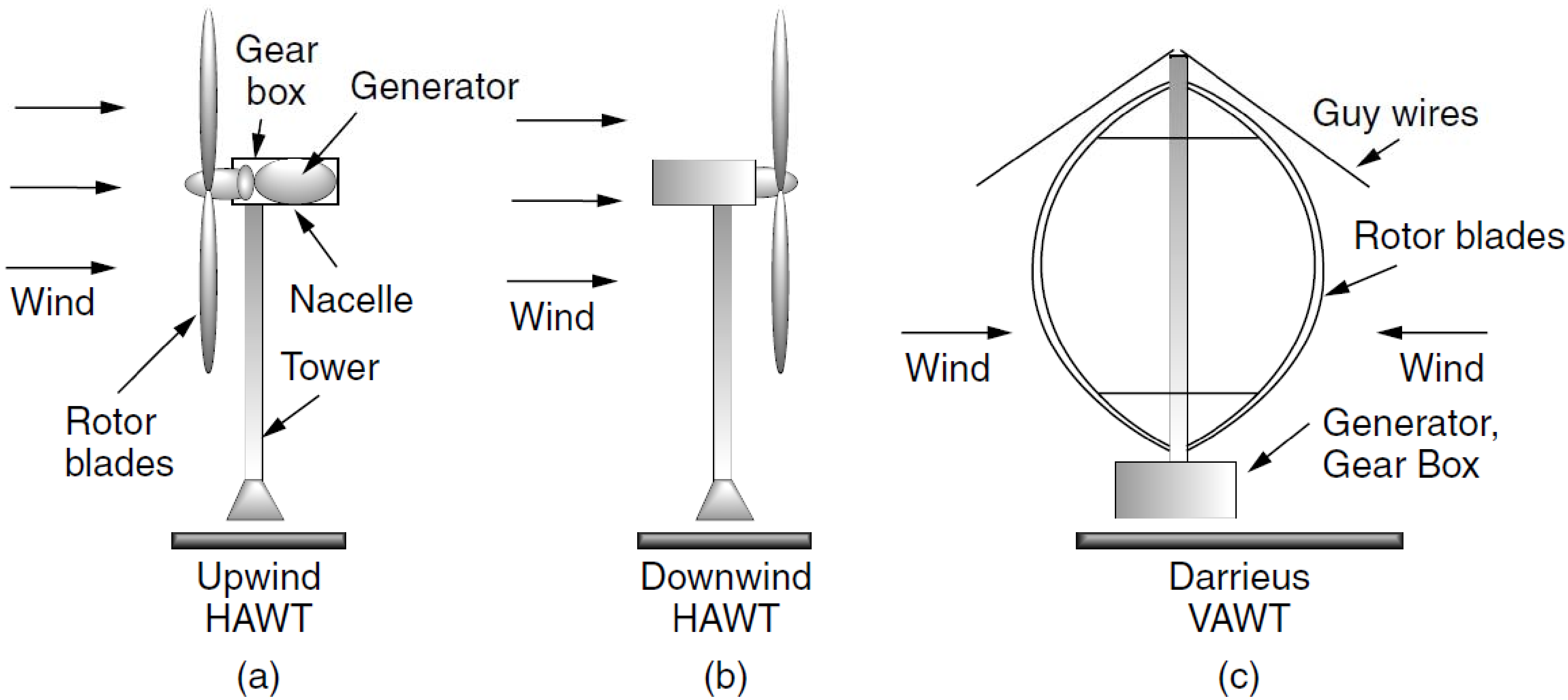
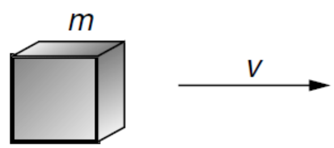
Renewable Energy

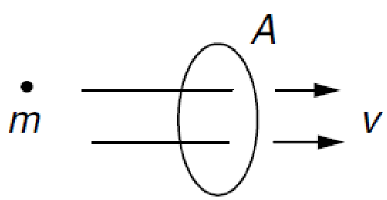
WIND POWER SYSTEMS



POWER IN THE WIND



$$K.E. = \frac{1}{2} m v^2$$



$$\text{Power through area } A = \frac{\text{Energy}}{\text{Time}} = \frac{1}{2}\left(\frac{\text{Mass}}{\text{Time}}\right) v^2$$

$$\left(\frac{\text{Mass passing through A}}{\text{Time}}\right) = \dot{m} = \rho A v$$

$$P_w = \frac{1}{2} \rho A v^3$$

In S.I. units; Pw is the power in the wind (watts); ρ is the air density (kg/m3) (at 15◦C and 1 atm, ρ = 1.225 kg/m3); A is the cross-sectional area through which the wind passes (m2); and v = windspeed normal to A (m/s) (a useful conversion: 1 m/s = 2.237 mph).

Temperature Correction for Air Density

ideal gas law $PV = nRT$

where *P is the absolute pressure (atm), V is the volume (m3), n is the mass* (mol), *R is the ideal gas constant = 8.2056 × 10−5 m3 · atm · K−1 · mol−1, and T is the absolute temperature (K), where K = ◦C + 273.15.*

$$\rho(\text{kg/m}^3) = \frac{\text{n(mol)} \cdot \text{M.W.(g/mol)} \cdot 10^{-3}\text{(kg/g)}}{V(\text{m}^3)}$$

$$\rho = \frac{P \times \text{M.W.} \times 10^{-3}}{RT}$$

Find the density of air at 1 atm and 30◦C (86◦F)

$$\rho = \frac{1\ \text{atm} \times 28.97\ \text{g/mol} \times 10^{-3}\ \text{kg/g}}{8.2056 \times 10^{-5}\text{m}^3 \cdot \text{atm/(K} \cdot \text{mol)} \times (273.15 + 30)\ \text{K}} = 1.165\ \text{kg/m}^3$$

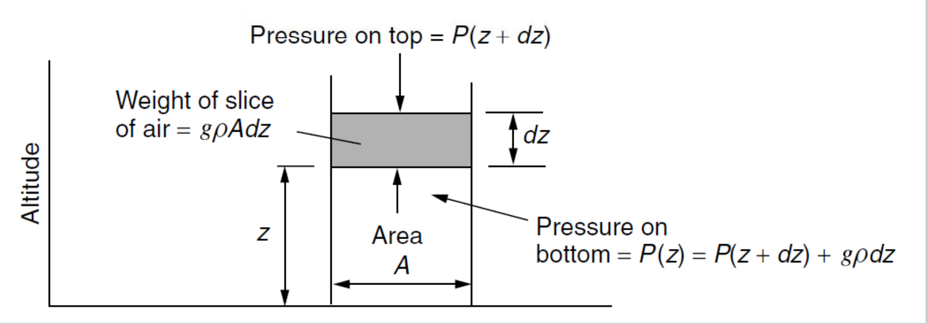
M.W. stand for the molecular weight of the gas (g/mol), Air=28.97 g/mol

Density of Dry Air at a Pressure of 1 Atmosphere

Temperature (°C)	Temperature (°F)	Density (kg/m^3)	Density Ratio (K_T)
−15	5.0	1.368	1.12
−10	14.0	1.342	1.10
−5	23.0	1.317	1.07
0	32.0	1.293	1.05
5	41.0	1.269	1.04
10	50.0	1.247	1.02
15	**59.0**	**1.225**	1.00
20	68.0	1.204	0.98
25	77.0	1.184	0.97
30	86.0	1.165	0.95
35	95.0	1.146	0.94
40	104.0	1.127	0.92

[a] The density ratio K_T is the ratio of density at T to the density at the standard (boldfaced) 15°C.

Altitude Correction for Air Density



$$dP = P(z + dz) - P(z) = -g\ \rho\, dz$$

$$\frac{dP}{dz} = -\rho g$$

$$\rho = \frac{P \times \text{M.W.} \times 10^{-3}}{RT}$$

$$\frac{dP}{dz} = -\left(\frac{g\ \text{M.W.} \times 10^{-3}}{R \cdot T}\right) \cdot P$$

$$\frac{dP}{dz} = -\left[\frac{9.806(\text{m/s}^2) \times 28.97(\text{g/mol}) \times 10^{-3}(\text{kg/g})}{8.2056 \times 10^{-5}(\text{m}^3 \cdot \text{atm} \cdot \text{K}^{-1} \cdot \text{mol}^{-1}) \times 288.15\ \text{K}}\right] \times \left(\frac{\text{atm}}{101,325\ \text{Pa}}\right) \cdot \left(\frac{1\ \text{Pa}}{\text{N/m}^2}\right)\left(\frac{1\ \text{N}}{\text{kg} \cdot \text{m/s}^2}\right) \cdot P$$

$$\frac{dP}{dz} = -1.185 \times 10^{-4} P$$

$$P = P_0 e^{-1.185 \times 10^{-4}\text{H}} = 1(\text{atm}) \cdot e^{-1.185 \times 10^{-4}\text{H}}$$

- Find the air density (a), at 15◦C (288.15 K), at an elevation of 2000 m (6562 ft). Then (b) find it assuming an air temperature of 5◦C at 2000 m.

$$P = 1\ \text{atm} \times e^{-1.185\times10^{-4}\times2000} = 0.789\ \text{atm}$$

$$\begin{aligned}\rho &= \frac{P \cdot \text{M.W.} \cdot 10^{-3}}{R \cdot T}\\ &= \frac{0.789(\text{atm}) \times 28.97(\text{g/mol}) \times 10^{-3}(\text{kg/g})}{8.2056 \times 10^{-5}(\text{m}^3 \cdot \text{atm} \cdot \text{K}^{-1} \cdot \text{mol}^{-1}) \times 288.15\ \text{K}}\\ &= 0.967\ \text{kg/m}^3\end{aligned}$$

At 5°C and 2000 m, the air density would be

$$\begin{aligned}\rho &= \frac{0.789(\text{atm}) \times 28.97(\text{g/mol}) \times 10^{-3}(\text{kg/g})}{8.2056 \times 10^{-5}(\text{m}^3 \cdot \text{atm} \cdot \text{K}^{-1} \cdot \text{mol}^{-1}) \times (273.15 + 5)\ \text{K}}\\ &= 1.00\ \text{kg/m}^3\end{aligned}$$

$$\rho = 1.225 K_T K_A$$

Altitude (meters)	Altitude (feet)	Pressure (atm)	Pressure Ratio (K_A)
0	0	1	1
200	656	0.977	0.977
400	1312	0.954	0.954
600	1968	0.931	0.931
800	2625	0.910	0.910
1000	3281	0.888	0.888
1200	3937	0.868	0.868
1400	4593	0.847	0.847
1600	5249	0.827	0.827
1800	5905	0.808	0.808
2000	6562	0.789	0.789
2200	7218	0.771	0.771

Temperature (°C)	Temperature (°F)	Density (kg/m^3)	Density Ratio (K_T)
−15	5.0	1.368	1.12
−10	14.0	1.342	1.10
−5	23.0	1.317	1.07
0	32.0	1.293	1.05
5	41.0	1.269	1.04
10	50.0	1.247	1.02
15	**59.0**	**1.225**	1.00
20	68.0	1.204	0.98
25	77.0	1.184	0.97
30	86.0	1.165	0.95
35	95.0	1.146	0.94
40	104.0	1.127	0.92

IMPACT OF TOWER HEIGHT

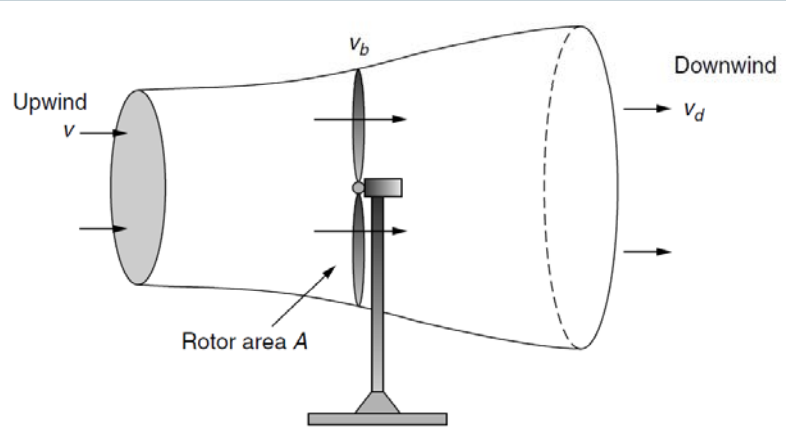
$$\left(\frac{v}{v_0}\right) = \left(\frac{H}{H_0}\right)^{\alpha}$$

Terrain Characteristics	Friction Coefficient α
Smooth hard ground, calm water	0.10
Tall grass on level ground	0.15
High crops, hedges and shrubs	0.20
Wooded countryside, many trees	0.25
Small town with trees and shrubs	0.30
Large city with tall buildings	0.40

$$\left(\frac{v}{v_0}\right) = \frac{\ln(H/z)}{\ln(H_0/z)}$$

Roughness Class	Description	Roughness Length $z(m)$
0	Water surface	0.0002
1	Open areas with a few windbreaks	0.03
2	Farm land with some windbreaks more than 1 km apart	0.1
3	Urban districts and farm land with many windbreaks	0.4
4	Dense urban or forest	1.6

MAXIMUM ROTOR EFFICIENCY



$$P_b = \frac{1}{2}\dot{m}(v^2 - v_d^2)$$

$$\dot{m} = \rho A v_b$$

$$P_b = \frac{1}{2}\rho A \left(\frac{v + v_d}{2}\right)(v^2 - v_d^2)$$

$$\lambda = \left(\frac{v_d}{v}\right)$$

$$P_b = \frac{1}{2}\rho A \left(\frac{v + \lambda v}{2}\right)(v^2 - \lambda^2 v^2) = \underbrace{\frac{1}{2}\rho A v^3}_{\text{Power in the wind}} \cdot \underbrace{\left[\frac{1}{2}(1 + \lambda)(1 - \lambda^2)\right]}_{\text{Fraction extracted}}$$

$$P_b = \frac{1}{2}\rho A v^3 \cdot C_p$$

$$\text{Rotor efficiency} = C_P = \frac{1}{2}(1 + \lambda)(1 - \lambda^2)$$

$$\frac{dC_p}{d\lambda} = \frac{1}{2}[(1 + \lambda)(-2\lambda) + (1 - \lambda^2)] = 0$$
$$= \frac{1}{2}[(1 + \lambda)(-2\lambda) + (1 + \lambda)(1 - \lambda)] = \frac{1}{2}(1 + \lambda)(1 - 3\lambda) = 0$$

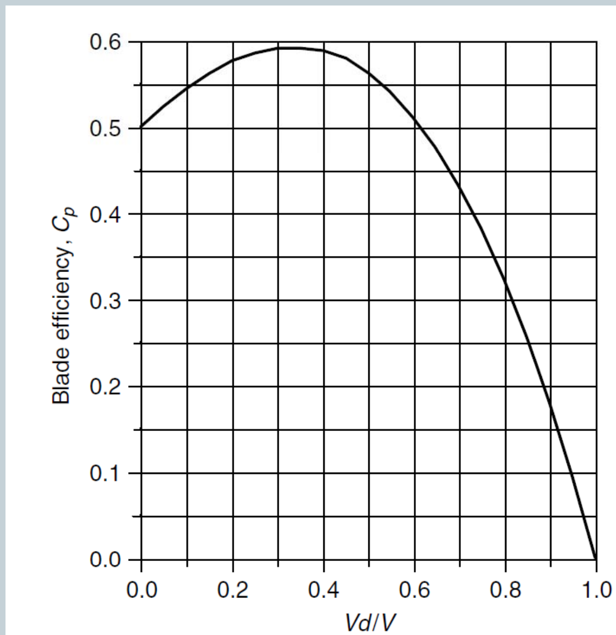
$$\lambda = \frac{v_d}{v} = \frac{1}{3}$$

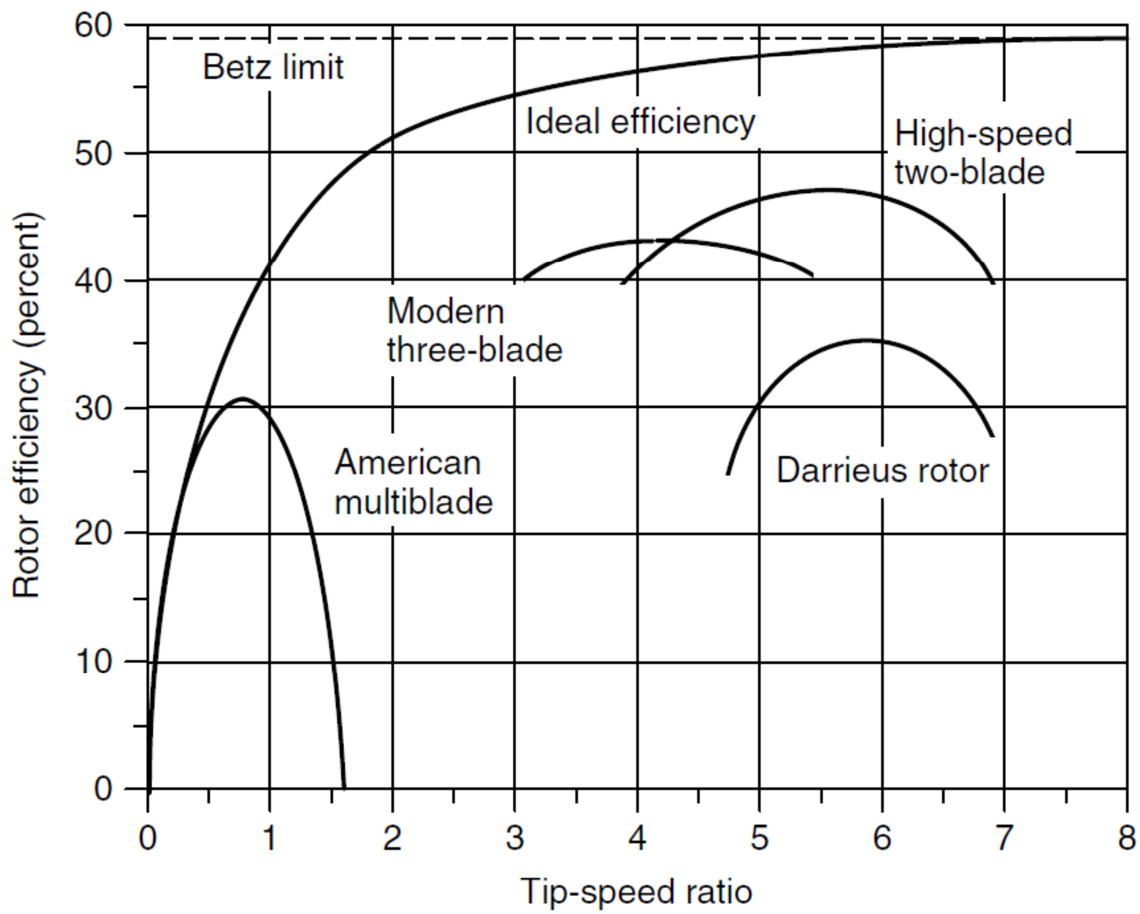
$$\text{Maximum rotor efficiency} = \frac{1}{2}\left(1 + \frac{1}{3}\right)\left(1 - \frac{1}{3^2}\right) = \frac{16}{27} = 0.593 = 59.3\%$$

tip-speed ratio

$$\text{Tip-Speed-Ratio (TSR)} = \frac{\text{Rotor tip speed}}{\text{Wind speed}} = \frac{\text{rpm} \times \pi D}{60\ v}$$

where rpm is the rotor speed, revolutions per minute; D is the rotor diameter (m); and v is the wind speed (m/s) upwind of the turbine.




60
50
40
30
20
10
0
Betz limit
Ideal efficiency
High-speed
two-blade
Modern
three-blade
American
multiblade
Darrieus rotor
Rotor efficiency (percent)
0
1
2
3
4
5
6
7
8
Tip-speed ratio