The Precision Era of Cosmology
or
How cosmologists fell in love with CMB

M. Farhang
"perfect" blackbody spectrum
Unbelievable agreement between theory and observation
The Cosmic Microwave Background Radiation

How did it form?

and

What does it tell us?
the cosmic soup!
at early times (hi z)

- photons
- electrons
- neutrinos
- dark matter
- protons

Compton scattering

Coulomb scattering

\[ e^- + p^+ \leftrightarrow \gamma + H \]
@ hi z

photon-baryon plasma

\[ \gamma + e^- \leftrightarrow \gamma + e^- \]
\[ e^- + p^+ \leftrightarrow \gamma + H \]

scattering rate \( \gg \) \(1/Hubble\) time scale

\[ \Gamma = \frac{c}{\lambda_{mfp}} \gg \frac{1}{t_H} \sim H \]

mean free path \( \ll \) horizon

Thermal equilibrium
blackbody distribution of photons
blackbody distribution of photons

\[
I(\nu, T) = \frac{2hc^2\nu^3}{e^{h\nu/kT} - 1}
\]

\[
n(\nu, T) = \frac{8\pi\nu^2}{e^{h\nu/kT} - 1}
\]

now

temperature \ T=2.726

number density of photons \ 410/cm^3
COBE

Cosmic Background Spectrum at the North Galactic Pole

The smooth curve is the best fit blackbody spectrum

- data with 1% error bar

Based on 9 minutes of data

universe expands and cools down
photons lose energy
\[ e^- + p^+ \rightarrow \gamma + H \]

mean free path >> horizon
recombination @ \( z \sim 1100 \)
last scattering of photons off electrons!
CMB forms!
Our goal

How do these CMB photons look in the sky now? We need to know the evolution of the cosmic soup! before and after recombination.

Boltzmann Equations

\[
\frac{Df}{Dt} = C[f]
\]

- distribution function
- collision term
Boltzmann Equations

set of equations describing the evolution of perturbations in the Universe

\[ \delta_b, \delta, \Phi, \Psi, \Theta, v, v_b \]

- baryon density
- dm density
- photons
- gravitational potentials
- baryon velocity
- dm velocity
need to deal with super horizon perturbations and relativistic particles therefore
general relativistic treatments.

in the newtonian limit, they reduce to
- mass conservation
- momentum conservation (Euler eqn)
...
Boltzmann (+Einstein) Equations

\[
\dot{\Theta} + i k \mu \Theta = -\dot{\Phi} - i k \mu \Psi - \dot{\tau} \left[ \Theta_0 - \Theta + \mu v_b - \frac{1}{2} \mathcal{P}_2(\mu) \Pi \right] \quad (4.100)
\]

\[\Pi = \Theta_2 + \Theta_{P2} + \Theta_{P0} \quad (4.101)\]

\[
\dot{\Theta}_{P} + i k \mu \Theta_{P} = -\dot{\tau} \left[ -\Theta_P + \frac{1}{2} (1 - \mathcal{P}_2(\mu)) \Pi \right] \quad (4.102)
\]

\[\dot{\delta} + ik \nu = -3\dot{\Phi} \quad (4.103)\]

\[\dot{\nu} + \frac{\dot{a}}{a} = -i k \Psi \quad (4.104)\]

\[\dot{\delta}_b + ik v_b = -3\dot{\Phi} \quad (4.105)\]

\[\dot{v}_b + \frac{\dot{a}}{a} v_b = -i k \Psi + \frac{\dot{\tau}}{R} [v_b + 3i \Theta_1] \quad (4.106)\]

\[\dot{N} + i k \mu N = -\dot{\Phi} - i k \mu \Psi. \quad (4.107)\]
for photons

temperature anisotropy

\[ \Theta = \frac{\delta T}{T} \]

\[ \Theta(\vec{x}) = \int \frac{d^3 k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{x}} \Theta(\vec{k}) \]

monopole

\[ \Theta_0(\vec{x}, t) = \frac{1}{4\pi} \int d\Omega' \Theta(p', \vec{x}, t) \]

dipole

\[ \Theta_1 \sim \int d\mu \Theta(\mu) \]

\( l \)-th multipole moment

\[ \Theta_l \sim \int d\mu P_l \mu \Theta(\mu) \]

polarization anisotropy

\[ \Theta_p \]
for photons

\[ \dot{\Theta} + i k \mu \Theta = -\dot{\Phi} - i k \mu \Psi - \dot{\tau} \left[ \Theta_0 - \Theta + \mu \nu_b - \frac{1}{2} P_2(\mu) \Pi \right] \]

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\[ \dot{\Theta}_P + i k \mu \Theta_P = -\dot{\tau} \left[ -\Theta_P + \frac{1}{2} (1 - P_2(\mu)) \Pi \right] \]

\[ \mu = \hat{k} \cdot \hat{p} \]

Legendre polynomials

optical depth

modern cosmology, S. Dodelson

Thursday, May 17, 18
optical depth

\[ \tau(\eta) \equiv \int_\eta^{\eta_0} d\eta' \ n_e \sigma_T a. \]

Boltzmann (+GR) Equations for (dark) matter

\[ \dot{\delta} + i k \nu = -3 \dot{\Phi} \]
\[ \dot{\nu} + \frac{\dot{a}}{a} \nu = -i k \Psi \]

\[ \delta_i = \frac{\delta \rho_i}{\bar{\rho}_i} \]
Boltzmann (+GR) Equations

for baryons

\[ \dot{\delta}_b + ik\nu_b = -3\dot{\Phi} \]
\[ \dot{\nu}_b + \frac{\dot{a}}{a}\nu_b = -ik\Psi + \frac{\dot{\nu}}{R} [v_b + 3i\Theta_1] \]
Boltzmann (+GR) Equations

for Neutrinos

\[ \dot{N} + ik\mu N = -\dot{\Phi} - ik\mu \Psi. \]
Boltzmann (+GR) Equations

\[
\begin{align*}
\dot{\Theta} + i k \mu \Theta &= -\dot{\Phi} - i k \mu \Psi - \dot{\tau} \left[ \Theta_0 - \Theta + \mu v_b - \frac{1}{2} P_2(\mu) \Pi \right] \quad \text{(4.100)} \\
\Pi &= \Theta_2 + \Theta_{P2} + \Theta_{P0} \quad \text{(4.101)} \\
\dot{\Theta}_P + i k \mu \Theta_P &= -\dot{\tau} \left[ -\Theta_P + \frac{1}{2} (1 - P_2(\mu)) \Pi \right] \quad \text{(4.102)} \\
\dot{\delta} + i k v &= -3 \dot{\Phi} \quad \text{(4.103)} \\
\dot{v} + \frac{\dot{a}}{a} v &= -i k \Psi \quad \text{(4.104)} \\
\dot{\delta}_b + i k v_b &= -3 \dot{\Phi} \quad \text{(4.105)} \\
\dot{v}_b + \frac{\dot{a}}{a} v_b &= -i k \Psi + \frac{\dot{\tau}}{R} [v_b + 3i \Theta_1] \quad \text{(4.106)} \\
\dot{N} + i k \mu N &= -\dot{\Phi} - i k \mu \Psi. \quad \text{(4.107)}
\end{align*}
\]
Boltzmann eqns. tell us how perturbations evolve, etc

but

where do perturbations come from in the first place?

most popular scenario
inflation seeds them.
primordial perturbations

adiabatic
- relative perturbations in various components have the same ratios at different points.
- total matter/energy content different at different points

\[ \delta_b = \delta_{dm} = \frac{3}{4} \delta_\nu = \frac{3}{4} \delta_\gamma \]
\[ \delta_i = \frac{\delta \rho_i}{\bar{\rho}_i} \]

iso-curvature
- same total matter/energy perturbations at different points

\[ b = \frac{3}{4} \]
\[ \phi = \frac{3}{4} \]

primordial perturbations

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primordial power spectrum

1- scalars

\[ P_s = A_s \left( \frac{k}{k_p} \right)^{n_s - 1} \]

2- tensors
aka gravitational waves

\[ P_t = A_t \left( \frac{k}{k_p} \right)^{n_t} \]

amplitudes

tilt
(in terms of slow-roll parameters)

\[ n_t = -2\epsilon \]
\[ n_s = 1 - 4\epsilon - 2\delta \]
Eqns can be solved numerically to get exact results. That is what is done by Boltzmann codes such as CAMB.

But let us get a feeling of eqns and their solutions by quantitative exploration.
perturbations

sub-horizon

\[ \lambda \ll d_H \]

\[ k\eta \gg 1 \]

super-horizon

\[ \lambda \gg d_H \]

\[ k\eta \ll 1 \]

@ hi z (b4 recombination) photons and baryons were tightly coupled photon-baryon fluid

only monopole and dipole contribute

equation for monopole to solve

\[ \left\{ \frac{d^2}{d\eta^2} + \frac{\dot{R}}{1 + R} \frac{d}{d\eta} + k^2 c_s^2 \right\} [\Theta_0 + \Phi] = \frac{k^2}{3} \left[ \frac{1}{1 + R} \Phi - \Psi \right] \]

oscillatory

sourced by gravity
solution

monopole contribution to photon at decoupling

\[ \Theta_0(\eta) + \Phi(\eta) = [\Theta_0(0) + \Phi(0)] \cos(kr_s) \]
\[ + \frac{k}{\sqrt{3}} \int d\eta' [\Phi(\eta') - \Psi(\eta')] \sin[k(r_s(\eta) - r_s(\eta'))] \]

location of acoustic peaks

similar equation for dipole perturbation
\[ \frac{k^{3/2}}{h} \mid \Theta_0 + \Phi(\eta_*) \]

- Undamped
- Analytic
- Exact

\[ \Omega_b h^2 = 0.015 \]

\[ k\eta_0 \]

Thursday, May 17, 18
perturbations to photon at decoupling

\[ \Theta_0(\eta) + \Phi(\eta) = [\Theta_0(0) + \Phi(0)] \cos(k r_s) \]
\[ + \frac{k}{\sqrt{3}} \int d\eta' [\Phi(\eta') - \Psi(\eta')] \sin[k(r_s(\eta) - r_s(\eta'))] \]

+ diffusion damping

photons decouple then from electrons and free stream till now.
free streaming maps inhomogeneities in photons at last scattering surface to anisotropies of the CMB sky today.

\[ \Theta_0(\eta^*) \rightarrow \Theta(\theta, \phi)_{\text{now}} \rightarrow a_{\ell m} \rightarrow C'_\ell \]
free-streaming

\[ \Theta_l(k, \eta_0) = \int_0^{\eta_0} d\eta S(k, \eta) j_l [k(\eta_0 - \eta)] \]

Spherical Bessel functions

monopole @ recomb

\[ \Theta_l(k, \eta_0) \simeq [\Theta_0(k, \eta_*) + \Psi(k, \eta_*)] j_l [k(\eta_0 - \eta_*)] \]

\[ + 3\Theta_1(k, \eta_*) \left( j_{l-1} [k(\eta_0 - \eta_*)] - \frac{(l + 1)j_l [k(\eta_0 - \eta_*)]}{k(\eta_0 - \eta_*)} \right) \]

dipole @ recomb

\[ + \int_0^{\eta_0} d\eta e^{-\tau} \left[ \dot{\Psi}(k, \eta) - \dot{\Phi}(k, \eta) \right] j_l [k(\eta_0 - \eta)] . \]

Integrated Sachs Wolf-effect (ISW)

perturbation with k contributes to angular scales \( l \sim k/\eta_0 \)
\[ a_{lm}(\tilde{x}, \eta) = \int \frac{d^3 k}{(2\pi)^3} e^{i \vec{k} \cdot \tilde{x}} \int d\Omega Y_{lm}^*(\hat{p}) \Theta(\vec{k}, \hat{p}, \eta). \]

\[ \langle a_{lm} \rangle = 0 \quad ; \quad \langle a_{lm} a_{l'm'}^* \rangle = \delta_{l l'} \delta_{m m'} C_l. \]

CMB power spectrum
So far

it was all **primordial** anisotropies, generated before the last scattering and free-streamed till now.

but ... the photons on their way to us, from the last scattering, encounter other phenomena, known as **secondaries**.

These include

- gravitational lensing
- tSZ
- reionization
- ISW
1- gravitational lensing

CMB photons are deflected due to matter in between.

http://sci.esa.int/planck/51606-gravitational-lensing-of-the-cosmic-microwave-background/
Such that the fields are remapped as:

\[ x(\hat{n}) \rightarrow x(\hat{n} + \nabla \phi), \]

\[
\phi(\hat{n}) = -2 \int_0^{\chi_*} d\chi \frac{f_K(\chi_* - \chi)}{f_K(\chi_*) f_K(\chi)} \Psi(\chi \hat{n}; \eta_0 - \chi). 
\]

**Imprint: Remaps primordial anisotropy**

- **Lensing potential**
- **Conformal time**
- **Gravitational potential**

Measures projected/integrated mass distribution back to the last scattering surface.
Seljak (1996) [see Challinor & Lewis (2006) for refinements]

http://background.uchicago.edu/~whu/Presentations/cmblens.pdf
2- thermal Sunyaev-Zeldovic effect

inverse Compton scattering of CMB photons by hot electrons along the line of sight in galaxy clusters

\[
\frac{\Delta T}{T} \propto \sigma_T \int n_e T_{edl} dl
\]
3- reionization

matter (hydrogen) in the universe reionized after the dark ages.

electrons are free again!

CMB photons re-scatter these free electrons!

re-scattering partially washes out the primordial anisotropies.
generates new polarization.
Cosmological parameters (standard model of Cosmology)

<table>
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<tr>
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<tbody>
<tr>
<td>$100\theta_{MC}$</td>
<td>1.04131 ± 0.00063</td>
<td>1.04126 ± 0.00047</td>
<td>1.04121 ± 0.00048</td>
<td>1.04094 ± 0.00048</td>
<td>1.04086 ± 0.00048</td>
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<td>$\Omega_b h^2$</td>
<td>0.02205 ± 0.00028</td>
<td>0.02234 ± 0.00023</td>
<td>0.02230 ± 0.00023</td>
<td>0.02225 ± 0.00023</td>
<td>0.02222 ± 0.00023</td>
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<tr>
<td>$\Omega_c h^2$</td>
<td>0.1199 ± 0.0027</td>
<td>0.1189 ± 0.0022</td>
<td>0.1188 ± 0.0022</td>
<td>0.1194 ± 0.0022</td>
<td>0.1199 ± 0.0022</td>
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<td>$H_0$</td>
<td>67.3 ± 1.2</td>
<td>67.8 ± 1.0</td>
<td>67.8 ± 1.0</td>
<td>67.48 ± 0.98</td>
<td>67.26 ± 0.98</td>
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<td>$n_s$</td>
<td>0.9603 ± 0.0073</td>
<td>0.9665 ± 0.0062</td>
<td>0.9655 ± 0.0062</td>
<td>0.9682 ± 0.0062</td>
<td>0.9652 ± 0.0062</td>
</tr>
<tr>
<td>$\Omega_m$</td>
<td>0.315 ± 0.017</td>
<td>0.308 ± 0.013</td>
<td>0.308 ± 0.013</td>
<td>0.313 ± 0.013</td>
<td>0.316 ± 0.014</td>
</tr>
<tr>
<td>$\sigma_8$</td>
<td>0.829 ± 0.012</td>
<td>0.831 ± 0.011</td>
<td>0.828 ± 0.012</td>
<td>0.829 ± 0.015</td>
<td>0.830 ± 0.015</td>
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<tr>
<td>$\tau$</td>
<td>0.089 ± 0.013</td>
<td>0.096 ± 0.013</td>
<td>0.094 ± 0.013</td>
<td>0.079 ± 0.019</td>
<td>0.078 ± 0.019</td>
</tr>
<tr>
<td>$10^3A_s e^{-2\tau}$</td>
<td>1.836 ± 0.013</td>
<td>1.833 ± 0.011</td>
<td>1.831 ± 0.011</td>
<td>1.875 ± 0.014</td>
<td>1.881 ± 0.014</td>
</tr>
</tbody>
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comparing the predictions of theory with observations
aka
finding the best fit
Polarization Anisotropies

so far: temperature anisotropies
but
CMB photons are also linearly polarized.

Thomson scattering
of photons off free electrons
--> linear polarization

described by Q and U
Q and U maps (coordinate dependent)

E and B modes

their power spectra $C_{\ell}^E$, $C_{\ell}^B$
B modes?

large scales
- direct probe of early universe
- can only be produced by primordial GWs, predicted by inflation.
- those early times: extremely high energy physics inaccessible to terrestrial accelerators.

small scales
- gravitational lensing turns E mode to B mode
CMB anisotropy power spectrum
results in a nutshell

fluctuations compatible with predictions of simplest inflationary models.

(Planck 2015 results XVII)
Anomalies?

- preferred direction?
- violation of isotropy?
- power deficit at large angular scales?
- large-scale dipolar power asymmetry?
- cold spot?
- hemispherical asymmetry?
- lack of large angle correlations?
- multipole alignment?
serious challenge:
look-elsewhere effect / posterior correction

take-home note:
largely consistent with isotropy
with a few possible mild anomalies

Planck 2015 results XVI
extensions to the standard model of Cosmology

model zoo

topological defects, dm annihilation, perturbed recombination history, extended models of inflations, constraints on BBN, DE scenarios, modified gravity, background geometry/topology

..., ...

data agrees extremely well with the standard model.
CMB has been great! precision cosmology would be non-existing without it! but does it have anything more to offer?
Every small step can yield the first detection of inflationary B modes.

Lorenz: Pocket for a major discovery (which could happen tomorrow, or in 20 years, or never!)

CMB is unique. GeÜng the best of it is a scientific imperative.

A comprehensive, sensitive and accurate space mission is needed for precision cosmology.

DILEMMA
THE(B(RACE(
THE(CMB(TASK(

J. Delabrouille, 2017
and more:

a lot from polarization and secondaries

- physics of inflation
  (r, n_s, running, n_t, non-Gaussianity, ...?)
- nature of dark matter?
- dark energy?
- neutrino physics? $N_{\text{eff}}, \Sigma m_\nu$
- cluster physics?

... 

besides, we have this treasure box and
would love to see all there is in it.
there could always be surprises.
What is coming next?