Mehrabadi's Conjecture

I have done several joint research with my friend, M.L. Mehrabadi(1969-1998). Unfortunately he passed away in a bus accident. During our joint research about chromatic sum he made an interesting conjecture about strength of a graph [1] and it is open by now. My friend, R. Tusserkani and me decided to call this conjecture, Mehrabadi's conjecture to commemorate him. We will pay $200 USD as prize to everybody prove or disprove this conjecture.

Let G be a graph, we denote by $\sum_k(G)$ the smallest possible sum among all proper k- colorings of G using natural numbers. The *vertex-chromatic sum* of G, denoted by $\sum(G)$, is defined as $\min_{k\geq\chi(G)}\sum_k(G)$. The *vertex-strength* of G denoted by $s(G)$, or briefly by s, is the smallest number s such that $\sum_s(G)=\sum(G)$. Clearly, $s(G)\geq\chi(G)$ and equality does not always hold. In fact, for every positive integer k, almost all trees satisfy $s\geq k$(for more about this concept see [1] and [2]).

Mehrabadi's Conjecture. *For every graph G, we have:*

$$s(G)\leq\lceil\frac{\chi(G)+\Delta(G)}{2}\rceil.$$

After Mehrabadi made his conjecture we have proved the following upper bound for strength of graphs.

Theorem.[1] *For every graph G, we have:*

$$s(G)\leq\lceil\frac{col(G)+\Delta(G)}{2}\rceil,$$

where $col(G)$ is the coloring number of graph G.

References

[1] H. Hajiabolhassan, M.L. Mehrabadi, and R. Tusserkani, Minimal colorings and strength of graphs, Discrete Mathematics, **215**(2000), 265–270.

[2] H. Hajiabolhassan, M.L. Mehrabadi, and R. Tusserkani, Tabular graphs and chromatic sum, , manuscript(1998).