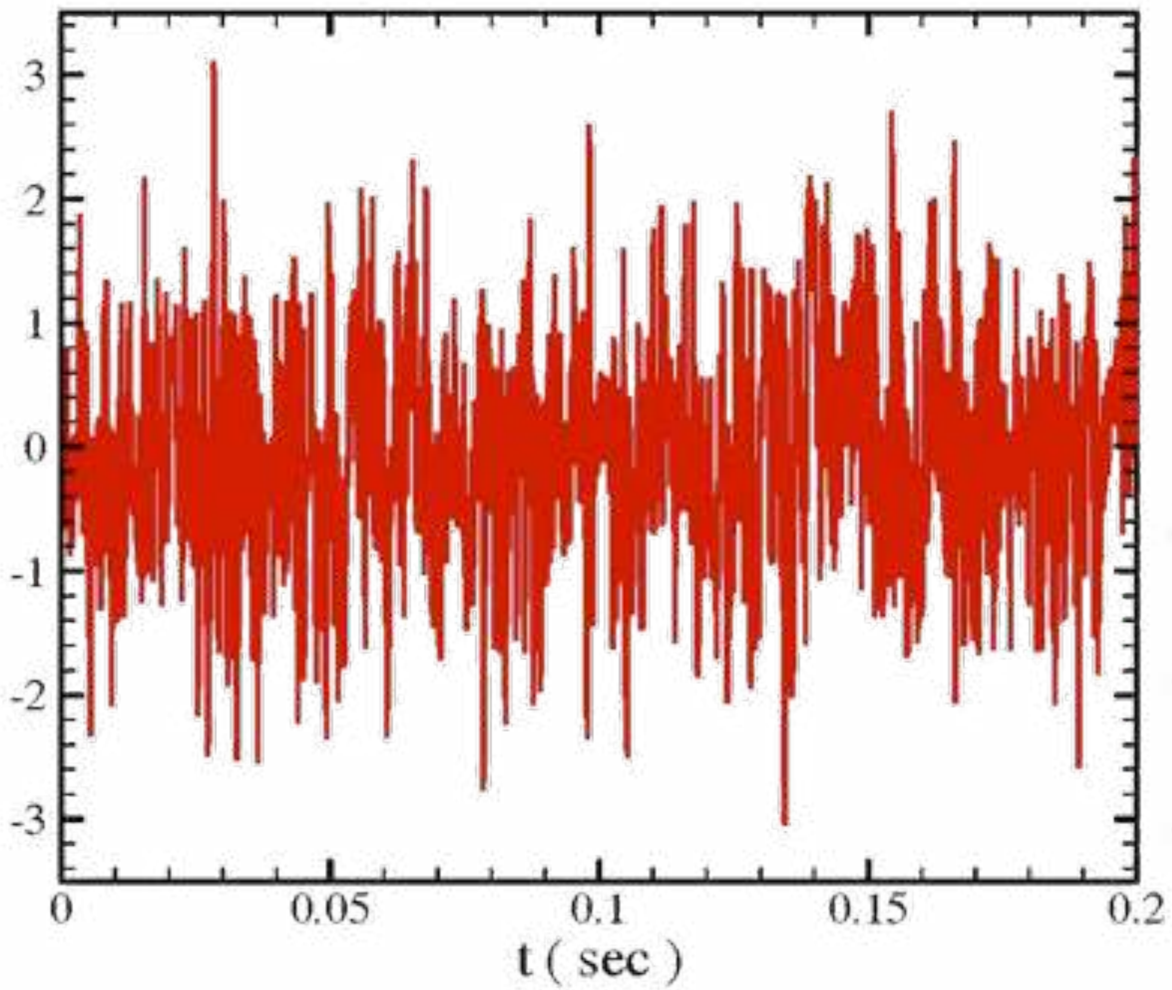
Figure 1. Two-point cross-correlation functions for pairs of positively and negatively biased critical points for the five different cosmologies as labeled. Correlation functions for the $\mathcal{FW}$, $\mathcal{FV}$, $\mathcal{PV}$, and $\mathcal{PW}$ are shown clockwise from the upper-left panel. The adopted Gaussian smoothing scale R_s is 6 h^{-1}Mpc. Vertical lines represent the exclusion zone radii and shaded regions show the standard errors around the fiducial cosmology. Note that the correlation function for the fiducial cosmology (red-solid) is nearly identical to those with the different equations of state dark energy models (blue). However, the two-point correlation function depends on Ω_m, see Fig. 2.

طیف توان و اندیسهای طیفی

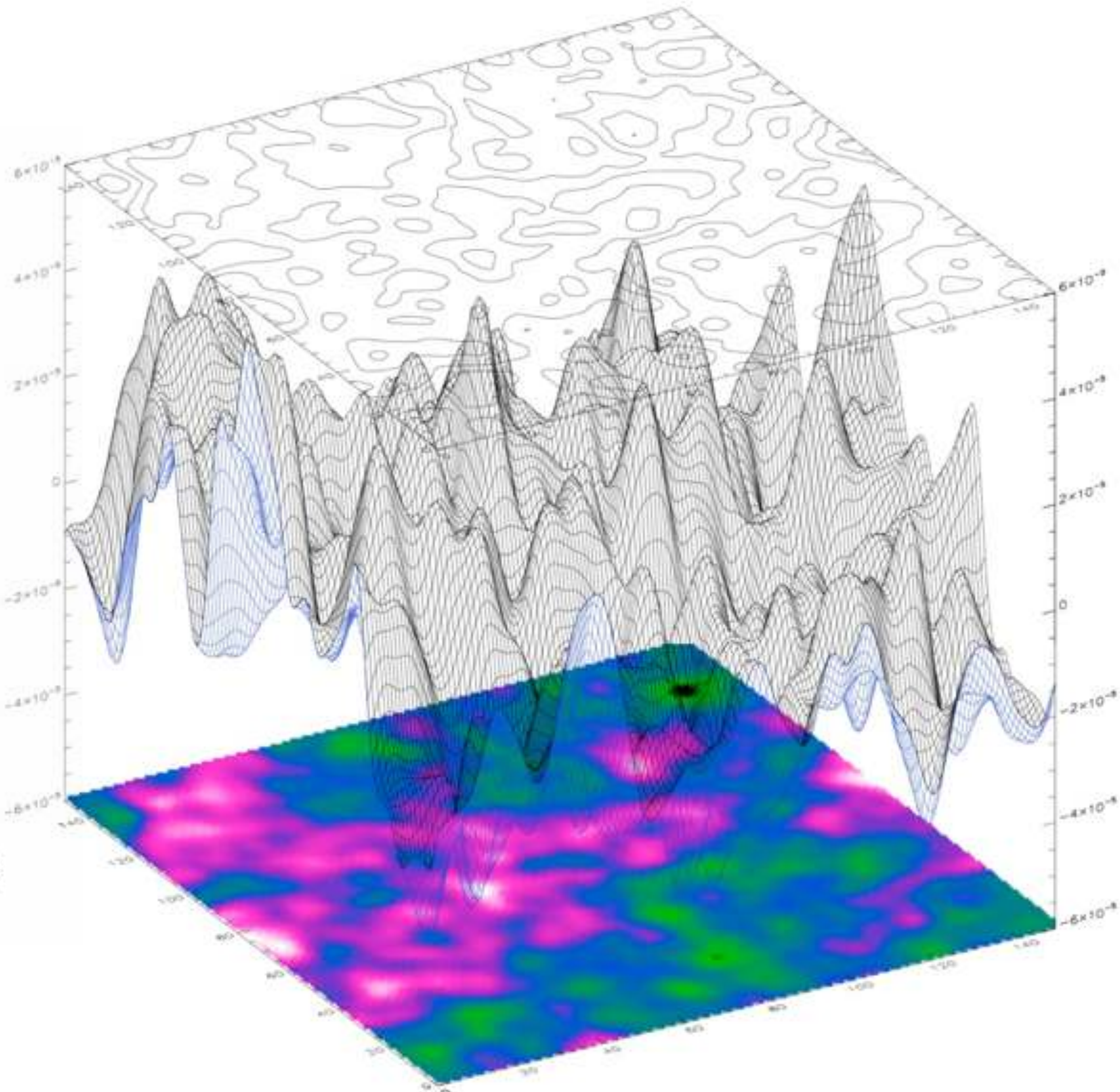
Power spectrum
&
Spectral Indices

Power spectrum

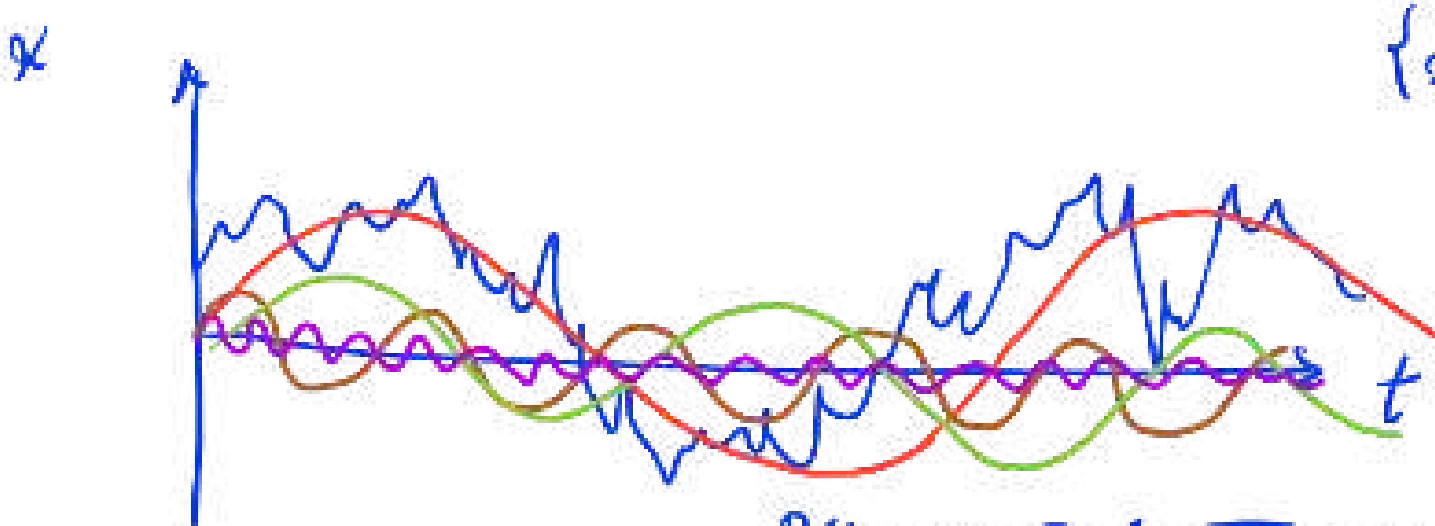
1+1 dimension



1+2 dimensions



$\{x\}: \{x_1, x_2, \dots \; x_N\}$ گسسته



Base

$$x(t) = A_1 F_1(t) + A_2 F_2 + A_3 F_3(t) + A_4 F_4(t) + \dots$$

$$S(\omega) = \int d\tau \, e^{-i\omega\tau} \, C_x(\tau)$$

Power spectrum

$$\underline{C_x(\tau)} = \int d\omega \, \underline{e^{i\omega\tau}} \, \underline{S(\omega)}$$

Some properties

$$\sigma^2 = C_x(0) = \int_{-T}^{T} S_x(\omega)d\omega$$

$$C_x(\tau) = C_x(-\tau)$$

$$S_x(\omega) = A(\omega) + iB(\omega)$$

$$B(\omega) = \frac{1}{2T}\int_{-T}^{T} C_x(\tau)\sin(\omega\tau)d\tau = 0$$

$$S_x(\omega) = |X(\omega)|^2$$

$$S_x(\omega) = \frac{1}{2T}\int_{-T}^{T} C_x(\tau)e^{i\omega\tau}d\tau = \frac{1}{2T}\int_{-T}^{T}\langle x(t)\cdot x(t+\tau)\rangle e^{i\omega\tau}d\tau$$

$$= \frac{1}{2T}\int_{-T}^{T}\frac{1}{T}\int_{-T}^{T} x(t)\cdot x(t+\tau)dt\, e^{i\omega\tau}d\tau$$

$$= \frac{1}{2T^2}\int_{-T}^{T}\int_{-T}^{T}\left(\int X(\omega')e^{-i\omega' t}d\omega'\right)\left(\int X(\omega'')e^{i\omega''(t+\tau)}d\omega''\right)dt\, e^{i\omega\tau}d\tau$$

$$= \frac{1}{2T^2}(2\pi)^2\delta(\omega-\omega'')\delta(\omega'+\omega)X(\omega)X^*(\omega)$$

How fast?

	CPU Time Required at 10^6 Flops	
N	Discrete Fourier Transform	Fast Fourier Trasform
10^3	1.0 sec	0.01 sec
10^6	10^6 sec = 12 days	20 sec
10^9	10^{12} sec = 32,000 years	3.0×10^5 sec = 8.3 hours

Fast Fourier Transformation (FFT)

$$x(0)\cdots x(N-1) \quad x(0)\cdots x((N-1)\Delta t) \quad T_{\min}=\Delta t \qquad \omega_{\max}=\frac{2\pi}{\Delta t}$$

$$T_{\max}=N\Delta t \qquad \omega_{\min}=\frac{2\pi}{N\Delta t} \qquad \Delta t\to\Delta\omega=\frac{2\pi}{N\Delta t} \quad \omega_k=\frac{2\pi k}{N\Delta t}$$

$$y_i=x(2j) \qquad z_j=x(2j+1)$$

$$Y_k=\frac{1}{N/2}\sum_{j=0}^{N/2-1} y_j e^{\frac{i2\pi kj}{N/2}}$$

$$Z_k=\frac{1}{N/2}\sum_{j=0}^{N/2-1} y_j e^{\frac{i2\pi kj}{N/2}} \qquad k=0\ldots N/2-1$$

$$X_k=\frac{1}{N}\sum_{j=0}^{N-1} x(j)e^{\frac{i2\pi kj}{N}} \quad k=0\cdots N-1$$

$$X_k=\frac{1}{N}\sum_{j=0}^{M-1} x(2j)e^{\frac{i2\pi kj}{N}}+x(2j+1)e^{\frac{i2\pi k(2j+1)}{N}}$$

$$=\frac{1}{N}\sum_{j=0}^{M-1} y_j e^{\frac{i2\pi kj}{N}}+z_j e^{\frac{i2\pi k(2j+1)}{N}}=\frac{1}{2}\left[Y_k+e^{\frac{i2\pi k}{N}}Z_k\right] \quad k=0\ldots N/2-1$$

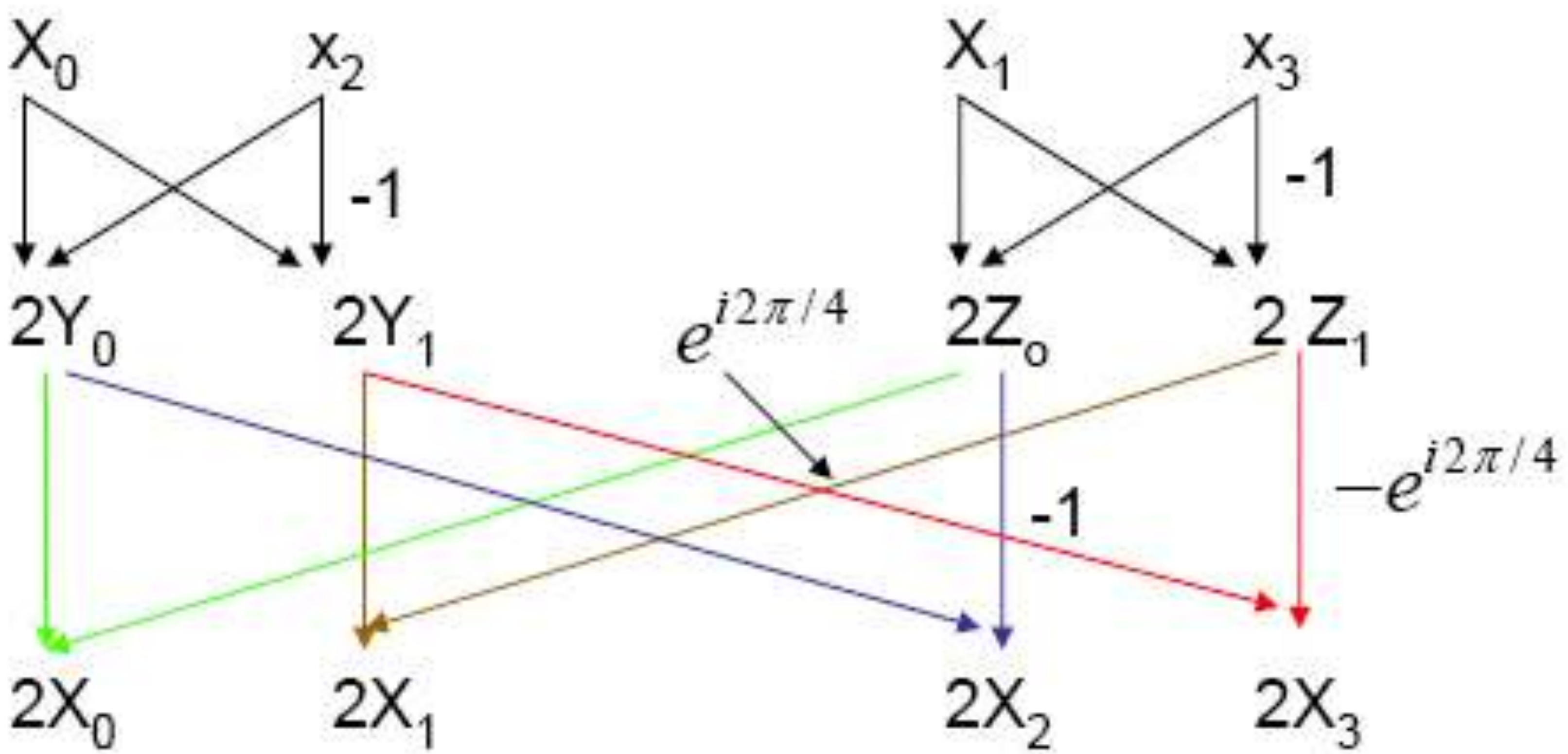
$$X_k=\frac{1}{2}\left[Y_k+e^{\frac{i2\pi k}{N}}Z_k\right] \quad k=N/2\ldots N-1$$

$$=\frac{1}{2}\left[Y_{k-\frac{N}{2}}+e^{\frac{i2\pi k}{N}}Z_{k-\frac{N}{2}}\right] \qquad k\to k+N/2 \quad k=0\ldots N/2-1$$

$$X_{k+\frac{N}{2}}=\frac{1}{2}\left[Y_k-e^{\frac{i2\pi k}{N}}Z_k\right] \quad k=0\ldots N/2-1$$

Butterfly diagram for FFT

X: {x_0,x_1,x_2,x_3}

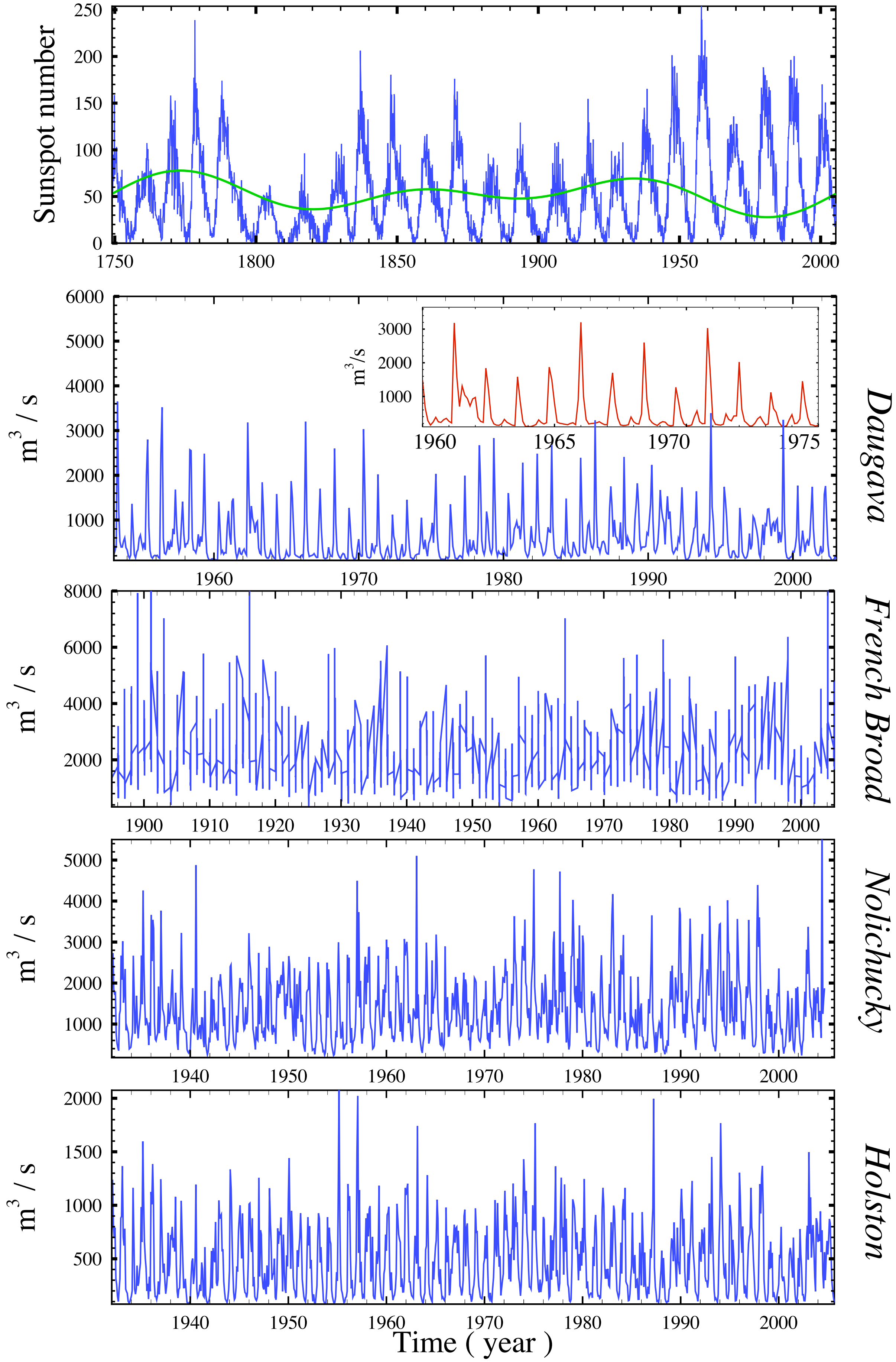


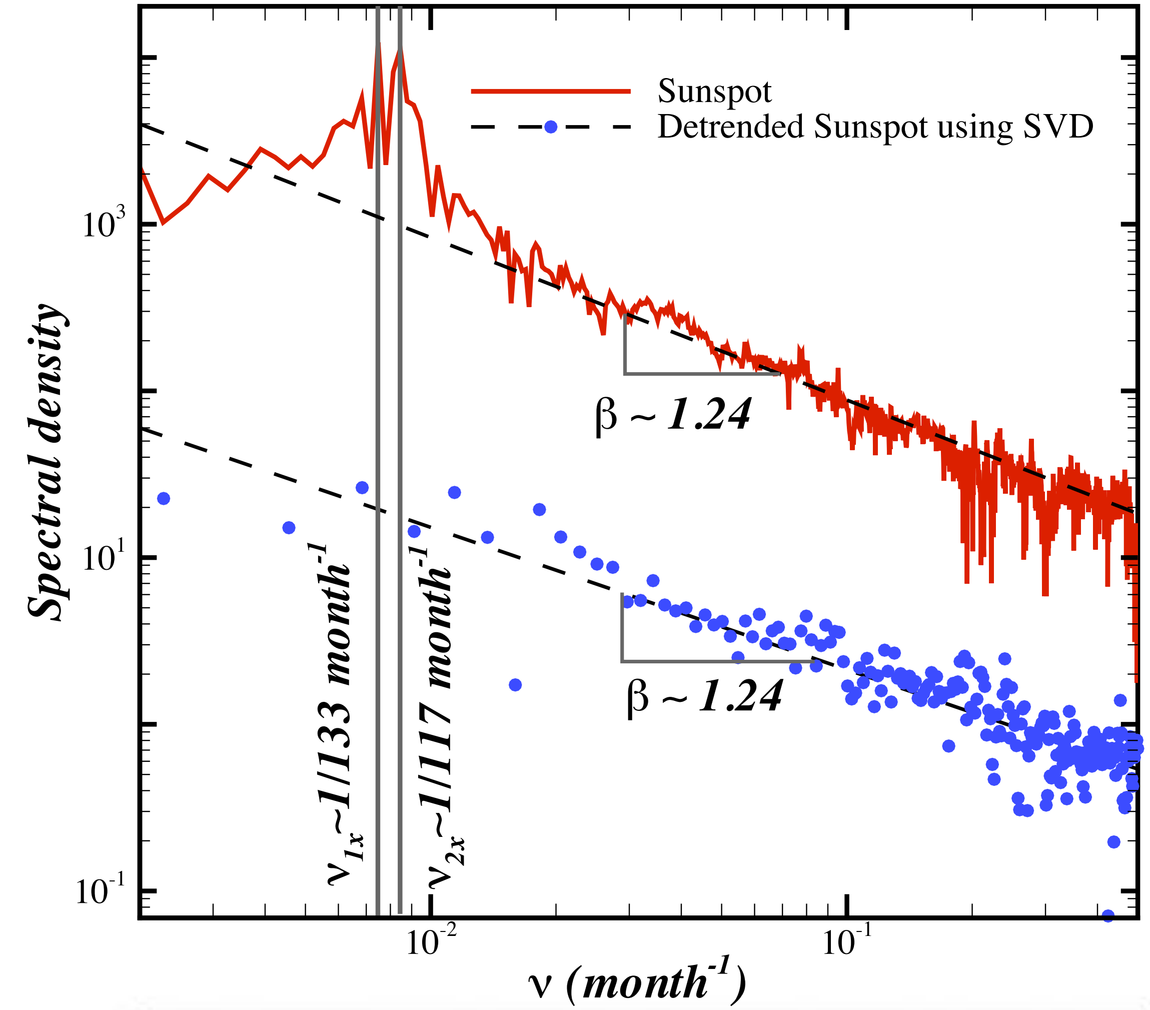
$$X_0 = \frac{1}{2}\{Y_0 + Z_0\}$$

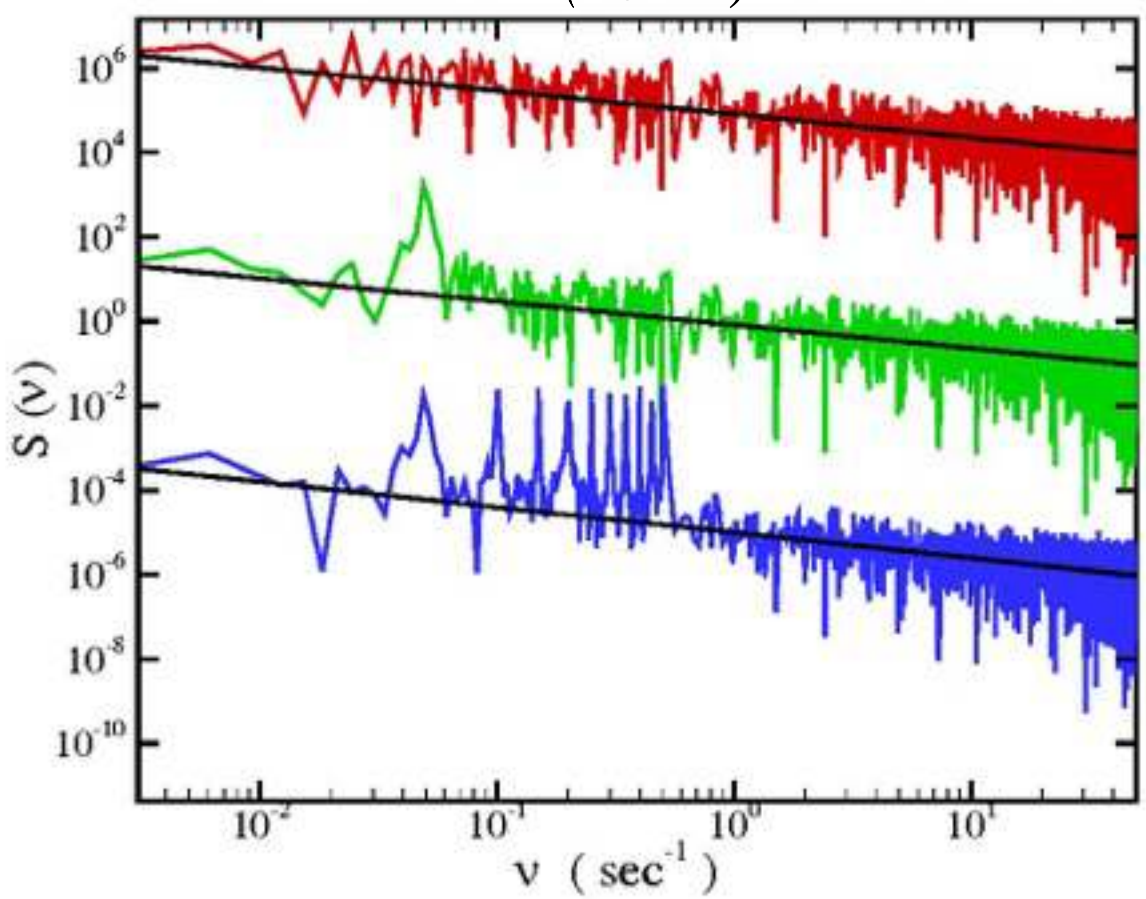
$$X_1 = \frac{1}{2}\{Y_1 + e^{i2\pi/4} Z_1\}$$

$$X_2 = \frac{1}{2}\{Y_0 + e^{i2\pi 2/4} Z_0\}$$

$$X_3 = \frac{1}{2}\{Y_1 - e^{i2\pi/4} Z_1\}$$

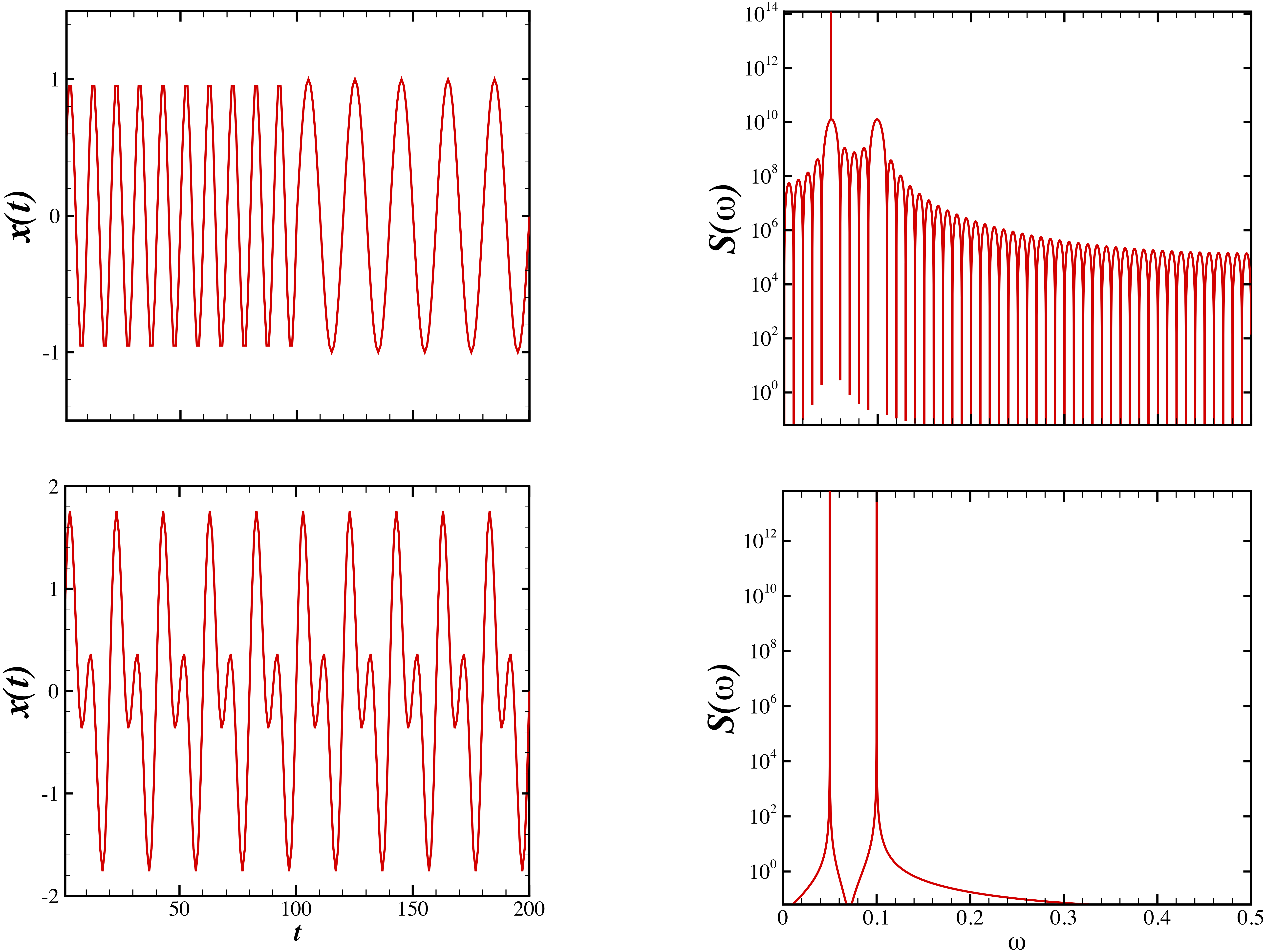




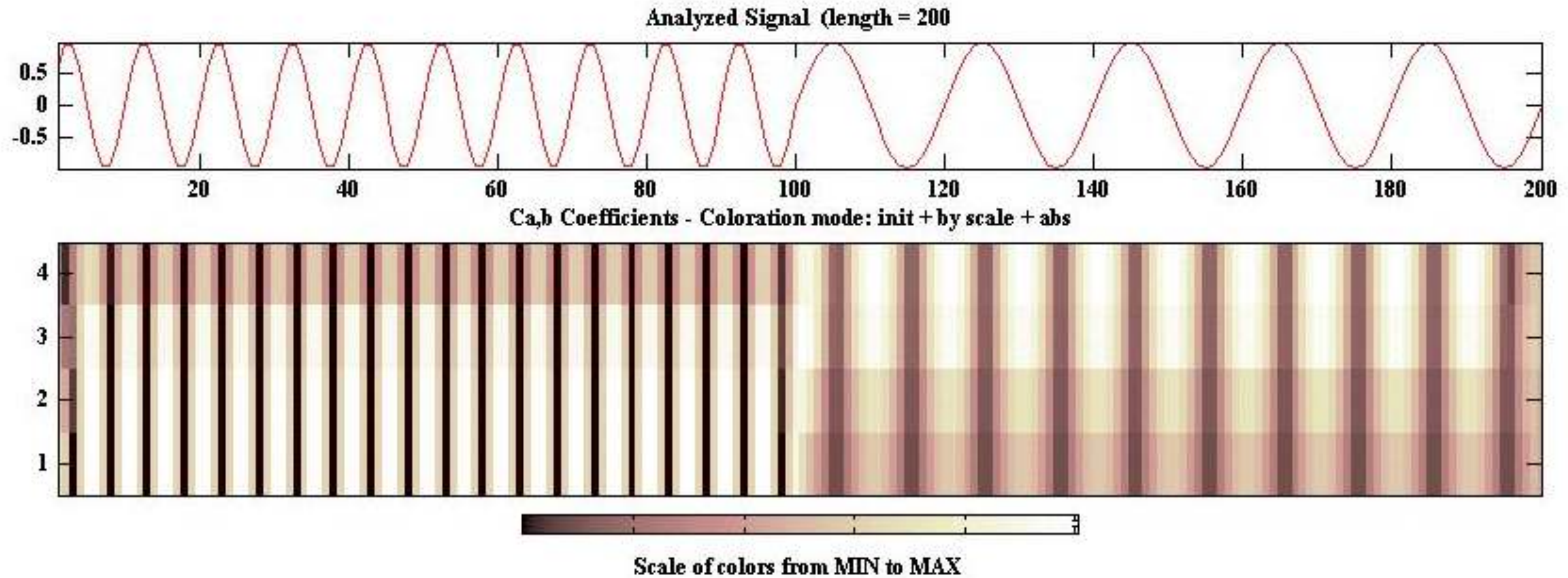


S.hajian and M.S. Movahed, Physica A 389 (2010) 4942 and S. Kimiagar, M.S. Movahed et .al., JSTAT/2009/P03020

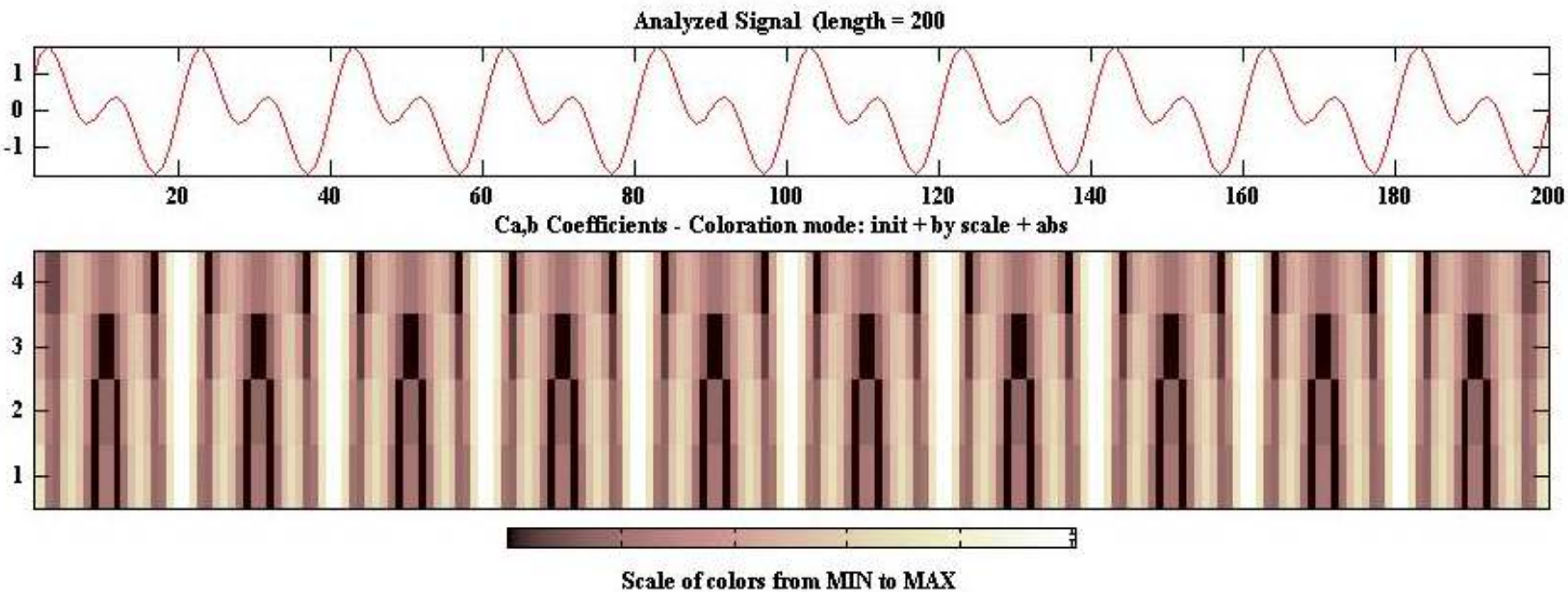
Wavelet versus FFT



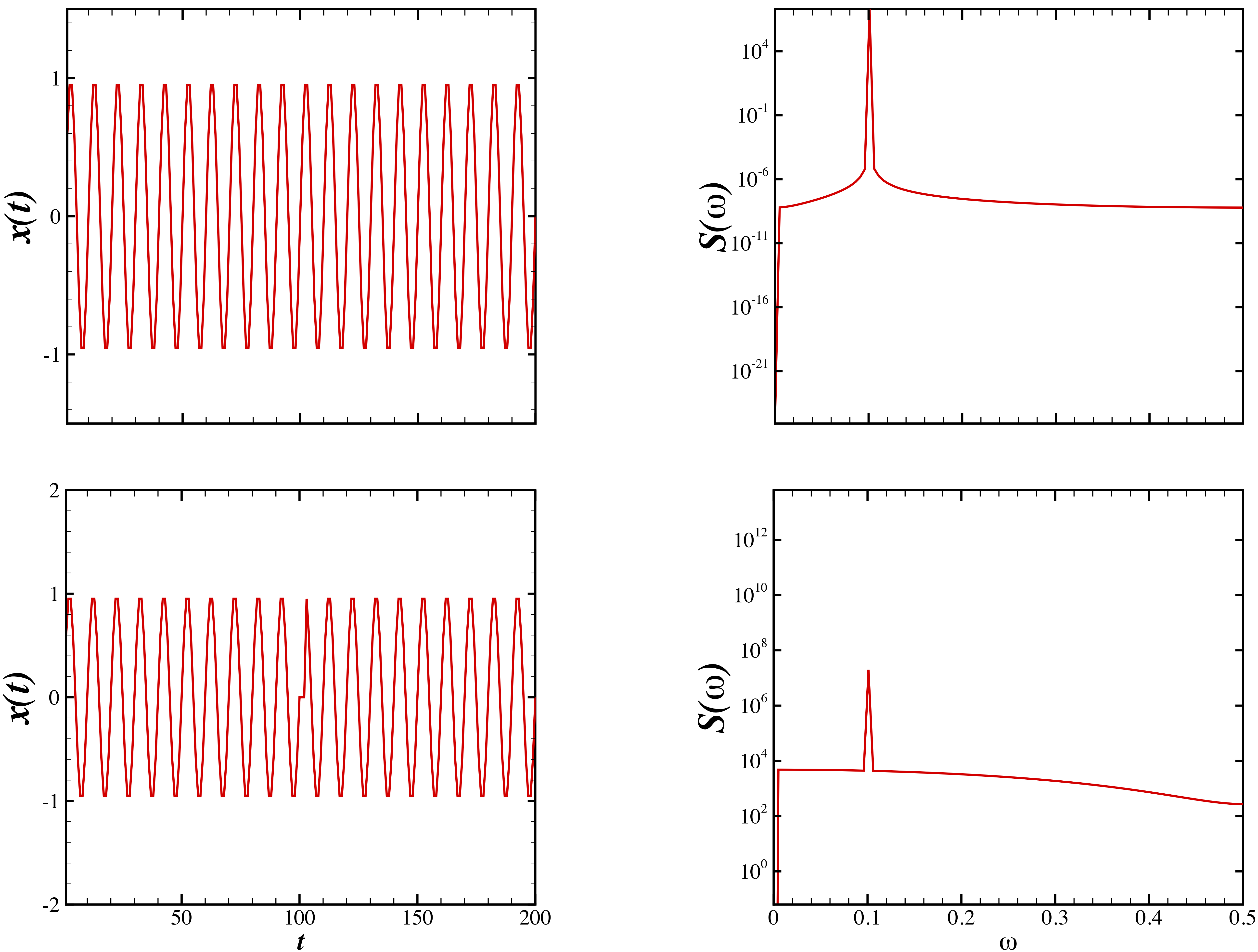
x(t)
S(ω)
x(t)
t
S(ω)
ω



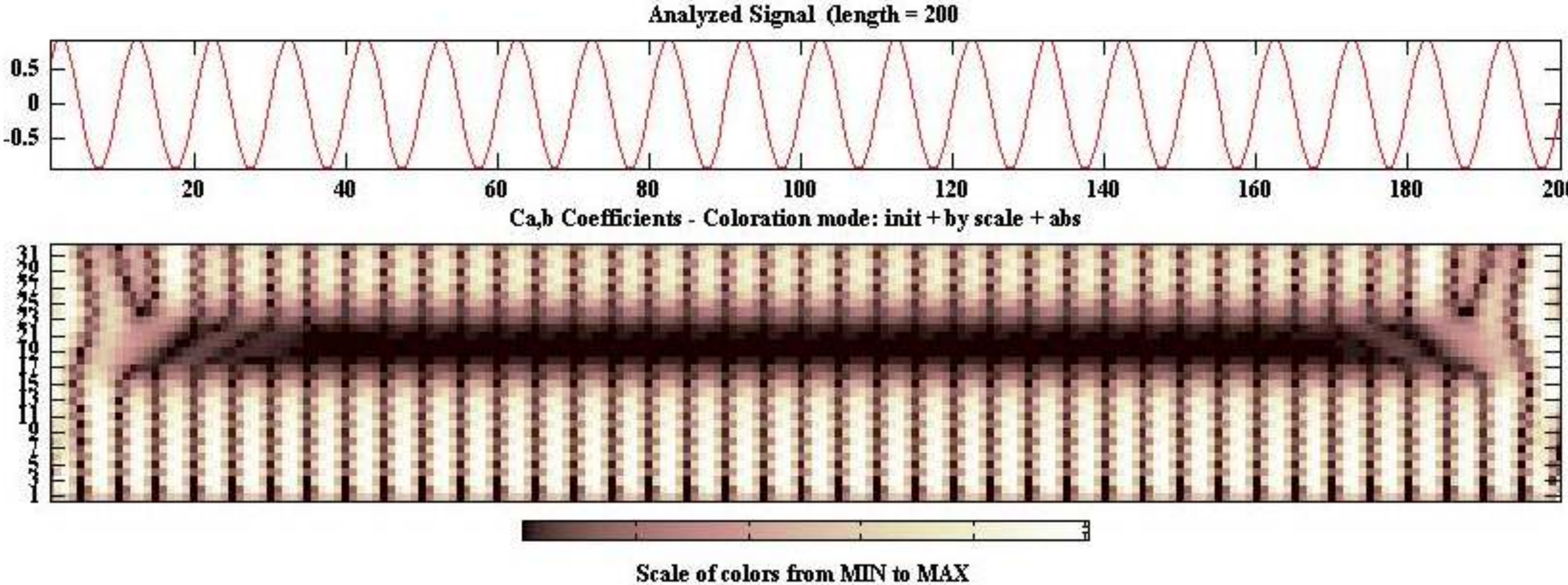
Analyzed Signal (length = 200
0.5
0
-0.5
20
40
60
80
100
120
140
160
180
200
Ca,b Coefficients - Coloration mode: init + by scale + abs
4
3
2
1
Scale of colors from MIN to MAX

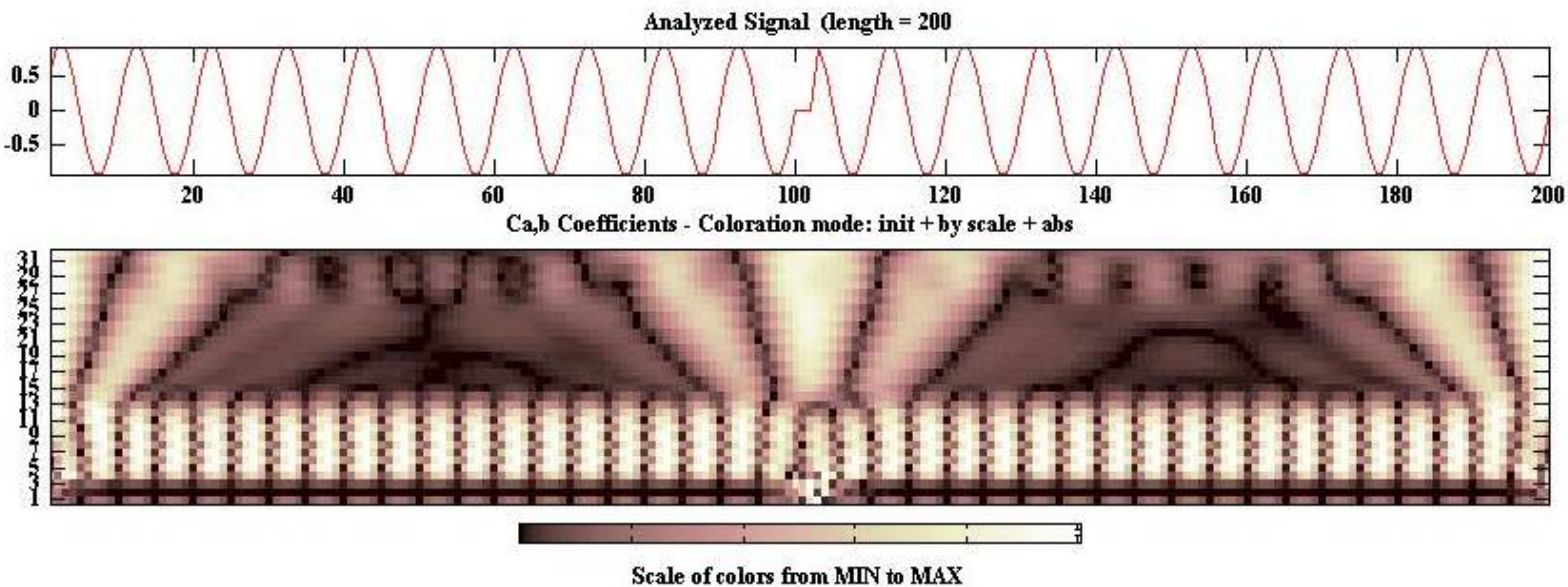


Analyzed Signal (length = 200
1
0
-1
20
40
60
80
100
120
140
160
180
200
Ca,b Coefficients - Coloration mode: init + by scale + abs
4
3
2
1
Scale of colors from MIN to MAX



x(t)
1
0
-1
S(ω)
10^4
10^-1
10^-6
10^-11
10^-16
10^-21
x(t)
2
1
0
-1
-2
50
100
150
200
t
S(ω)
10^12
10^10
10^8
10^6
10^4
10^2
10^0
0
0.1
0.2
0.3
0.4
0.5
ω


Analyzed Signal (length = 200
0.5
0
-0.5
20
40
60
80
100
120
140
160
180
200
Ca,b Coefficients - Coloration mode: init + by scale + abs
31
1
Scale of colors from MIN to MAX


Analyzed Signal (length = 200
0.5
0
-0.5
20
40
60
80
100
120
140
160
180
200
Ca,b Coefficients - Coloration mode: init + by scale + abs
31
1
Scale of colors from MIN to MAX

$$\frac{\Delta T(\theta,\varphi)}{\bar{T}} = \sum_{lm} a_{lm} Y_{lm}(\theta,\varphi)$$

$$a_{lm} = \int d\Omega \frac{\Delta T(\theta,\varphi)}{\bar{T}} Y_{lm}(\theta,\varphi)$$

$$\left\langle a_{lm} a_{l'm'} \right\rangle = \int d\Omega \int d\Omega' Y_{lm}(\theta,\varphi) Y_{l'm'}(\theta',\varphi') \left\langle \frac{\Delta T(\theta,\varphi)}{\bar{T}} \frac{\Delta T(\theta',\varphi')}{\bar{T}} \right\rangle$$

$$C(\hat{n},\hat{n}') = C(\hat{n} \bullet \hat{n}') = \frac{1}{4\pi} \sum_{l} (2l+1) C_l P_l \left(\left| \hat{n} \bullet \hat{n}' \right| \right)$$

$$\left\langle a_{lm} a_{l'm'} \right\rangle = \delta_{ll'} \delta_{mm'} C_l$$

Cosmic Variance

$$C_l = \frac{1}{(2l+1)} \sum_{m=-l}^{m=+l} \left| a_{lm} \right|^2$$

$$\sigma_l^2 = \left\langle C_l^2 \right\rangle - \left\langle C_l \right\rangle^2 \rightarrow \left\langle C_l^2 \right\rangle = \frac{3}{(2l+1)^2} \sum_{mm'} \left\langle a_{lm} a_{lm} \right\rangle \left\langle a_{lm'} a_{lm'} \right\rangle = 3 \left\langle C_l \right\rangle^2$$

$$\sigma_l^2 = 2 \left\langle C_l \right\rangle^2 \rightarrow \left(\sigma_l^2 \right)_m = \frac{\sigma_l^2}{N} = \frac{2 \left\langle C_l \right\rangle^2}{2l+1} : \frac{2C_l^2}{2l+1}$$

Spectral Indices

$$\sigma_n^2 = \left(\frac{d}{d\tau}\right)^{2n} C_{xx}(\tau) = \int d^D\omega \, \omega^m S(\omega)$$

$$m = \begin{cases} 2n+2 & \text{for} \quad D=3 \\ 2n+1 & \text{for} \quad D=2 \\ 2n & \text{for} \quad D=1 \end{cases}$$

کاربردهای این تعاریف را در درس بعدی خواهیم دید

جمع بندی در مورد تابع توزیع و تابع همبستگی

۱) تابع توزیع در حقیقت اطلاعاتی از فراوانی مقادیر منتسب به کمیت مورد نظر به دست می دهد

۲) تابع همبستگی اطلاعاتی از چگونگی ارتباط مقادیر منتسب به کمیت مورد نظر در زمان (مکان)های مختلف به دست می دهد.

۳) اعداد کاتوره ای یعنی اینکه $\langle x(t)x(t')\rangle = g\delta(t-t')$

ولی می تواند همزمان فرآوانی های مختلفی داشته باشد.

۴) اعدادی که با تابع توزیع گوسی وجود دارند در حالت کلی می توانند تابع همبستگی های مختلفی داشته باشند

Computational Cosmology Group
Of Shahid Beheshti University

از توجه شما سپاسگزارم

COMPLEXLAB
SHAHID BEHESHTI UNIVERSITY