

# Subject: Transformation (3)

بِسْمِ اللّٰهِ الرَّحْمٰنِ الرَّحِیْمِ

## ① Summary on Transformation (Projection, ...) So far

### ☆ Continuous Fourier Transformation (F.T.) (1+1)-Dimension

Time Series  
 سری زمانی  
 Independent Parameter  
 Dependent Parameter

$$x(t) = \int d\omega e^{+i\omega t} X(\omega) \quad (1+1) \quad \omega = \frac{2\pi}{T}$$

$$X(\omega) = \int dt e^{-i\omega t} x(t)$$

مغز اندیش

\* (R+D)-dimension  $\{ x^{(j)}(t^{(k)}) \}$   $j=1, \dots, R$   
 $k=1, \dots, D$

Ex 1: (2+3)-Dimension

→ Temperature  
 → Pressure

$(r_x, r_y, r_z)$

Independent Parameters

$$A^{(j)}(r^{(k)})$$

$j=1, 2 \quad (T, P)$

$k=1, 2, 3 \quad (x, y, z)$

$$A^{(1)}(x, y, z)$$

Discrete Form

$$A(x_i, y_i, z_i)$$

$$A^{(2)}(x, y, z)$$

$i=1, \dots, N$

$$A^{(2)}(x_i, y_i, z_i)$$

$i=1, \dots, N$

$x$	$y$	$z$	$T(x,y,z)$	$P(x,y,z)$
$x_1$	$y_1$	$z_1$	$T_1$	$P_1$
$x_2$	$y_2$	$z_2$	$T_2$	$P_2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

}  $N$

$$A^{(R)}(t^{(D)}) \longleftarrow A(\vec{r})$$

$$\phi(\vec{r}) \longrightarrow (1+3)$$

$$\phi(\vec{r}) = \int d^3k e^{i\vec{k} \cdot \vec{r}} \tilde{\phi}(\vec{k})$$

$$|\vec{k}| = \frac{2\pi}{\lambda} = \text{Wave Number} \quad \text{عبره}$$

$$\vec{E}(\vec{r}) \longrightarrow (3+3)\text{-Dimens.}$$

$$\vec{E}^{(D)}(\vec{r}) = \int d^3k e^{i\vec{k} \cdot \vec{r}} \tilde{E}_e(\vec{k})$$

$$l=x \quad E_x(\vec{r}) = \int d^3k e^{i\vec{k} \cdot \vec{r}} \tilde{E}_x(\vec{k})$$

$$l=y \quad E_y(\vec{r}) = \int d^3k e^{i\vec{k} \cdot \vec{r}} \tilde{E}_y(\vec{k})$$

$$l=z \quad E_z(\vec{r}) = \int d^3k e^{i\vec{k} \cdot \vec{r}} \tilde{E}_z(\vec{k})$$

$$\tilde{E}_x(\vec{k}) = \int d^3r e^{-i\vec{k}\cdot\vec{r}} E_x(\vec{r})$$

Coefficient

$$\tilde{E}_y(\vec{k}) = \int d^3r e^{-i\vec{k}\cdot\vec{r}} E_y(\vec{r})$$

$$\tilde{E}_z(\vec{k}) = \int d^3r e^{-i\vec{k}\cdot\vec{r}} E_z(\vec{r})$$

$$e^{i\vec{k}\cdot\vec{r}} \quad e^{ik\cdot x}$$

$$\vec{E}(\vec{r}) = \begin{pmatrix} E_x(x) \\ E_y(y) \\ E_z(z) \end{pmatrix} \begin{matrix} E_x(x, y, z) \\ E_y(x, y, z) \\ E_z(x, y, z) \end{matrix}$$

$$e^{i\vec{k}\cdot\vec{r}} = \sum_j \dots$$

Ex2:  $T(x, y, z)$  این اسکالر است در ۳ بعد ✓  
(1+3)-Dimens.

Ex3:  $i\hbar \frac{\partial}{\partial t} \psi = H\psi$  3-D spatial coordinate  
4-D Temporal.

$\psi(x, y, z, t)$   
(1 + 3)-Dimens.

(1+3)

$$\psi(\vec{r}, t) = \int d^3k \int d\omega e^{i\vec{k}\cdot\vec{r}} e^{i\omega t} \tilde{\psi}(\vec{k}, \omega)$$

$$\tilde{\psi}(\vec{k}, \omega) \leftarrow \begin{aligned} P &= \chi_E E \rightarrow \frac{\partial P}{\partial E(t, r)} \\ M &= \chi_H B \end{aligned}$$

$$\chi(\vec{r}, t) \rightarrow \tilde{\chi}(\vec{k}, \omega)$$

Static Response function  $\rightarrow \tilde{\chi}(0, 0)$

$\omega \rightarrow 0 \quad T \rightarrow \infty$

$k=0 \rightarrow \lambda \rightarrow \infty$

★ Discrete F.T. (Regular Data (Equidistant Sampling))

$\Delta t = \text{cts}$

$$x(t) \rightarrow \{x_j\}, j=1, \dots, N \text{ or } \{x(t_1), x(t_2), \dots, x(t_N)\}$$

$$t_{j+1} - t_j = \Delta t \quad \forall j \in [1, N]$$

$$\int d\omega \rightarrow \sum \frac{\Delta\omega}{\left(\frac{2\pi}{T}\right)}$$

$$\rightarrow t_{\min} = \Delta t$$

$$t_j = j \Delta t \quad \begin{matrix} j=1, \dots, N \\ = 0, \dots, N-1 \end{matrix}$$

$$t_{\max} = N \Delta t$$

$$\omega_{\max} = \frac{2\pi}{t_{\min}} = \frac{2\pi}{\Delta t}$$

$$\omega = \frac{k \cdot 2\pi}{N \Delta t} = k \omega_{\min} = k \Delta \omega \quad k=1, \dots, N \quad (0, \dots, N-1)$$

$$\Delta \omega \equiv \omega_{\min} = \frac{2\pi}{t_{\max}} = \frac{2\pi}{N \Delta t}$$

$$f_N \gg 2f_s$$

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بافتش  $f_s$

$$f_s \ll f_N/2$$

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★ Recall that

$$\langle x \rangle = 0$$

$$C_x(\tau) = \langle x(t) x(t+\tau) \rangle = \int d\omega e^{i\omega\tau} S_x(\omega)$$

↑  
Power Spectrum

Ⓐ Random Number  $C_x(\tau) = \langle x(t) x(t+\tau) \rangle = \delta_D(\tau)$

$$\delta_D(\tau) = \int d\omega e^{i\omega\tau} S(\omega)$$

$$S(\omega) = \int d\tau e^{-i\omega\tau} \delta_D(\tau) = cts$$

$S(\omega)$

$cts$

White Noise

→ Frequency-Independent

$\omega$

(B)  $C_x(\tau) \neq \delta_D(\tau)$

Power-law  $\tau^{-\alpha}$   
 $e^{-\tau}$

Colored Noise  
 نویزی

Blue Tilt  
 Red Tilt

\*

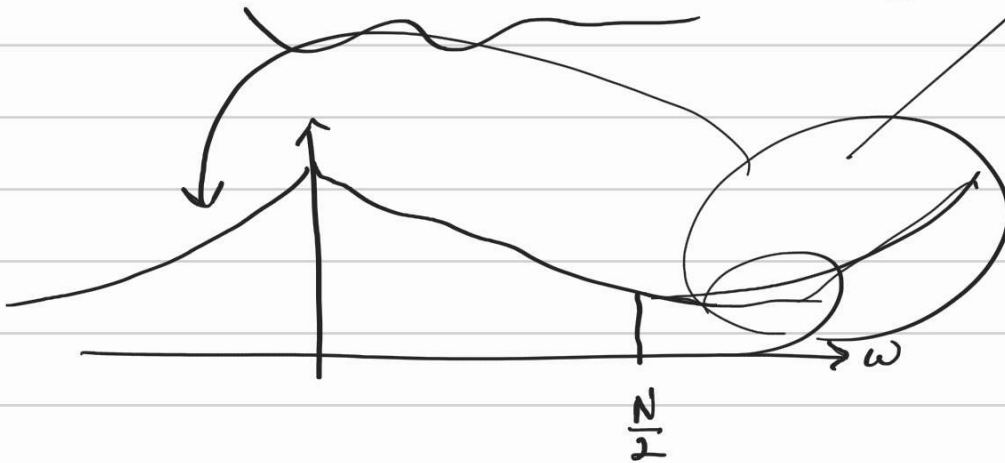
$$e^{i\omega t}$$

$$e = C_u(\omega t) + i \sin(\omega t)$$

$$C(\tau) = \int_{-\infty}^{+\infty} d\omega [C_u(\omega t) + i \sin(\omega t)] S_x(\omega)$$

$$= \int d\omega C_u(\omega t) S(\omega) + i \int d\omega \sin(\omega t) S(\omega)$$

$\uparrow$   
 even

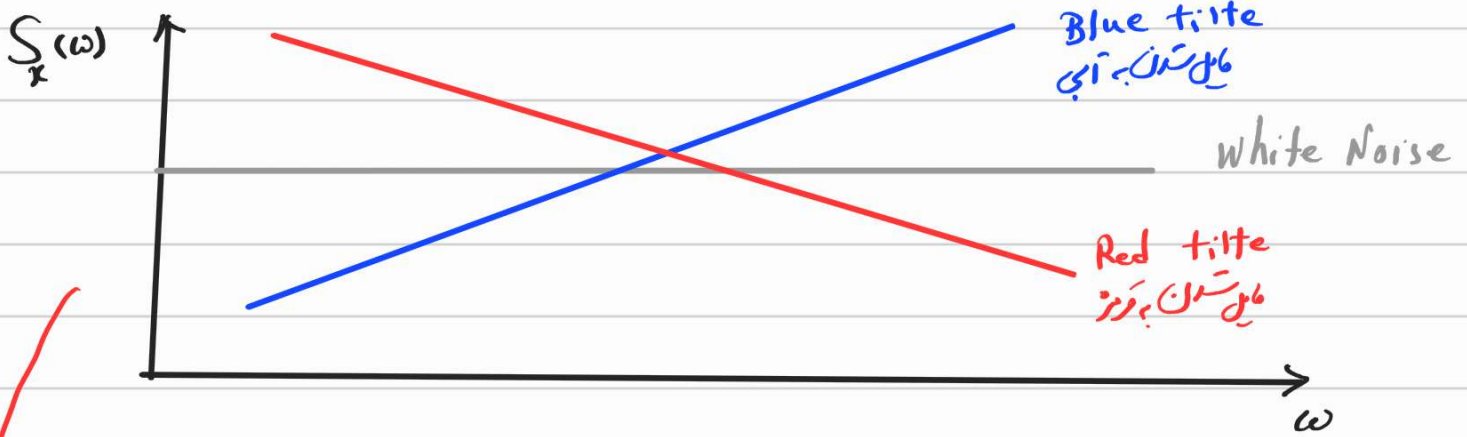


$$S_x(\omega_k) = S_x\left(\frac{k2\pi}{N\Delta t}\right) \quad \begin{matrix} k=0, N-1 \\ 1, N \end{matrix}$$

$$S_x(k) = S_x\left(-\left(k - \frac{N}{2}\right)\right) \quad k \geq \frac{N}{2}$$

Nyquist phenomenon ... فرایند

\*



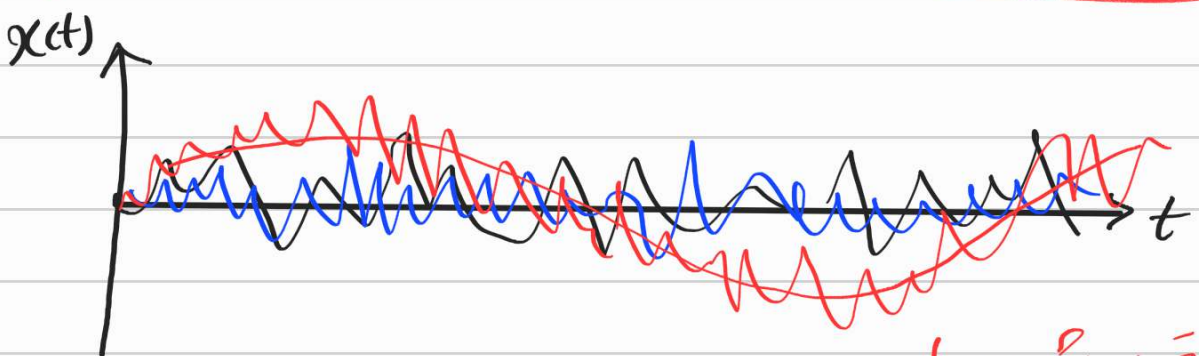
$$H = \int_{-\infty}^{\infty} dx \left[ t m^2 + K (\nabla m)^2 + L (\nabla^2 m)^2 \right]$$

Field

$$\tilde{H} = \int_{-\frac{2\pi}{L}}^{\frac{2\pi}{L}} \frac{d^d q}{(2\pi)^d} \left[ t |\tilde{m}(q)|^2 + K q^2 |\tilde{m}(q)|^2 + L q^4 |\tilde{m}(q)|^2 \right]$$

$$a_1 \ll L < \infty \longrightarrow 0 < q \leq \frac{2\pi}{a} \longrightarrow \text{UV-cutoff}$$

$$q = \frac{2\pi}{\lambda} \quad \frac{2\pi}{L} \leq q < \frac{2\pi}{a} \longrightarrow \begin{matrix} \text{UV-cutoff} \\ \text{IR-cutoff} \end{matrix}$$



تصویر نمودی از داده



## ② Surrogate Data

$$\text{کوسی سره} \rightarrow P(x_j) = N(\bar{x}, \sigma)$$

Shuffled Data

$$\text{برخورد} \rightarrow C_x(\tau) = \delta_0(\tau)$$

$$S_x(\omega) = c + \Delta$$

Import Data set  $\{x(t)\} \{x_j\}, j=1, N$

loop MC

$l = \text{Call Random Number}(1, N)$

$m = \text{Call Random Number}(1, N)$

$$\Delta = x(l)$$

$$x(l) = x(m)$$

$$x(m) = \Delta$$

End loop

loop  $\tau$

$$C_{sk} = 0$$

loop  $j=1, N-\tau$

$$C_{sk} = C_{sk} + x(j)x(j+\tau)$$

End loop

$$C_{sk} = \frac{C_{sk}}{N-\tau}$$

Write  $\tau, C_{sk}$

End loop





To make a Surrogate series, we use

Phase-randomized algorithm

$$\{x(t)\} \xrightarrow{\text{F.T.}} \{X(\omega)\}$$

Random phase

$$\rightarrow X(\omega) \rightarrow X'(\omega) = X(\omega) e^{i\phi(\omega)}$$

$$X'(\omega) \xrightarrow{\text{I.F.T.}} \{x'(t)\}$$

$$P(x') = \mathcal{N}$$

$$C_{x'}(\tau) = C_x(\tau)$$

$$X(\omega) = \frac{1}{N} \sum_{j=1}^N e^{i\omega t_j} x(t_j)$$

Random Number

$$\downarrow (R \times 2\pi)$$

$$\downarrow \phi(\omega_k) \in [0, 2\pi]$$

$$X'(\omega) = X(\omega) e^{i\phi(\omega)}$$

$$x'_j(t) = \frac{1}{N} \sum_{\omega_k} e^{-i\omega_k t_j} X(\omega) e^{i\phi(\omega)}$$

← ساخته شده از داده های اصلی  
صفحه ۱۰

$x'(t_j)$

تالیق توزیع کو اسرار دارد.

$$S_x(\omega) = |X(\omega)|^2$$

$$S_{x'}(\omega) = |X'(\omega)|^2 = (X(\omega) e^{i\phi}) (X(\omega) e^{i\phi})^* = |X(\omega)|^2 = S_x(\omega)$$

$$= \left( C_x(\tau) \int d\omega e^{i\omega\tau} S_x(\omega) \right) \left( C_{x'}(\tau) \int d\omega e^{i\omega\tau} S_{x'}(\omega) \right)$$