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Answer to Exercise set 2

1.**Part A**: We have:

$$\begin{aligned}\bar{p}(E) &= \int dv' \frac{1}{|\frac{dg}{dv}|_{v=v'}} \delta_D(v-v')p(v') \\ &= p(v)\Big|\frac{dg}{dv}\Big|^{-1}_{v=v'} \\ &= p(g^{-1}(E))\Big|\frac{dg}{dv}\Big|^{-1}_{v=g^{-1}(E)}\end{aligned}$$

$$\begin{aligned}\bar{p}(E) &= p(\sqrt{\frac{2E}{m}})\frac{1}{\sqrt{(2mE)}} \\ &\propto \frac{1}{\sqrt{(2mE)}}e^{-E/kT}.\end{aligned}$$

v and -v have same energy so we should multiply above equation in two, so:

$$\bar{p}(E) \propto \frac{2}{\sqrt{(2mE)}}e^{-E/kT}.$$

Part B: For oscillator equation of motion is $x = A\cos(\omega t)$, t is function of x in form $t = (1/\omega)cos^{-1}\big(x/A\big)$. We see this oscillator in T seconds so we can guss that $p(t) = 1/T$. With this equations we can get:

$$|\frac{dg}{dt}|_{t=g^{-1}(x)} = A\omega \sin\Big(cos^{-1}(\frac{y}{A})\Big)$$

so:

$$\begin{aligned}p(x) &= p(t)\Big|\frac{dg}{dt}\Big|^{-1}_{t=g^{-1}(x)} \\ &= \frac{1}{T}\frac{1}{A\omega}\frac{1}{\sin\Big(cos^{-1}(\frac{y}{A})\Big)} \\ &= \frac{1}{T\omega}\frac{1}{\sqrt{A^2-x^2}}\end{aligned}$$

x and -x have same PDF so we should multiply it by two, we have:

$$p(x) = \frac{2}{T\omega}\frac{1}{\sqrt{A^2-x^2}}$$

2. You should get something like below: