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Answer to Exercise set 3

1. **Part A**: We have conditional probability like below:

$$p(on; t_n|off; t_{n-1}) = A \qquad p(on; t_n|on; t_{n-1}) = C$$

$$p(off; t_n|on; t_{n-1}) = B \qquad p(off; t_n|off; t_{n-1}) = D$$

From Bayse theorem we have constrain on above parameter and finally we have transformation matrix, so:

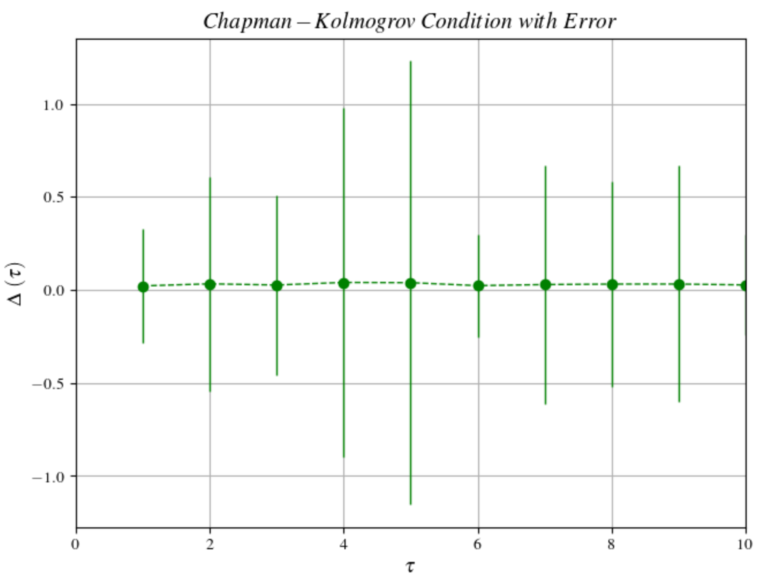
$$T = \begin{bmatrix} 1-B & A \\ B & 1-A \end{bmatrix}$$

Now we diagonalize above matrix. Eigenvalues are 1 and $1-A-B$. Use this website to diagonal matrix with parameters. [1]. After N step diagonal matrix become like this:

$$T^N = \begin{bmatrix} (1-A-B)^N & 0 \\ 0 & 1 \end{bmatrix}$$

So your answer is $(1-A-B)^N$.

Part B: This data is Markovian because Δ became zero after $\tau = 1$, so your plot of $\Delta(\tau)$ should be like this:



[1] http://wims.unice.fr/~wims/en_tool~linear~matrix.en.phtml

2. We should calculate this integral, result is inverse of normalization constant, so we have:

$$\int_{-\infty}^{\infty} d^N X \exp\Big(-\frac{X^\dagger \cdot A \cdot X}{2}\Big)$$

If B is diagonalized matrix of A with transformation $A = QBQ^T$ and $A = QBQ^{-1}$ because $Q^{-1} = Q^T$, with change of variable in form $X' = Q^T X$ we can get:

$$\int_{-\infty}^{\infty} d^N X \exp\Big(-\frac{X^\dagger \cdot A \cdot X}{2}\Big) = \int_{-\infty}^{\infty} d^N X' \Big|\frac{d^N X}{d^N X'}\Big| \exp\Big(-\frac{X'^\dagger \cdot B \cdot X'}{2}\Big)$$

$$\int_{-\infty}^{\infty} d^N X' \Big|\frac{d^N X}{d^N X'}\Big| \exp\Big(-\frac{X'^\dagger \cdot B \cdot X'}{2}\Big) = \int_{-\infty}^{\infty} \prod_{i=1}^{N} dX'_i \Big|\frac{dX_i}{dX'_i}\Big| \exp\Big(-\frac{X'^2_i B_{ii}}{2}\Big)$$

$$= \prod_{i=1}^{N} \int_{-\infty}^{\infty} dX'^2_i \Big|\frac{dX_i}{dX'_i}\Big| \exp\Big(-\frac{X'^2_i B_{ii}}{2}\Big)$$

Finally:

$$\prod_{i=1}^{N} \int_{-\infty}^{\infty} dX'_i \exp\Big(-\frac{X'^2_i B_{ii}}{2}\Big)^3 = \prod_{i=1}^{N} \Big[\frac{2\pi}{B_{ii}}\Big]^{1/2}$$

$$= \Big[(2\pi)^N \prod_{i=1}^{N} \frac{1}{B_{ii}}\Big]^{1/2}$$

$$= \Big[(2\pi)^N \frac{1}{|B|}\Big]^{1/2}$$

$$\int d^N X \Big[\frac{Det(B)}{(2\pi)^N}\Big]^{1/2} \exp\Big(-\frac{X^\dagger \cdot A \cdot X}{2}\Big)$$

Determinant is invariant constant so we can do this:

$$\int d^N X \Big[\frac{Det(A)}{(2\pi)^N}\Big]^{1/2} \exp\Big(-\frac{X^\dagger \cdot A \cdot X}{2}\Big).$$

[2] We have $Det(1/Q^T) = Det(1/Q) = Det(Q^{-1}) = Det(Q^T) = +1$.

[3] This integral is helpful:

$$\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}}$$